CS6015 : Quiz 1 - Solutions

1.

a) True. $A(I+A) = I$

b) True. $(I-A)(I+A+A^2) = I$

c) True. Suppose $\alpha_1(u+v) + \alpha_2(v+w) + \alpha_3(w+u) = 0$

Then, $\alpha_1 + \alpha_3 = 0$ — (*)

$\alpha_1 + \alpha_2 = 0$ — (**)

$\alpha_2 + \alpha_3 = 0$ — (***)

$\Rightarrow$ $\alpha_1 = \alpha_2$ (Subtract (***) from (*))

From (**), $\alpha_1 = \alpha_2 = 0$ & $\alpha_3 = 0$.

d). ~~True~~ False

$$\begin{bmatrix} 0 & -1 \\ -1 & 4 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -11 \\ 3 \end{bmatrix}.$$

Do Gauss-elimination on $\left[\begin{array}{cc|c} 0 & -1 & 2 \\ -1 & 4 & -4 \\ 5 & 9 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -4 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & \boxed{6} \end{array}\right]$.

~~to obtain $x = 3, y = 2$~~

to obtain an "inconsistent" system.

e) False. $A^{-1}BA = B$ holds whenever $AB = BA$.

f) False.

2)

$$A = \begin{bmatrix} 1 & -5 & 2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = E_{12} A$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = E_{32} E_{12} A$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_{13} E_{32} E_{12} A$$

So, $E_{13} E_{32} E_{12} A = I$

$$A = E_{12}^{-1} E_{32}^{-1} E_{13}^{-1}$$

$$= \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & -5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$