

# Pointer Analysis

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# Outline

- Introduction
- Pointer analysis as a DFA problem
- Design decisions
- Andersen's analysis, Steensgaard's analysis
- Pointer analysis as a graph problem
  - Optimizations
- Pointer analysis as graph rewrite rules
- Applications
- Parallelization
  - Constraint based
  - Replication based

# What is Pointer Analysis?

```
a = &x;
```

```
b = a;
```

```
if (b == *p) {
```

```
    ...
```

```
} else {
```

```
    ...
```

```
}
```

# What is Points-to Analysis?

`a = &x;`



a points to x

`b = a;`

`if (b == *p) {`

`...`

`} else {`

`...`

`}`

# What is Points-to Analysis?

`a = &x;`

a points to x

`b = a;`

a and b are aliases

`if (b == *p) {`

`...`

`} else {`

`...`

`}`

# What is Points-to Analysis?

`a = &x;`

a points to x

`b = a;`

a and b are aliases

`if (b == *p) {`

Is this condition always satisfied?

`...`

`} else {`

`...`

`}`

# What is Points-to Analysis?

`a = &x;`

a points to x

`b = a;`

a and b are aliases

`if (b == *p)`

Is this condition always satisfied?

...

`} else {`

...

`}`

Pointer Analysis is a mechanism to **statically** find out run-time values of a pointer.

# Why Points-to Analysis?

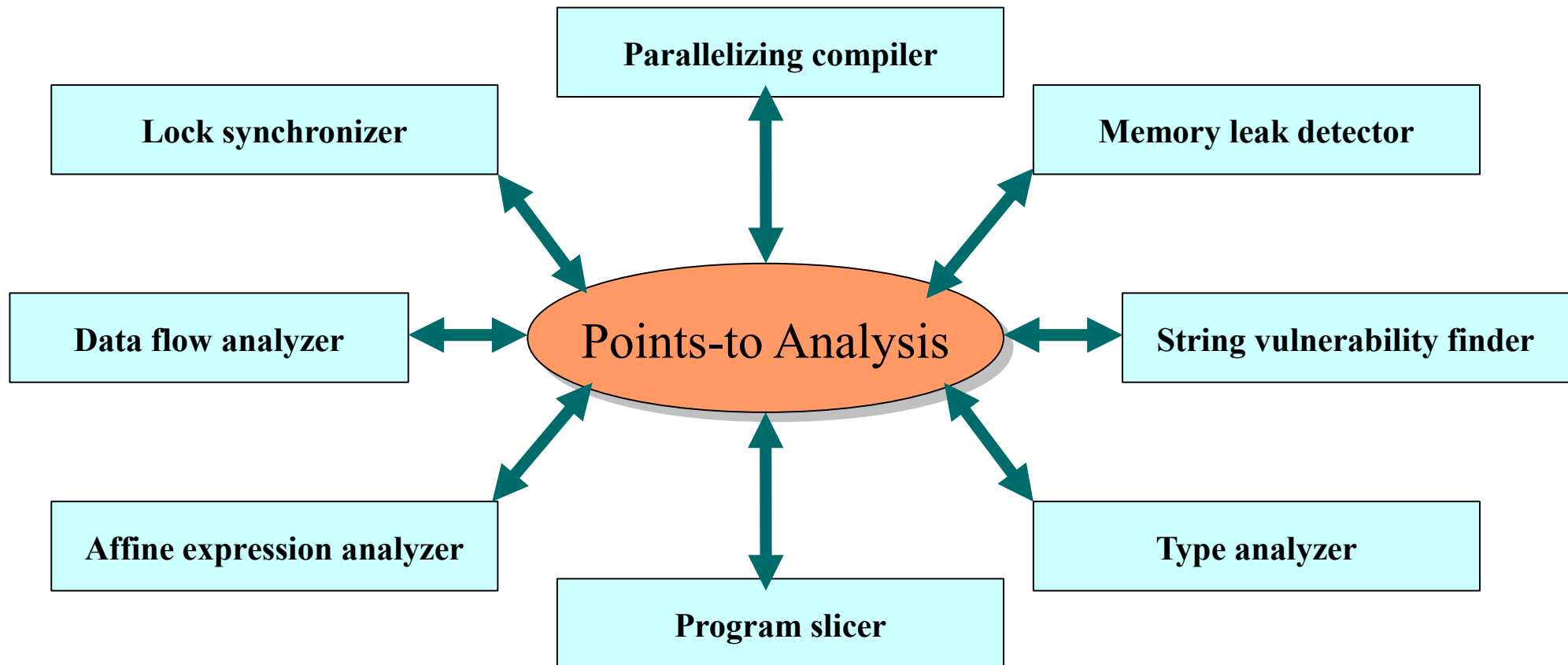
- for Parallelization
  - `fun(p) || fun(q)`
- for Optimization
  - `a = p + 2;`
  - `b = q + 2;`
- for Bug-Finding
- for Program Understanding
- ...



