CS6015 Linear Algebra and Random Process
Tutorial 1

1. Under what condition on y_1, y_2, y_3 do the points $(0, y_1), (1, y2), (2, y3)$ lie on a straight line?
2. The first of these equations plus the second equals the third:

$$x+y+z=2$$
$$x+2y+z=3$$
$$2x+3y+2z=5.$$

   The first two planes meet along a line. The third plane contains that line, because if x, y, z satisfy the first two equations then they also . The equations have infinitely many solutions (the whole line L). Find three solutions.
3. Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent:

$$v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad v_2 = \begin{bmatrix}1\\1\\0\end{bmatrix} \quad v_3 = \begin{bmatrix}1\\1\\1\end{bmatrix} \quad v_4 = \begin{bmatrix}2\\3\\4\end{bmatrix}.$$

   Solve $c_1v_1 + \ldots\ldots + c_4v_4 = 0$ or $Ac = 0$. The v's go in the columns of A.
4. If w_1, w_2, w_3 are independent vectors, show that the differences $v_1 = w_2 - w_3$, $v_2 = w_1 - w_3$, and $v_3 = w_1 - w_2$ are dependent. Find a combination of the v's that gives zero.
5. Let x be the column vector (1,0,...,0). Show that the rule (AB)x = A(Bx) forces the first column of AB to equal A times the first column of B.
6. If B is a square matrix, show that $A = B + B^T$ is always symmetric and $K = B - B^T$ is always skew-symmetric-which means that $K^T = -K$. Find these matrices A and K when

$$B = \begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}$$

   Write B as the sum of a symmetric matrix and a skew-symmetric matrix.
7. Invert these matrices A by the Gauss-Jordan method starting with [A I]:

$$A = \begin{bmatrix}1 & 0 & 0\\2 & 1 & 3\\0 & 0 & 1\end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix}1 & 1 & 1\\1 & 2 & 2\\1 & 2 & 3\end{bmatrix}.$$

8. The first row of AB is a linear combination of all the rows of B. What are the coefficients in this combination, and what is the first row of AB, if

$$A = \begin{bmatrix}2 & 1 & 4\\0 & -1 & 1\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}1 & 1\\0 & 1\\1 & 0\end{bmatrix}?$$

9. Choose a right-hand side which gives no solution and another right-hand side which gives infinitely many solutions. What are two of those solutions?

$$3x + 2y = 10$$
$$6x + 4y = \_\_\_\_\_$$

10. Apply elimination and back-substitution to solve

$$2u + 3v = 0$$
$$4u + 5v + w = 3$$
$$2u - v - 3w = 5.$$

What are the pivots? List the three operations in which a multiple of one row is subtracted from another.