Module 8.3 : True error and Model complexity

Using Stein's Lemma (and some trickery) we can show that

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- Hence, we can say that
  true error = empirical train error + small constant + Ω(model complexity)
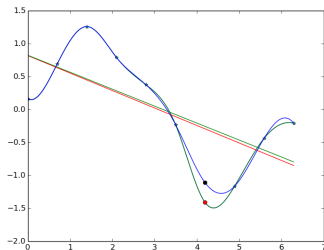
- Let us verify that indeed a complex model is more sensitive to minor changes in the data
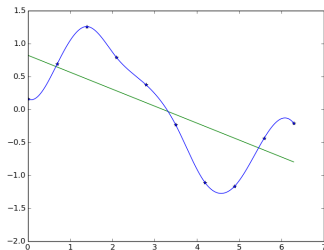
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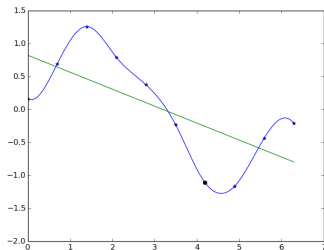
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- The simple model does not change much as compared to the complex model

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- We can show that l_1 regularization, l_2 regularization, early stopping and injecting noise in input are all instances of this form of regularization.
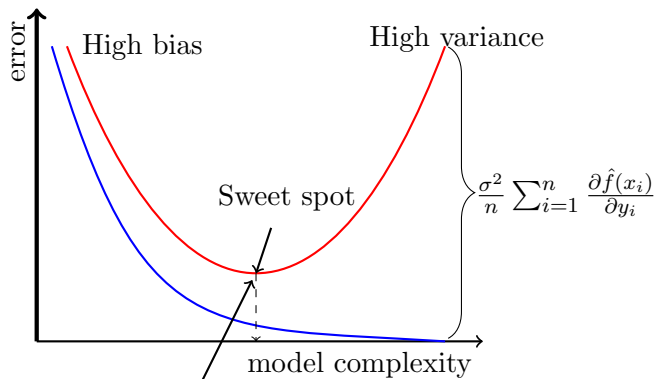
$\Omega(\theta)$ should ensure
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- Hence we need some form of regularization.

Different forms of regularization

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- Dataset augmentation

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- Dropout