Data Flow Analysis

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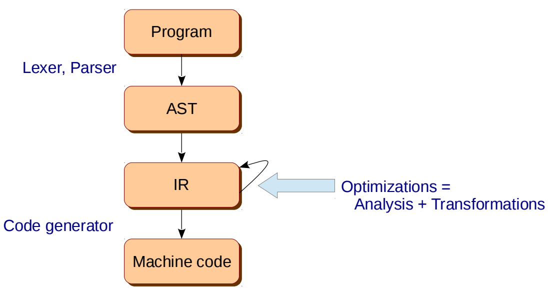
Outline

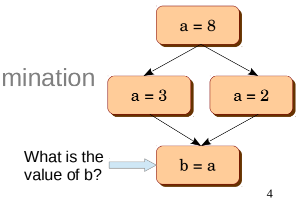
- What is DFA?
  - Reaching definitions
  - Live variables
- DFA framework
  - Monotonicity
  - Confluence operator
  - MFP/MOP solution
- Analysis dimensions

Compiler Organization



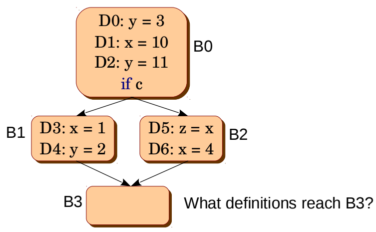
Data Flow Analysis

- Flow-sensitive: Considers the control-flow in a function
- Operates on a flow-graph with nodes as basic-blocks and edges as the control-flow
- Examples
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination



Reaching Definitions

- Every assignment is a definition
- A definition d reaches a program point p if there exists a path from the point immediately following d to p such that d is not killed along the path.

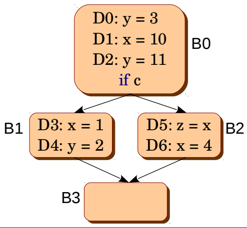


DFA Equations

- in(B) = set of data flow facts entering block B
- out(B) = …
- gen(B) = set of data flow facts generated in B
- kill(B) = set of data flow facts from the other blocks killed in B

DFA for Reaching Definitions

- in(B) = U out(P) where P is a predecessor of B
- out(B) = gen(B) U (in(B) – kill(B))
- Initially, out(B) = { }



gen(B0) = {D1, D2} kill(B0) = {D3, D4, D6}
gen(B1) = {D3, D4} kill(B1) = {D0, D1, D2, D6}
gen(B2) = {D5, D6} kill(B2) = {D1, D3}
gen(B3) = { } kill(B3) = { }

	in1	out1	in2	out2	in3	out3
B0	{}	{D1, D2}	{}	{D1, D2}	{}	{D1, D2}
B1	{}	{D3, D4}	{D1, D2}	{D3, D4}	{D1, D2}	{D3, D4}
B2	{}	{D5, D6}	{D1, D2}	{D2, D5, D6}	{D1, D2}	{D2, D5, D6}
B3	{}	{}	{D3, D4, D5, D6}	{D3, D4, D5, D6}	{D2, D3, D4, D5, D6}	{D2, D3, D4, D5, D6}

DFA for Reaching Definitions

Domain	Sets of definitions
Transfer function	in(B) = U out(P) out(B) = gen(B) U (in(B) - kill(B))
Direction	Forward
Meet / confluence operator	U
Initialization	out(B) = { }

Algorithm for Reaching Definitions

for each basic block B

  compute gen(B) and kill(B)

  out(B) = {}

Can you do better?
Hint: Worklist

do {

  for each basic block B

    in(B) = U out(P) where P \in pred(B)

    out(B) = gen(B) U (in(B) - kill(B))

} **while** in(B) changes for any basic block B

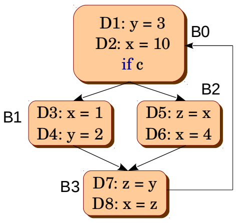
DFA for Live Variables

Domain	Sets of variables
Transfer function	in(B) = use(B) U (out(B) - def(B)) out(B) = U in(S) where S is a successor of B
Direction	Backward
Meet / confluence operator	U
Initialization	in(B) = { }

A variable v is live at a program point p if v is used along some path in the flow graph starting at p.
Otherwise, the variable v is dead.

