Parallelization

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Speedup

- Speedup = Ts / Tp
- Amdahl's Law: Speedup is limited by the sequential part of the task.

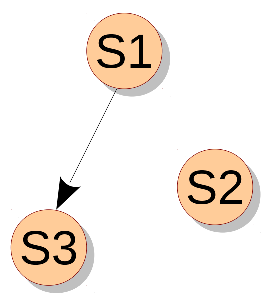
- If 20% of the task is sequential, program's speedup is limited to 5 (irrespective of the number of cores or amount of effort).

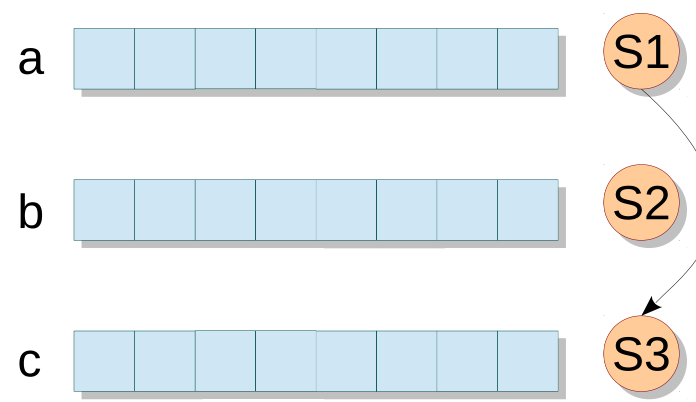
Task Parallel vs. Data Parallel

- Parallelism extracted from multiple instructions on the data items.

- Parallelism extracted from the same task on different data items.

S1: a[ii] = 2
S2: b[ii] = 4
S3: c[ii] = a[ii]





S1 is the source and S3 is the sink of the dependence.

Control Dependence

- if (x == 4) y = 10; else y = 1;

Data Dependence

- $pi = 3.142; r = 5.0; area = pi * r * r;$
- Types
  - True / Flow: S1 δ S2 (x = ...; ... = x;)
  - Anti: S1 δ^{-1} S2 (... = x; x = ...)
  - Output: S1 δ^{0} S2 (x = ...; x = ...)

Program Order vs. Dependence

- Sequential order imposed by the program is too restrictive.
- Only the partial order of all dependences need to be maintained by the compiler to guarantee program correctness.
- So, reorder flow; maintain dependence.

Advantages of Reordering

- Improved locality
  - Spatial: matrix operations
  - Temporal: xinit(); yinit(); xcompute(); ycompute();
- Improved load balance
  - small1(); big1(); small2(); big2();
- Improved parallelism
  - xuse(); xdef(); yuse(); ydef();

Let's Focus on Loops

- Iteration vector: Sequence of outer loops.
  - $\vec{iv}$ = (ioutermost, ..., imiddle, ..., iinnermost)
  - For instance (i, j, k).
- Iteration space: Set of all possible iteration vectors for a statement.
- Statement instance: $S(\vec{i})$
- $S(\vec{i})\ \delta\ S(\vec{j})$ iff

  (a) i < j or (i == j and S1 ⇨ ⇨ ⇨ S2 path in loop-body)