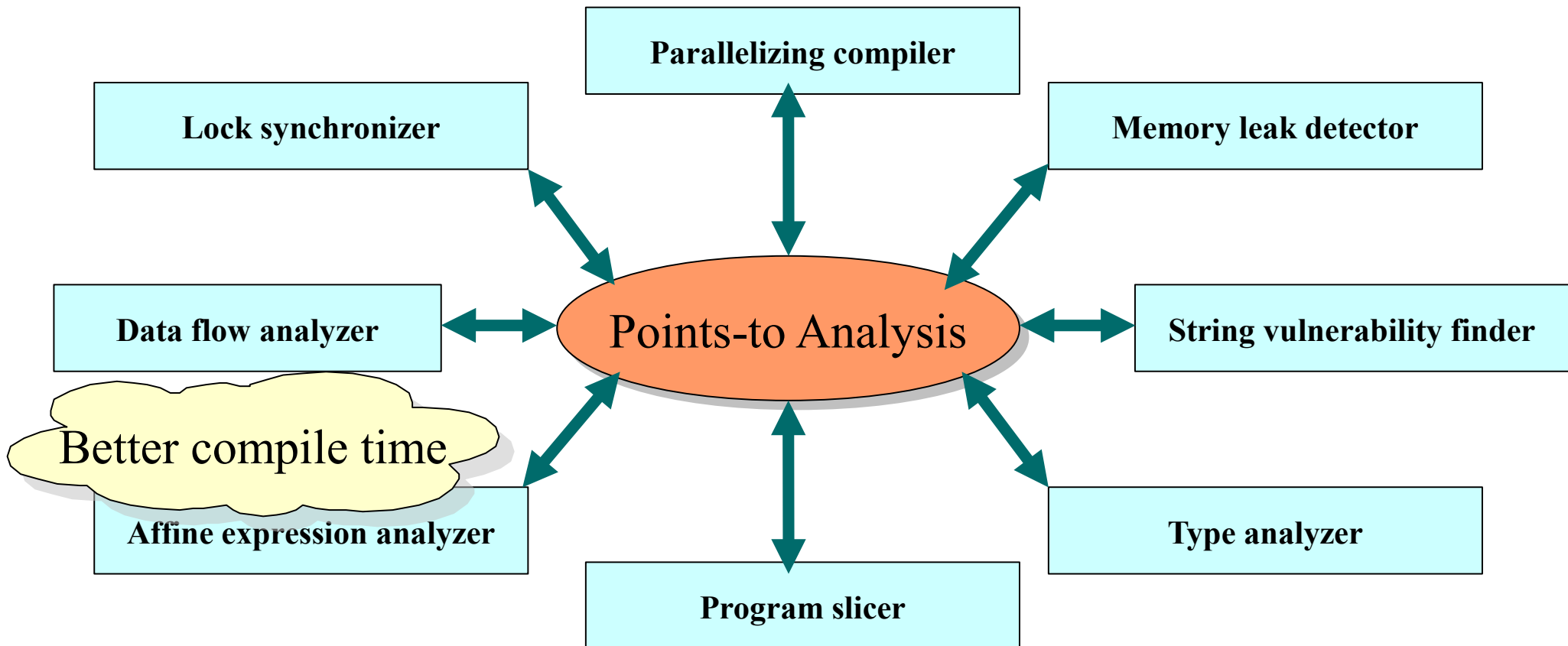
**Clients of  
Points-to Analysis**



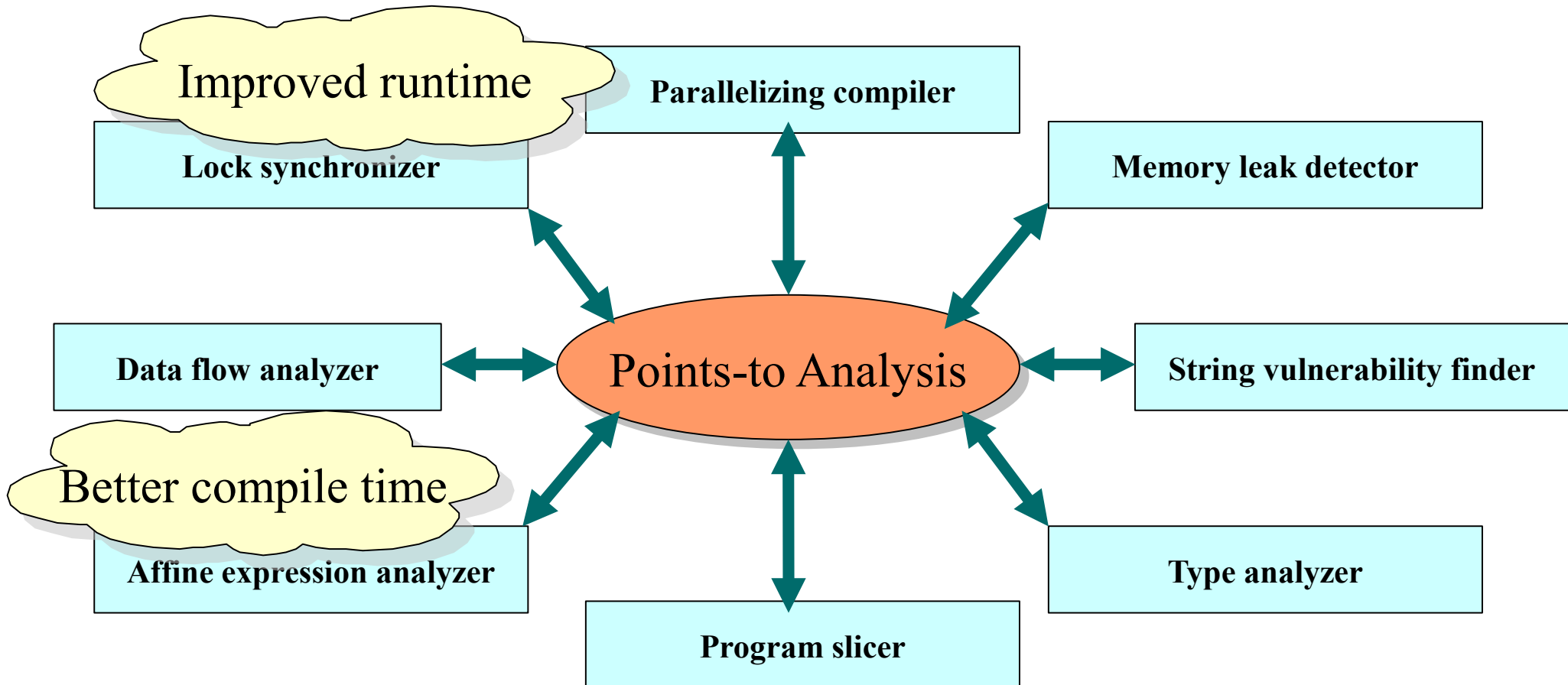
