CS6015: Linear Algebra and Random Processes
Quiz - 4
Course Instructor : Prashanth L.A.
Date : Sep-18, 2017 **Duration** : 30 minutes

Name of the student :
Roll No :

INSTRUCTIONS: For true/false questions, you do not have to justify the answer. For the rest, provide proper justification for the answers. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. DO NOT use pencil for writing the answers.

Notation: For a square matrix A, $|A|$ denotes its determinant.

1. True or False? Answer any five. $(2+2+2+2+2$ marks)

   Note: 2 marks for the correct answer and -1 for the wrong answer.

   (a) If 0 is the eigenvalue of a matrix A, then A cannot be invertible.

   (b) For any square matrix, $|-A| = -|A|$.

   (c) If $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $S : \mathbb{R}^8 \to \mathbb{R}^3$ are linear transformations, then ST is a linear transformation from $\mathbb{R}^8$ to $\mathbb{R}^2$.

   (d) Let A, B, C, D be square matrices. Then, $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} A & C \\ B & D \end{vmatrix}$.

   (e) For any $a, b, c, d, e, f, g, h, i \in \mathbb{R}$, $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, $B = \begin{bmatrix} c & a & b \\ f & d & e \\ i & g & h \end{bmatrix}$ are similar.

   (f) For a matrix A, let λ_1 and λ_2 be two different eigenvalues with corresponding eigenvectors x_1 and x_2. Then, $x_1 + x_2$ is an eigenvector for A.

   (g) Let A, B be $n \times n$ matrices. Then, $|AB| = |BA|$.

2. Let $A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y - z \\ -y \\ x + 7z \end{bmatrix}$. (3+4+3 marks)

   (a) What is the matrix of A w.r.t. the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$?

   (b) What is the matrix of A w.r.t. the basis $\mathcal{B}' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$?

   (c) Relate the matrices, say $[A]_{\mathcal{B}}, [A]_{\mathcal{B}'}$, obtained in the answers to the parts (a) and (b) above using an invertible S such that

$$[A]_{\mathcal{B}'} = S^{-1}[A]_{\mathcal{B}}S.$$