  (b) both access same memory location

  (c) at least one of the accesses is a write

Safe Transformations

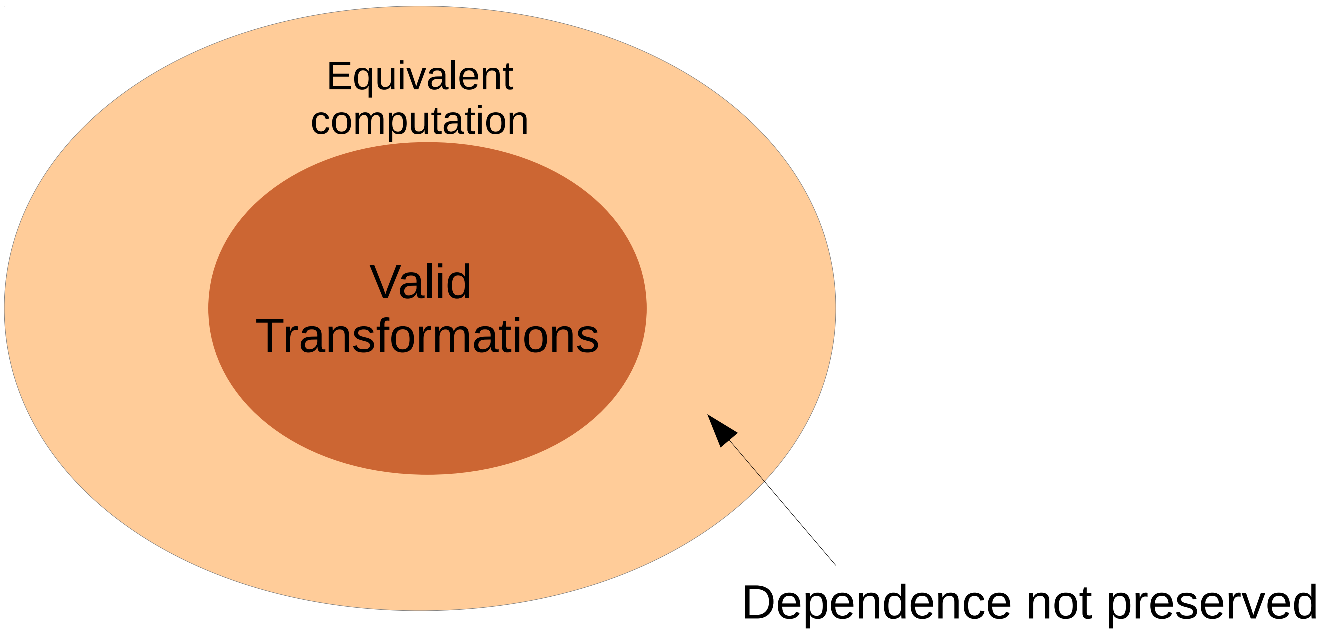
- **Loop Dependence Theorem**
  - There exists a dependence from statement S1 to statement S2 in a common nest of loops iff there exist two iteration vectors $\vec{i}$ and $\vec{j}$ for the nest, such that S1($\vec{i}$) δ S2($\vec{j}$).
- Two computations are equivalent if on the same inputs they produce the same output.
- A transformation is safe if it leads to an equivalent program.

Reordering Transformations

- A reordering transformation is any program transformation that merely changes the execution order of the code, without adding or deleting any executions of any statements.
- A reordering transformation preserves a dependence if it preserves the relative execution order of the source and the sink of that dependence.
- **Theorem:** Any reordering transformation that preserves every dependence in a program leads to an equivalent computation.

Valid Transformations

- A transformation is valid for the program to which it applies if it preserves all the dependences in the program.



Loop Parallelization

- **Theorem**: It is valid to convert a sequential loop to a parallel loop if the loop carries no dependence.

```
for (k = 0; k < n; ++k) {
  S1: a[k] = b[k];
  S2: b[k] = a[k] + 1;
}
```

```
for (k = 0; k < n; ++k) {
  S1: a[k] = a[k + 1];
}
```

General Strategy

```
for (ii = 0; ii < n; ++ii) {
    for (jj = 0; jj < m; ++jj) {
        a[f(ii, jj)][g(ii, jj)] = ...
        ... = ... a[h(ii, jj)][k(ii, jj)]...
    }
}
```

Conditions for flow dependence from iteration (ii_w, jj_w) to (ii_r, jj_r):

$0 <= ii_w < n$

$0 <= jj_w < m$

$0 <= ii_r < n$

$0 <= jj_r < m$

$(ii_w, jj_w) <= (ii_r, jj_r)$

$\mathbf{f}(ii_w, jj_w) = \mathbf{h}(ii_r, jj_r)$

$\mathbf{g}(ii_w, jj_w) = \mathbf{k}(ii_r, jj_r)$

If f, g, h, k are affine functions of loop variables, then dependence testing can be formulated as an ILP.

ILP Formulation

```
for (ii = 0; ii < 10; ++ii) {
    a[2 * ii] = ... a[2 * ii + 1] ...
}
```

Is there a flow dependence between different iterations?

Dependence equations

$0 <= ii_w < ii_r < 10$

$2 * ii_w = 2 * ii_r + 1$

which can be written as

$0 <= ii_w$

$ii_w <= ii_r - 1$

$ii_r <= 9$

$2 * ii_w <= 2 * ii_r + 1$

$2 * ii_r + 1 <= 2 * ii_w$

$$\begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 2 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} ii_w \\ ii_r \end{pmatrix} <= \begin{pmatrix} 0 \\ -1 \\ 9 \\ 1 \\ -1 \end{pmatrix}$$

Dependence exists if the system has a solution.