# Placement of Points-to Analysis



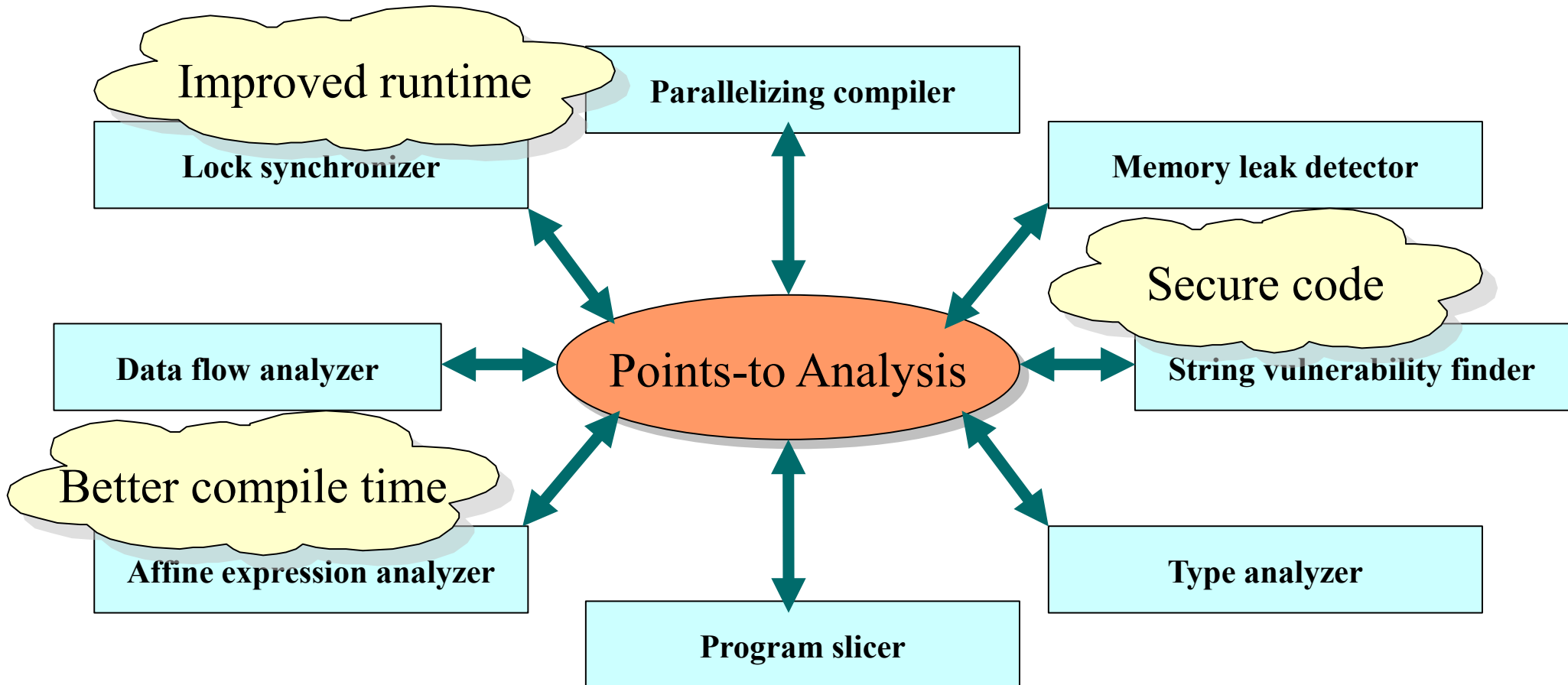