Classwork

- in(B) = U out(P) where P is a predecessor of B
- out(B) = gen(B) U (in(B) – kill(B))
- Initially, out(B) = { }



gen(B0) = {D1, D2} kill(B0) = {D3, D4, D6, D8}
gen(B1) = {D3, D4} kill(B1) = {D1, D2, D6, D8}
gen(B2) = {D5, D6} kill(B2) = {D2, D3, D7, D8}
gen(B3) = {D7, D8} kill(B3) = {D2, D3, D5, D6}

	in1	out1	in2	out2	in3	out3	in4	out4
B0	{}	{D1, D2}	{D7, D8}	{D1, D2, D7}	{D4, D7, D8}	{D1, D2, D7}	{D1,4,7}	{D1,2,7}
B1	{}	{D3, D4}	{D1, D2}	{D3, D4}	{D1, D2, D7}	{D3, D4, D7}	{D1,2,7}	{D3,4,7}
B2	{}	{D5, D6}	{D1, D2}	{D1, D5, D6}	{D1, D2, D7}	{D1, D5, D6}	{D1,2,7}	{D1,5,6}
B3	{}	{D7, D8}	{D3, D4, D5, D6}	{D4, D7, D8}	{D1, D3, D4, D5, D6}	{D1, D4, D7, D8}	{D1,3,4,5,6,7}	{D1,4,7,8}

Classwork

- Write an algorithm for Live Variable Analysis

for each basic block B
  compute gen(B) and kill(B)
  out(B) = {}
do {
  for each basic block B
    in(B) = U out(P) where P \in pred(B)
    out(B) = gen(B) U (in(B) - kill(B))
} **while** in(B) changes for any basic block B

Algo for **reaching definitions**

Domain	Sets of variables
Transfer function	in(B) = use(B) U (out(B) - def(B)) out(B) = U in(S) where S is a successor of B
Direction	Backward
Meet / confluence operator	U
Initialization	in(B) = { }

Parameters for **live variable analysis**

Direction and Confluence

	Forward	Backward
∪	Reaching Definitions	Live Variables
∩	Common Subexpressions	Very Busy Expressions

Data Flow Framework

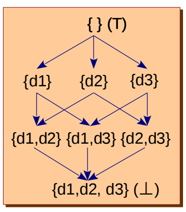
- Point: start or end of a basic block
- Information flow direction: forward / backward
- Transfer functions
- Meet / confluence operator

- One can define a transfer function over a path in the CFG $f_k(f_{k-1}(\ldots f_2(f_1(f_0(T))\ldots))$
- $MOP(x) = \sqcap f_Q(T)$

Meet over all paths
Path enumeration is expensive

Structure in Data Flow Framework

- A semilattice $\mathcal{L}$ with a binary meet operator $\sqcap$, such that a, b, c $\in \mathcal{L}$
  - Idempotency: $a \sqcap a = a$
  - Commutativity: $a \sqcap b = b \sqcap a$
  - Associativity: $a \sqcap (b \sqcap c) = (a \sqcap b) \sqcap c$
- $\sqcap$ imposes an order on $\mathcal{L}$
  - a >= b ⇔ a ⊓ b = b
- $\mathcal{L}$ has a bottom element ⊥, a ⊓ ⊥ = ⊥
- $\mathcal{L}$ has a top element T, a ⊓ T = a



Reaching Definitions Lattice

Monotone Framework

- A framework <$\mathcal{L}$, ⊓, $\mathcal{F}$> is monotone if $\mathcal{F}$ is monotonic, i.e.,

  $$(\forall f \in F)(\forall x, y \in \mathcal{L}),\ x \geq y \Rightarrow f(x) \geq f(y)$$

- If a data-flow framework is monotonic, the convergence (termination) is guaranteed for finite height lattices.

Distributive Framework

- A framework <$\mathcal{L}$, ⊓, $\mathcal{F}$> is distributive if $\mathcal{F}$ is distributive, i.e.,

  $$(\forall f \in F)(\forall x, y \in \mathcal{L})\ f(x \sqcap y) \leq f(x) \sqcap f(y)$$

- Maximal fixed point (MFP) solution is obtained with our iterative DFA.
- MFP is unique and order independent.
- The best we can do is MOP (most feasible, but undecidable).
- In general, MFP ≤ MOP ≤ Perfect solution.
- If distributive, MFP = MOP.
- Every distributive function is also monotonic.

Outline

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  - Reaching definitions
  - Live variables
- DFA framework
  - Monotonicity
  - Confluence operator
  - MFP/MOP solution
- Analysis dimensions

Analysis Dimensions

An analysis's precision and efficiency is guided by various design decisions.

- Flow-sensitivity
- Context-sensitivity
- Path-sensitivity
- Field-sensitivity

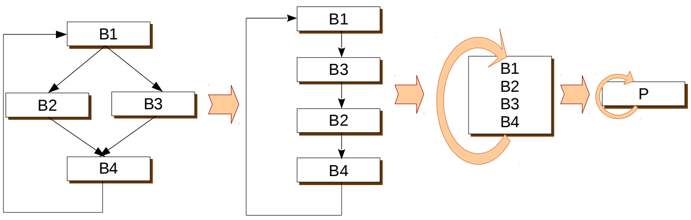
How many hands are required to know the time precisely?

Flow-sensitivity

```
L0:  a = 0;
L1:  a = 1;
L2:  ...
```

Flow-sensitive solution: *at L1 a is 0, at L2 a is 1*
Flow-insensitive solution: *in the program a is in {0, 1}*

Flow-insensitive analyses ignore the control-flow in the program.