ILP Formulation

```
for (ii = 0; ii < 10; ++ii) {
    a[2 * ii] = ... a[2 * ii + 1] ...
}
```

Is there an anti-dependence between different iterations?

Dependence equations

$0 <= ii_r < ii_w < 10$

$2 * ii_w = 2 * ii_r + 1$

which can be written as

$0 <= ii_r$

$ii_r <= ii_w - 1$

$ii_w <= 9$

$2 * ii_w <= 2 * ii_r + 1$

$2 * ii_r + 1 <= 2 * ii_w$

$$\begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} ii_r \\ ii_w \end{pmatrix} <= \begin{pmatrix} 0 \\ -1 \\ 9 \\ 1 \\ -1 \end{pmatrix}$$

The system is not satisfiable, so anti-dependence does not exist.

ILP Formulation

```
for (ii = 0; ii < 10; ++ii) {
    a[2 * ii] = ... a[ii + 1] ...
}
```

Is there an anti-dependence between different iterations?

Dependence equations

$0 <= ii_r < ii_w < 10$

$2 * ii_w = ii_r + 1$

which can be written as

$$
\left.\begin{array}{l}
0 <= ii_r \\
ii_r <= ii_w - 1 \\
ii_w <= 9 \\
2 * ii_w <= ii_r + 1 \\
ii_r + 1 <= 2 * ii_w
\end{array}\right\}
\begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ -1 & 2 \\ 1 & -2 \end{pmatrix}
\begin{pmatrix} ii_r \\ ii_w \end{pmatrix}
<=
\begin{pmatrix} 0 \\ -1 \\ 9 \\ 1 \\ -1 \end{pmatrix}
$$

The system is not satisfiable, so anti-dependence does not exist.

ILP Formulation

```
for (ii = 0; ii < 10; ++ii) {
    a[2 * ii] = ... a[ii + 1] ...
}
```

Is there a true dependence between different iterations?

Dependence equations

$0 <= ii_w < ii_r < 10$

$2 * ii_w = ii_r + 1$

which can be written as

$0 <= ii_w$

$ii_w <= ii_r - 1$

$ii_r <= 9$

$2 * ii_w <= ii_r + 1$

$ii_r + 1 <= 2 * ii_w$

$$\begin{pmatrix} 0 & -1 \\ -1 & 1 \\ 1 & 0 \\ -1 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} ii_r \\ ii_w \end{pmatrix} <= \begin{pmatrix} 0 \\ -1 \\ 9 \\ 1 \\ -1 \end{pmatrix}$$

ii_r	ii_w
0	--
1	--
2	--
3	2
4	--
5	3
6	--
7	4
8	--
9	5

The system is satisfiable, so true dependence exists.

Managing Races

- Data-race between iterations p and q for element a[f(i)].
- Critical section
  - Locks
  - Atomics
  - Barriers

Inserting Locks

- Data-race between iterations p and q for element a[f(i)].

```
if (i == p || i == q) {
    lock(f(i));
    ... perform operation ...
    unlock(f(i));
}
```

This operation could be same or different for the involved threads.

- e.g., Producer-consumer

```
produce() {
    while (...) {
        items.add(...);
    }
}
```

```
consume() {
    e = items.remove();
}
```

Inserting Locks

- For multiple data items a[f(i)] and a[g(i)]
  - Single lock
  - Multiple locks
- Multiple locks may lead to deadlock
  - may allow deadlock if it improves parallelism
- Deadlock avoidance may lead to livelock
  - may allow livelock if rare

Inserting Locks

- Sometimes, a lock may be for a simple operation

```
if (i == p || i == q) {
    lock(f(i));
    sum += a[i];
    unlock(f(i));
}
```

- A simple critical section may be convertible to atomics.