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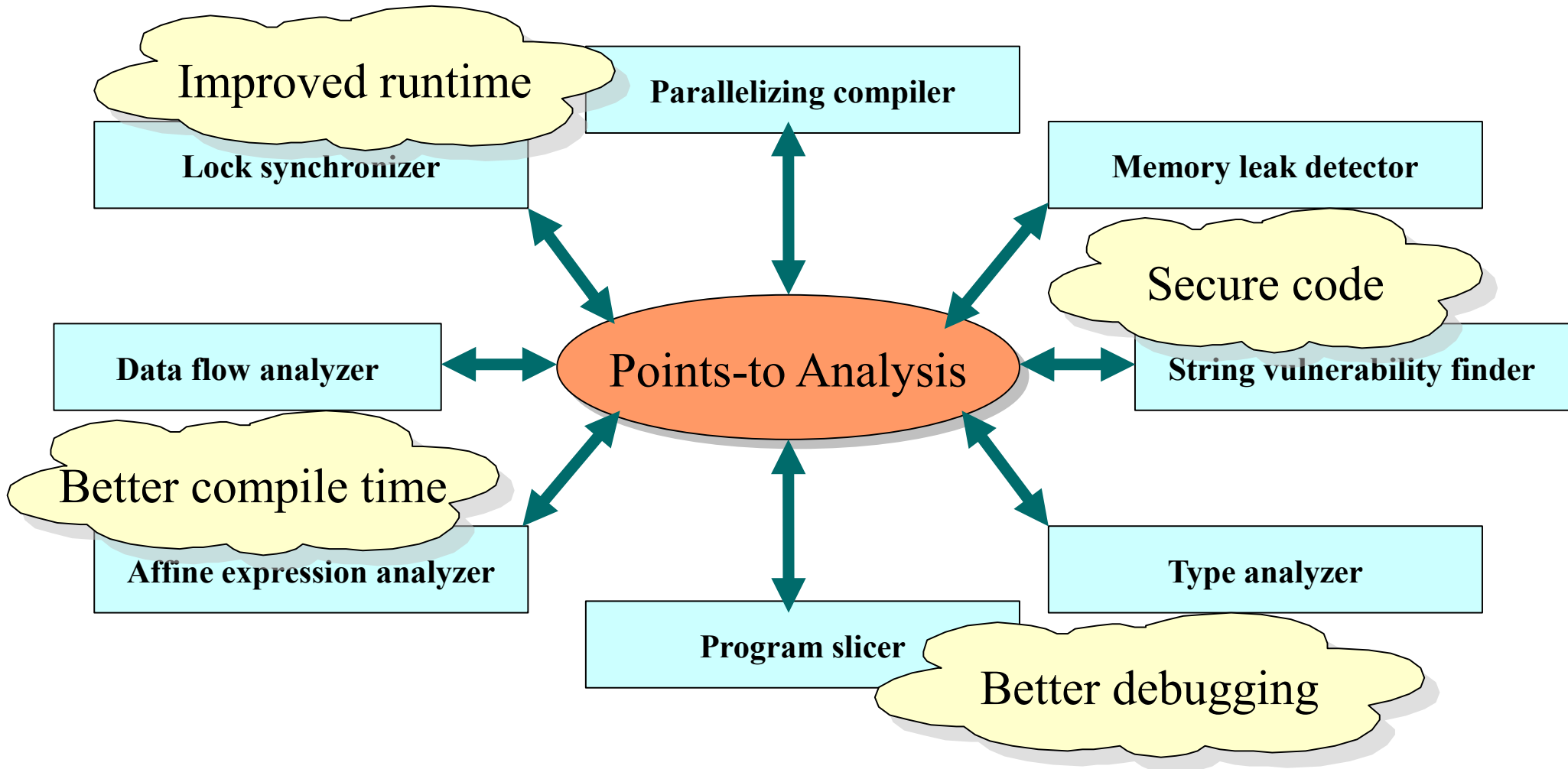
# Placement of Points-to Analysis



