CS6015: Linear Algebra and Random Processes
Quiz - 6
Course Instructor : Prashanth L.A.
Date : Oct-17, 2017   **Duration** : 30 minutes

Name of the student :
Roll No :

INSTRUCTIONS: For true/false questions, you do not have to justify the answer. For the rest, provide proper justification for the answers. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. DO NOT use pencil for writing the answers.

1. True or False? Answer any five. $(2 + 2 + 2 + 2 + 2$ marks)

   Note: 2 marks for the correct answer and -1 for the wrong answer.

   (a) In $B \subset A$ and $\mathbb{P}(B) \neq 0$, then $\mathbb{P}(A \mid B) = 1$.

   (b) If $\mathbb{P}(A) = \mathbb{P}(A \mid B) = \mathbb{P}(A \mid C)$, then $\mathbb{P}(A) = \mathbb{P}(A \mid B \cap C)$.

   (c) Let X be a random variable with distribution F and $a < b$. Then,

   $$\mathbb{P}(X \in (a, b)) = F(b) - F(a).$$

   (d) If F is a distribution function, then so is G, where $G(x) = 1 - F(x)$, $\forall x \in \mathbb{R}$.

   (e) If A, B, C are independent events and $\mathbb{P}(C) > 0$, then A and B are conditionally independent given C.

   (f) If X, Y are random variables on $(\Omega, \mathcal{F}, \mathbb{P})$, then so is $\min\{X, Y\}$.

2. Suppose there are $N + 1$ urns, each containing a total of N red and white balls. The urn number k contains k red and $N - k$ white balls $(k = 0, 1, \ldots, N)$. An urn is chosen at random and n random drawings are made from it, the ball being replaced each time.

   Answer the following: (4+6 marks)

   (a) Let A be the event that all n balls turn out to be red. What is $\mathbb{P}(A)$?

   (b) Given event A, what is the (conditional) probability that the next drawing will also yield a red ball?