Inserting Atomics

- If the operation is simple
  - Primitive type
  - Single element
  - Relative update / read-write
- Example
  - Producer-consumer with single element update
- Types
  - increment, decrement
  - add, sub
  - min, max
  - exch, CAS

Inserting Atomics

- **Classwork:** convert the following example from locks to atomics

```
if (i == p || i == q) {
    lock(f(i));
    sum += a[i];
    unlock(f(i));
}
```

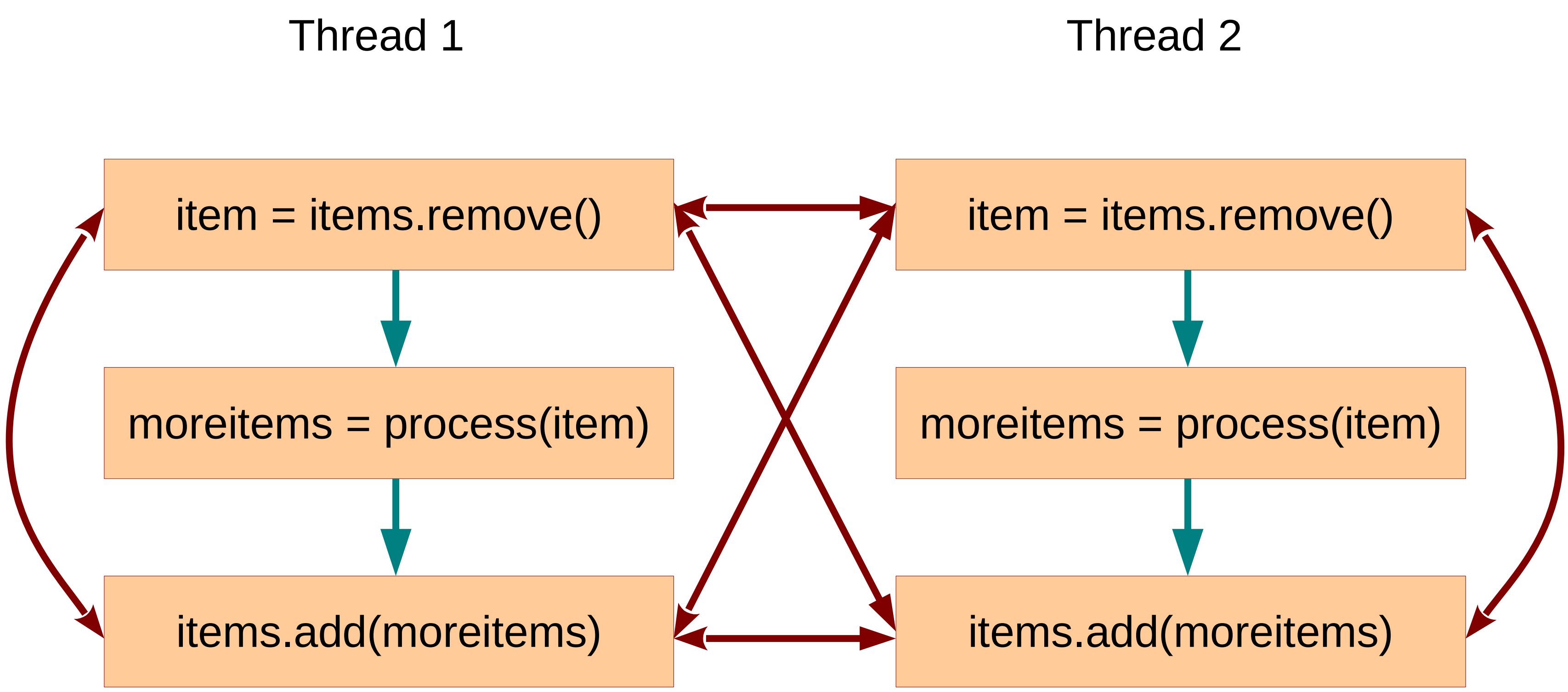
- **Classwork:** write parallel slist insertion and deletion routines using atomics
- **Homework:** write parallel dlist insertion routine using atomics