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# Points-to Analysis

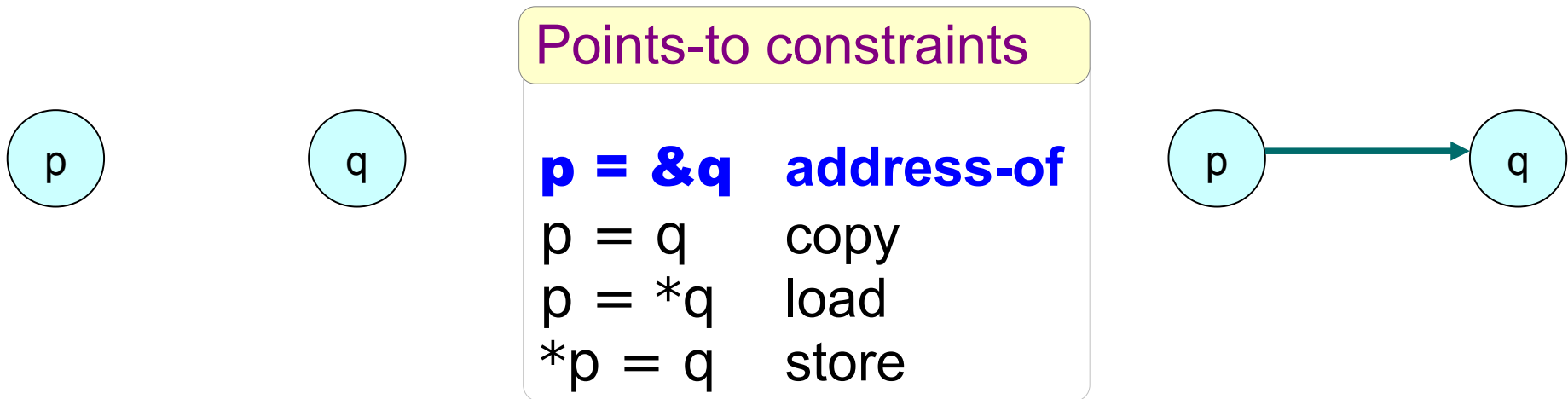
A C program can be **normalized** to contain only four types of pointer-manipulating statements or constraints.

## Points-to constraints

<code>p = &amp;q</code>	address-of
<code>p = q</code>	copy
<code>p = *q</code>	load
<code>*p = q</code>	store

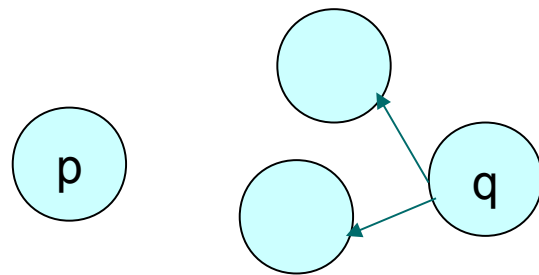
# Points-to Analysis

A C program can be **normalized** to contain only four types of pointer-manipulating statements or constraints.



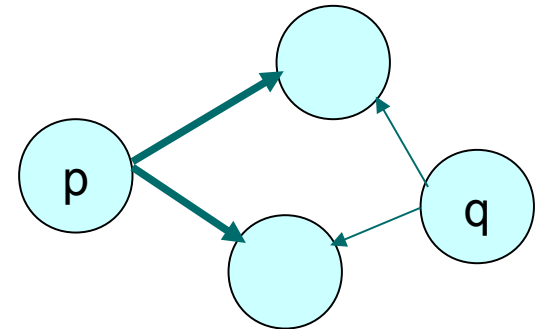
# Points-to Analysis

A C program can be **normalized** to contain only four types of pointer-manipulating statements or constraints.



## Points-to constraints

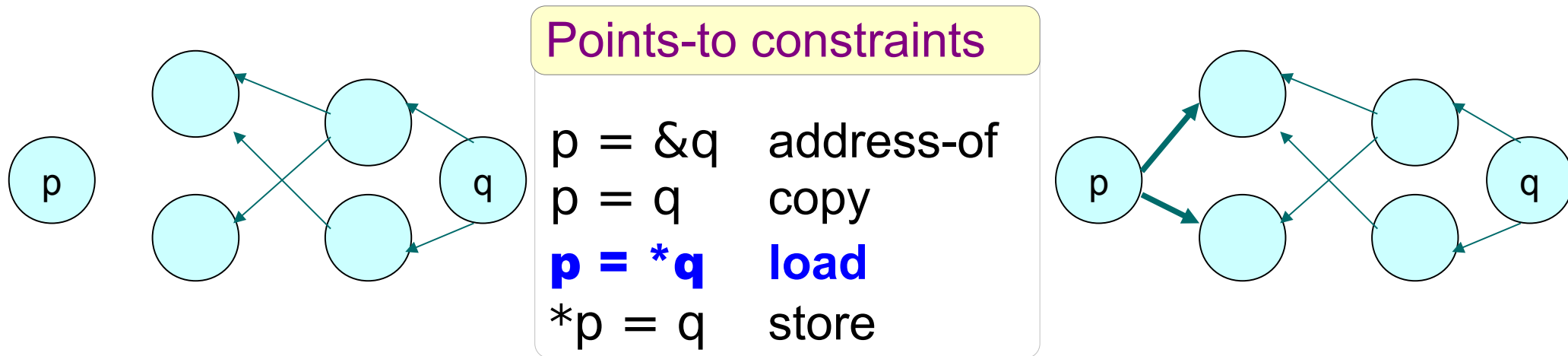
$p = \&q$	address-of
<b><math>p = q</math></b>	<b>copy</b>
$p = *q$	load
$*p = q$	store