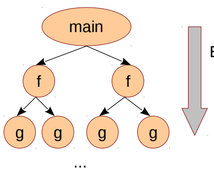


Context-sensitivity

```
main() {                fun(int x) {
    L0:  fun(0);            y = x;
    L1:  fun(1);        }
}
```

Context-sensitive solution:
y is 0 along L0, y is 1 along L1

Context-insensitive solution:
y is in {0, 1} in the program



Exponential Number of contexts

Along main-f1-g1, …
Along main-f1-g2, …
Along main-f2-g1, …
Along main-f2-g2, ...

Exponential time requirement

Exponential storage requirement

Context-sensitivity

```
main() {                fun(int x) {
    L0:  fun(0);            y = x;
    L1:  fun(1);        }
}
```

Context-sensitive solution:
y is 0 along L0, y is 1 along L1

Context-insensitive solution:
Inter-procedural → *y is in {0, 1} in the program*
intra-procedural → *y is in {-∞, +∞} in the program*

Path-sensitivity

```
if (a == 0)
    b = 1;
else
    b = 2;
```

Path-sensitive solution:
b is 1 when a is 0, b is 2 when a is not 0

Path-insensitive solution:
b is in {1, 2} in the program

```
if (c1)
  while (c2) {
    if (c3)
      ...
    else
      for (; c4; )
        ...
  }
else
  ...
```

c1 and c2 and c3, …
c1 and c2 and !c3 and c4, …
c1 and c2 and !c3 and !c4, …
c1 and !c2, …
!c1 ...
...

Field-sensitivity

```
struct T s;

s.a = 0;
s.b = 1;
```

Field-sensitive solution:
s.a is 0, s.b is 1

Field-insensitive solution:
s is in {0, 1}

Aggregates are collapsed into a single variable.
e.g., arrays, structures, unions.

This reduces the number of variables tracked during the analysis and reduces precision.

A Note on Abstraction

Maintain one bit for x == 0
Initialized to F (false)

```
?
x = 0;
T
++x;
F
--x;
?
```


A Note on Choosing Abstraction

Maintain one bit for x == 0
Initialized to F (false)

```
?
x = 0;
T
++x;
F
--x;
?
```

Maintain two bits for value of x
Initialized to 00

```
??
x = 0;
00
++x;
01
--x;
00
```

Maintain one bit for x == 0
Another bit for x < 2
Initialized to 00

```
??
x = 0;
11
++x;
01
--x;
11
```

If type information available, then {01} --x {11} possible.
Otherwise, {01} --x {00}

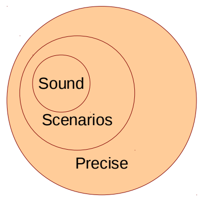
Abstraction Storage

- Saturating counters
- Number of values stored faithfully with log(n) bits – (n-2)
- Additional information may help increase the range, e.g., type information as unsigned.

Conservative Analysis

- Being safe versus being precise
  - Relation with lattice
  - Initialiations and confluence
  - Constructive versus destructive operators
- Safety versus liveness property
  - Absence of bugs versus presence of a bug

Soundness and Precision



- Analyses enable optimizations.
- An optimization is sound if it maintains the functionality of the original code.
- A program may be optimized in certain scenarios.
- An analysis is sound if it leads to sound optimization.
  - The analysis does not enable optimization outside the above set of scenarios.
- An analysis is precise if it does not disable optimization for any possible scenario.

On Soundness

- Usually, multiple optimizations expect same information-theoretic behavior from analyses.
  - If more information means analysis A1 is less precise according to optimization O1, often optimization O2 also sees A1 that way.
  - This allows us to argue about analysis soundness without talking about optimizations.
- But this is not always true.
  - Soundness depends upon optimization enabling.
  - And two opposite optimizations may see the information from the same analysis in opposing ways.

Optimization-specific Soundness

- Consider O1 that changes *p to x if p points to only x.
- Consider O2 that makes p volatile if p points to multiple variables at different program points.
- Analysis A computes points-to information p → {x, y}
  - If A computes more information p → {x, y, z}, O1 is suppressed but O2 is enabled.
  - If A computes less information p → {x}, O1 is enabled and O2 is suppressed.
  - Thus, conservative for one is precise for another.
  - And sound for one is unsound for another.

Optimization-specific Soundness

- Consider O1 that converts multiplication by 2 to a left-bit-shift operation (x * 2 to x << 1).
- Consider O2 that has uses a special circuit (fast operation) when there is a sum of reciprocals of powers of 2 (1 + ½ + ¼ + …)
- Analysis A is used to compute values of arithmetic expressions.
  - Converting 1.98 to 2 enables O1, disables O2.
  - Converting 1.98 to 1.96875 enables O2, disables O1.
  - Precise for one is imprecise for another.
  - Sound for one is unsound for another.

Acknowledgements

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