Inserting Locks

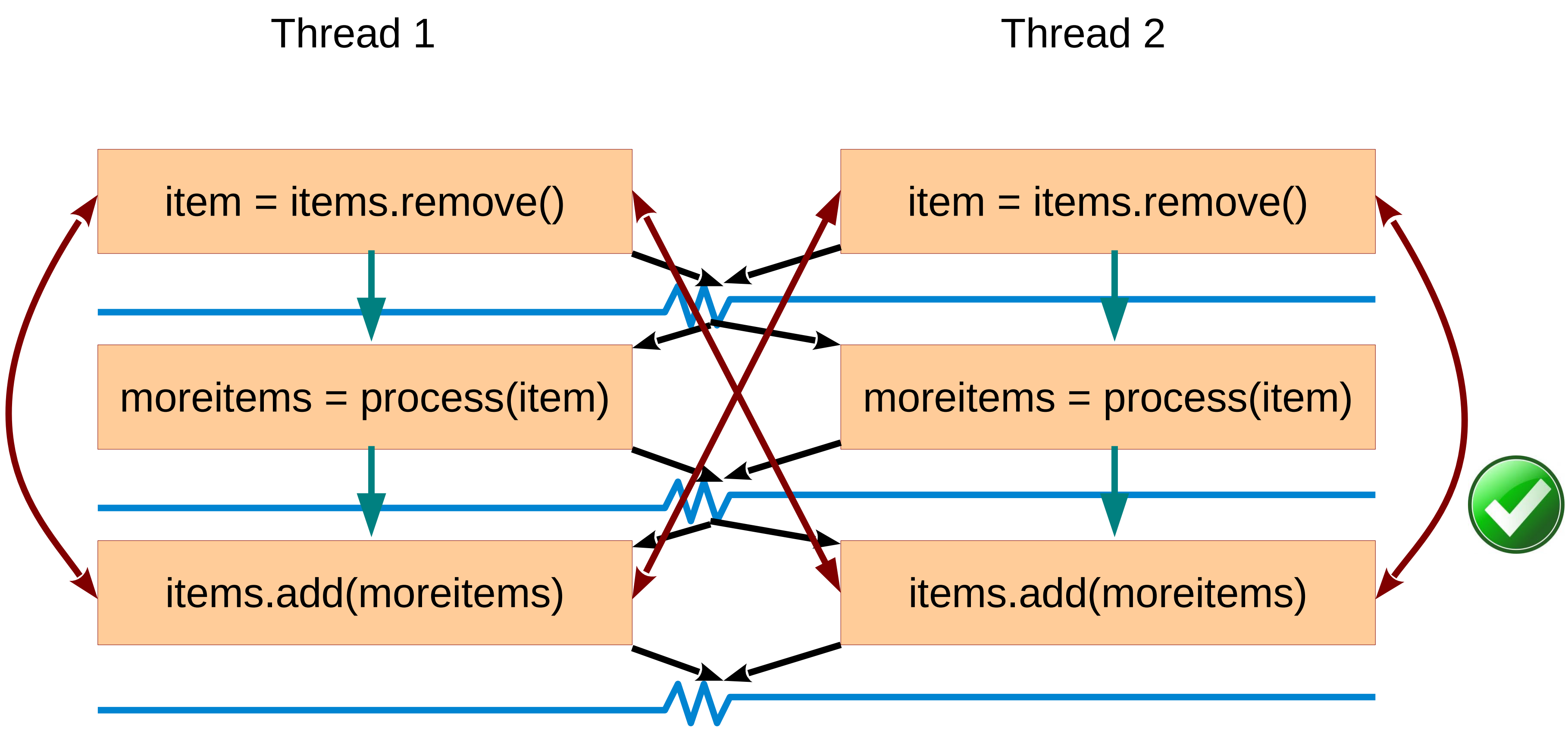
```
if (i == 1 || i == 2 || i == 4 || ...) {
    lock(f(i));
    item = items.remove();
    moreitems = process(item);
    items.add(moreitems);
    unlock(f(i));
}
```

- If there are many threads involved in the if(...) condition and the operation is multi-step, overapproximate the dependences.