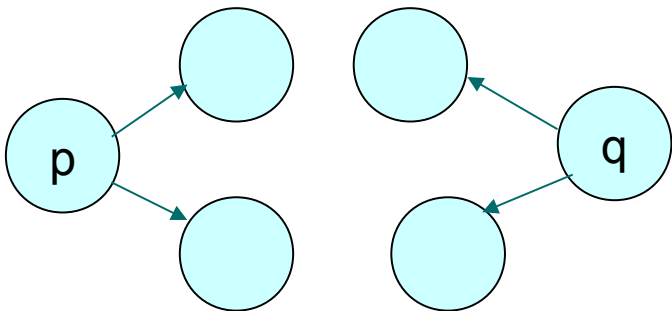
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A C program can be **normalized** to contain only four types of pointer-manipulating statements or constraints.



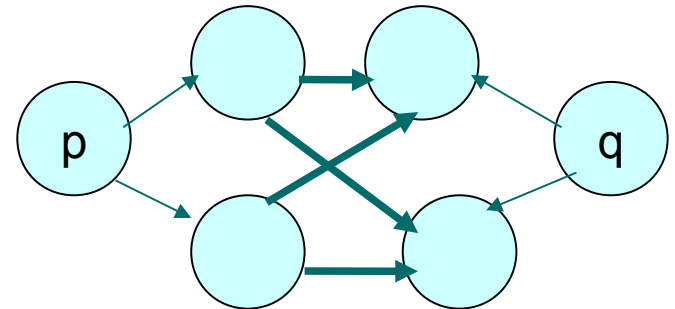
# Points-to Analysis

A C program can be **normalized** to contain only four types of pointer-manipulating statements or constraints.



## Points-to constraints

$p = \&q$  address-of  
 $p = q$  copy  
 $p = *q$  load  
 **$*p = q$  store**



# Definitions

- **Points-to analysis** computes points-to information for each pointer.
- **Alias analysis** computes aliasing information for all pointers.
- Aliasing information can be computed using points-to information, but not vice versa.
- Clients often query for aliasing information, but storing it is expensive  $O(n^2)$ , hence frameworks store points-to information.
- If  $a \rightarrow x$ ,  $x$  is often called a **pointee** of  $a$ .

Points-to information

$a \rightarrow \{x, y\}$
$b \rightarrow \{y, z\}$
$c \rightarrow \{z\}$

Aliasing information

	a	b	c
a	--	Yes	No
b	--	--	Yes
c	--	--	--

# Nomenclature

- **Pointer analysis**: Ambiguous usage in literature. We will use it to refer to both **points-to analysis** and **alias analysis**.
- In the context of Java-like languages, it is called **reference analysis**.
- Also called as **heap analysis**.

# Algebraic Properties

- **Aliasing** relation is reflexive, symmetric, but not transitive.
- **Points-to** relation is neither reflexive, nor symmetric, not even transitive.
- The points-to relation induces a restricted DAG for **strictly typed** languages.

# Cyclic Dependence

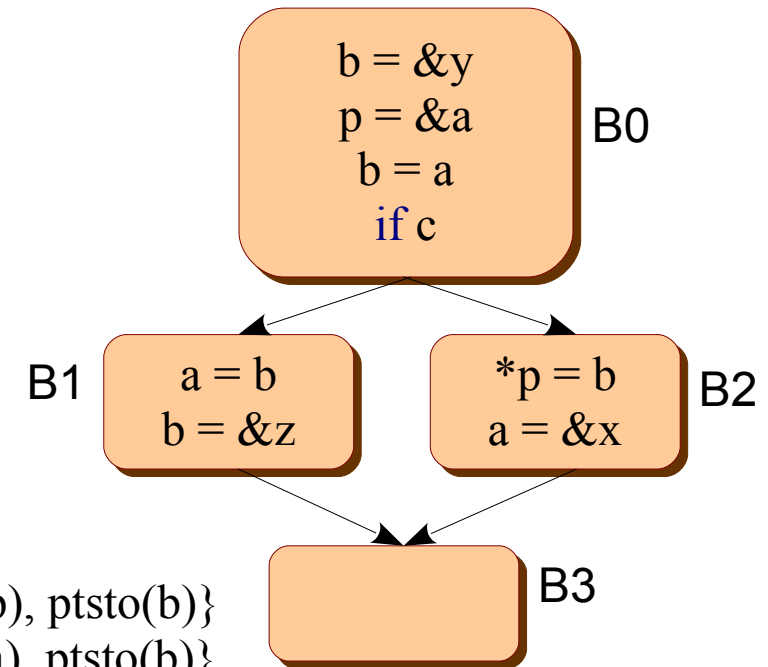
- Call graph  $\leftrightarrow$  function pointers
- Optimization  $\leftrightarrow$  points-to information

# As a DFA

$a = \&x:$      $\text{gen}\{a \rightarrow x\}$   
 $a = b:$          $\text{gen}\{a \rightarrow x\}$  if  $\{b \rightarrow x\}$   
 $a = *p:$        $\text{gen}\{a \rightarrow x\}$  if  $\{p \rightarrow b \rightarrow x\}$   
 $*p = a:$        $\text{gen}\{b \rightarrow x\}$  if  $\{p \rightarrow b \text{ and } a \rightarrow x\}$   
                   $\text{kill}\{b \rightarrow x\}$  if  $\{p \rightarrow b \text{ and } b \rightarrow x\}$

$\text{In}(B) = \bigcup \text{Out}(P)$  where  $P \in \text{Pred}(B)$

$\text{Out}(B) = \text{Gen}(B) \cup (\text{In}(B) - \text{Kill}(B))$



$\text{gen}(B0) = \{p \rightarrow a, b \rightarrow x \text{ if } a \rightarrow x\}$

$\text{gen}(B1) = \{a \rightarrow x \text{ if } b \rightarrow x, b \rightarrow z\}$

$\text{gen}(B2) = \{a \rightarrow x, m \rightarrow n \text{ if } p \rightarrow m \text{ and } b \rightarrow n \text{ and } m \neq a\}$

$\text{gen}(B3) = \{ \}$

$\text{kill}(B0) = \{\text{ptsto}(p), \text{ptsto}(b)\}$

$\text{kill}(B1) = \{\text{ptsto}(a), \text{ptsto}(b)\}$

$\text{kill}(B2) = \{\text{ptsto}(\text{ptsto}(p)), \text{ptsto}(a)\}$

$\text{kill}(B3) = \{ \}$

	in1	out1	in2	out2	in3	out3
<b>B0</b>	{}	{p→a}	{}	{p→a, b→{x,z}}	{}	{p→a, b→{x,z}}
<b>B1</b>	{}	{b→z}	out1(B0)	{p→a, a→{x,z}, b→{z}}	out2(B0)	{p→a, a→{x,z}, b→{z}}
<b>B2</b>	{}	{a→x}	out1(B0)	{p→a, a→{x}, b→{x,z}}	out2(B0)	{p→a, a→{x}, b→{x,z}}
<b>B3</b>	{}	{}	out1(B1) U out1(B2)	{p→a, a→{x,z}, b→{x,z}}	out2(B1) U out2(B2)	{p→a, a→{x,z}, b→{x,z}}

# As a DFA: Notes

- Gen and Kill are dynamic (not fixed before analysis).
- Gen/Kill and Points-to Information are cyclically dependent.
- Single copy of a variable leads to imprecision.
  - e.g., a's points-to set doesn't reach B0 in any execution, but the analysis treats it otherwise.

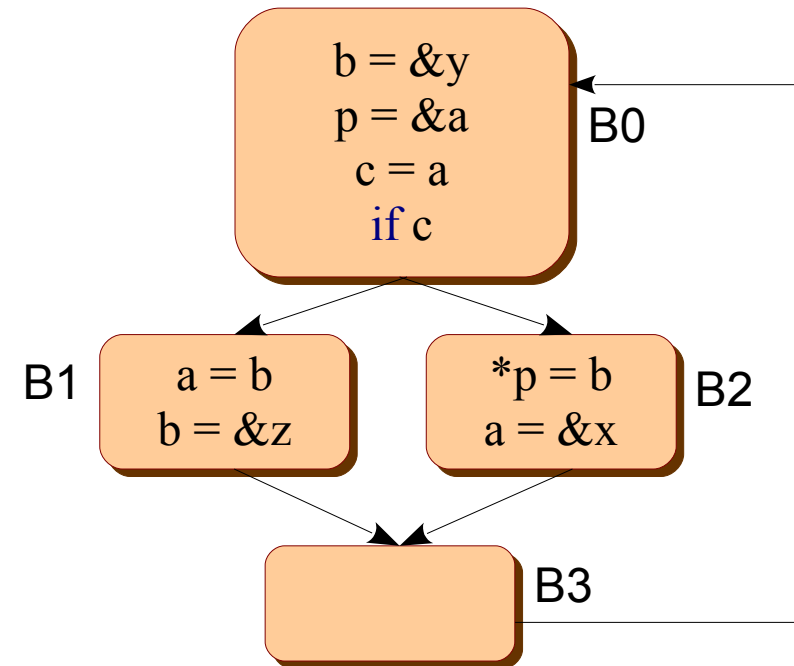


# Classwork

$a = \&x:$      $\text{gen}\{a \rightarrow x\}$   
 $a = b:$          $\text{gen}\{a \rightarrow x\}$  if  $\{b \rightarrow x\}$   
 $a = *p:$         $\text{gen}\{a \rightarrow x\}$  if  $\{p \rightarrow b \rightarrow x\}$   
 $*p = a:$         $\text{gen}\{b \rightarrow x\}$  if  $\{p \rightarrow b \text{ and } a \rightarrow x\}$   
                   $\text{kill}\{b \rightarrow x\}$  if  $\{p \rightarrow b \text{ and } b \rightarrow x\}$