Dependences

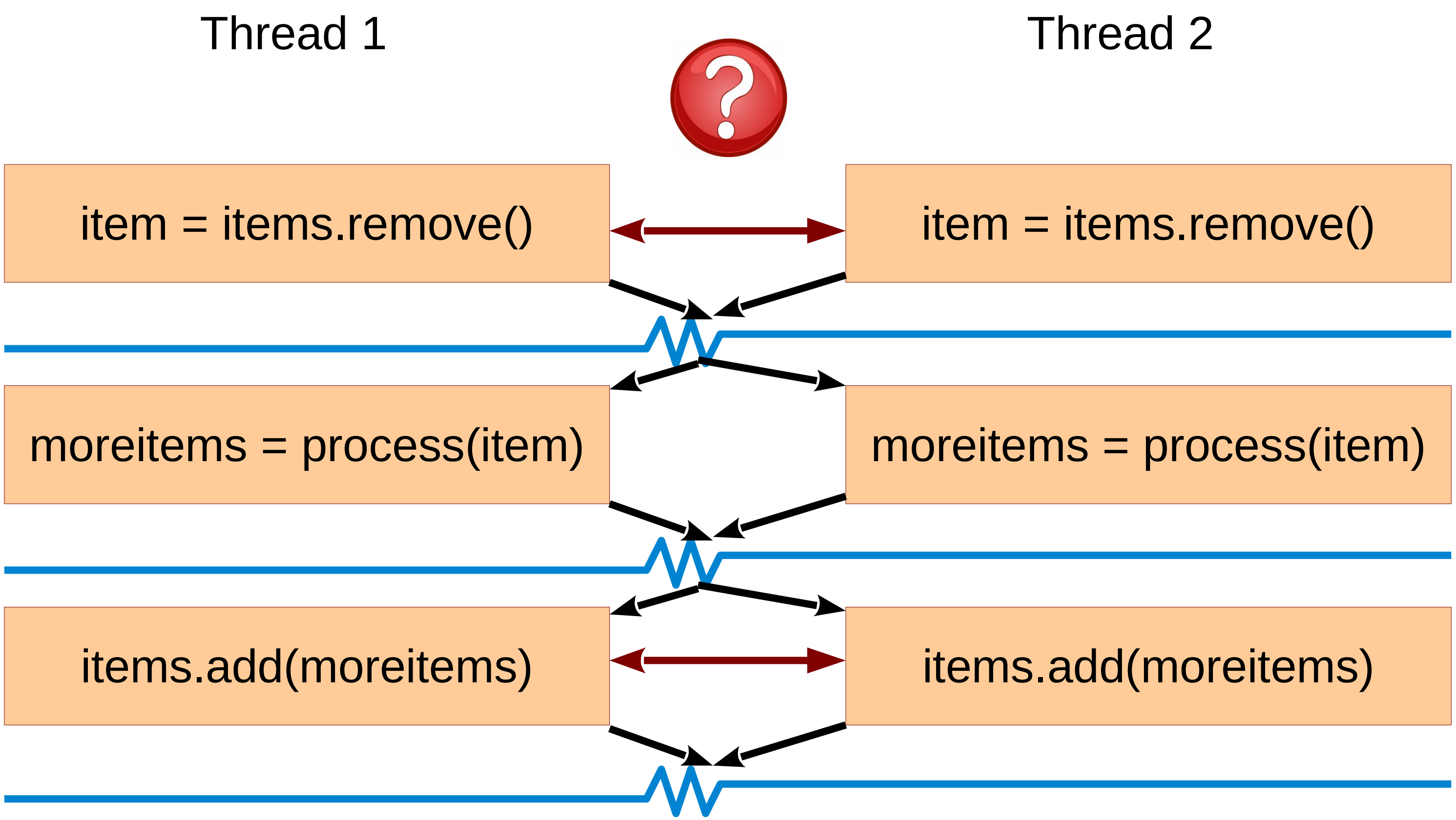


Barriers



Thread 1
Thread 2
item = items.remove()
item = items.remove()
moreitems = process(item)
moreitems = process(item)
items.add(moreitems)
items.add(moreitems)

Barriers


Thread 1
Thread 2
item = items.remove()
item = items.remove()
moreitems = process(item)
moreitems = process(item)
items.add(moreitems)
items.add(moreitems)

Inserting Barriers

```
if (i == 1 || i == 2 || i == 4 || ...) {
    lock(f(i));
    item = items.remove();
    unlock(f(i));
    -- barrier --

    moreitems = process(item);
    -- barrier --

    lock(f(i));
    items.add(moreitems);
    unlock(f(i));
    -- barrier --
}
```

Can be converted to atomics. (item = items.remove();)

Can lead to good parallelism. (moreitems = process(item);)

Can be converted to atomics. (items.add(moreitems);)

Inserting Barriers

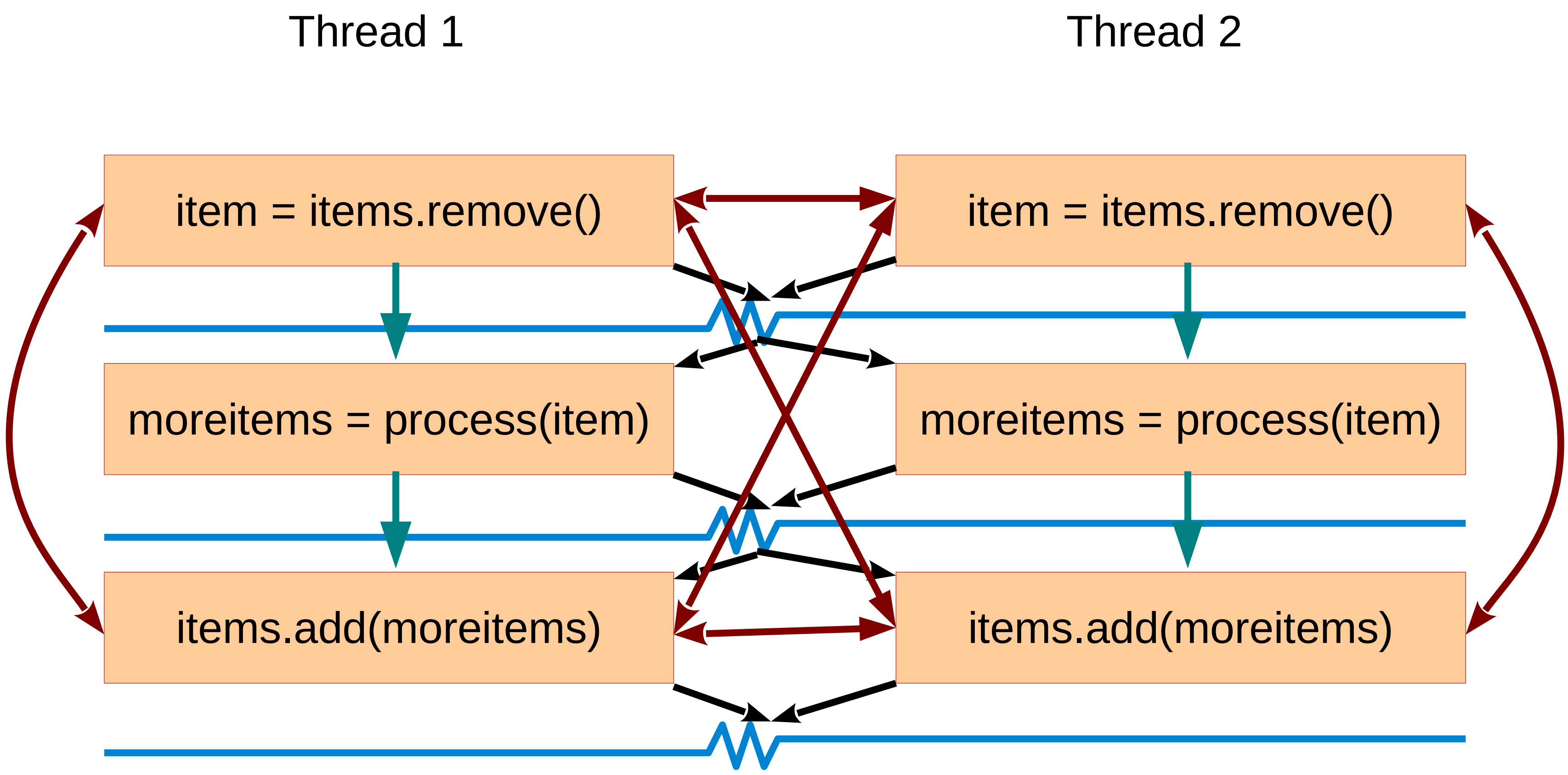
```
if (i == 1 || i == 2 || i == 4 || ...) {
    atomicDec(items[f(i)]);
    -- barrier --

    moreitems = process(item);
    -- barrier --

    atomicAdd(items[f(i)], size(moreitems));
    items.addunsync(moreitems);
    -- barrier --
}
```

If the barrier is emulated, one can combine these operations.

Barriers and Dependences

- A barrier may be considered in effect similar to loop distribution.
- If dependences are sparse, use atomics/locks; otherwise barriers work well.
- A barrier may add more dependences than required.
- But it must preserve all the existing dependences.

Barriers and Dependences



Did we add any extra dependences?

Barriers and Dependences

- **Homework:** Using the mechanisms we studied, construct an example wherein inserting a barrier may lead to incorrect parallel execution.