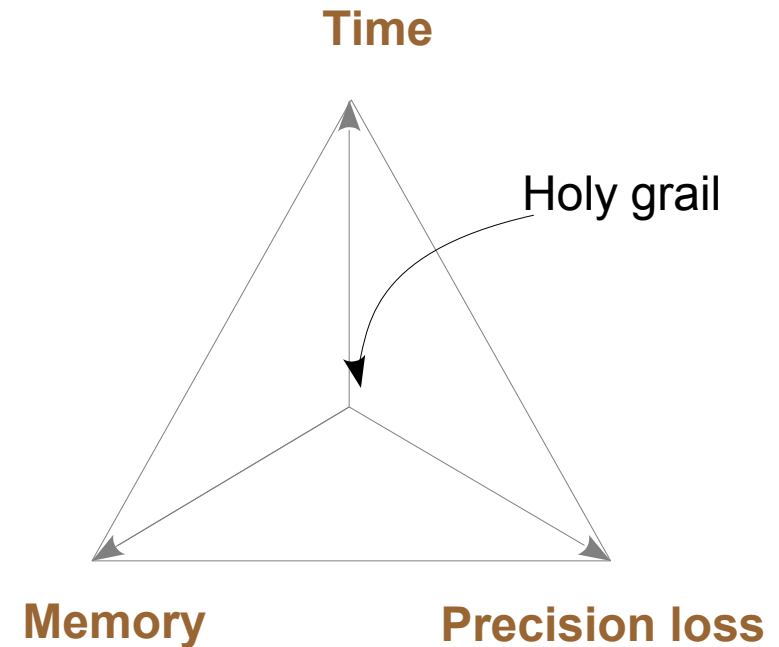
$In(B) = \bigcup Out(P)$  where  $P \in Pred(B)$

$Out(B) = Gen(B) \cup (In(B) - Kill(B))$



# Design Decisions

- Analysis dimensions
- Heap modeling
- Set implementation
- Call graph, function pointers
- Array indices



# Analysis Dimensions

An analysis's precision and efficiency is guided by various design decisions.

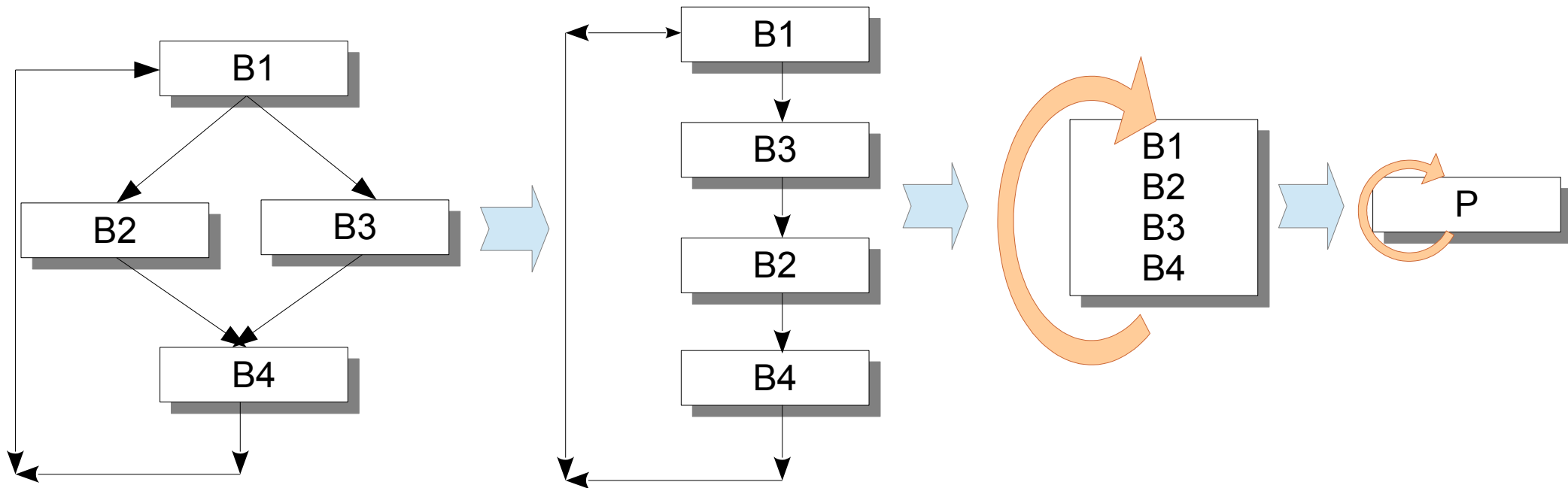
- Flow-sensitivity
- Context-sensitivity
- Path-sensitivity
- Field-sensitivity

# Flow-sensitivity

L0:  $a = \&x;$   
L1:  $a = \&y;$   
L2: ...

Flow-sensitive solution: *at L1 a points to x, at L2 a points to y*  
Flow-insensitive solution: *in the program a's points-to set is  $\{x, y\}$*

Flow-insensitive analyses ignore the control-flow in the program.



# Context-sensitivity

```
main() {  
  L0: fun(&x);  
  L1: fun(&y);  
}
```

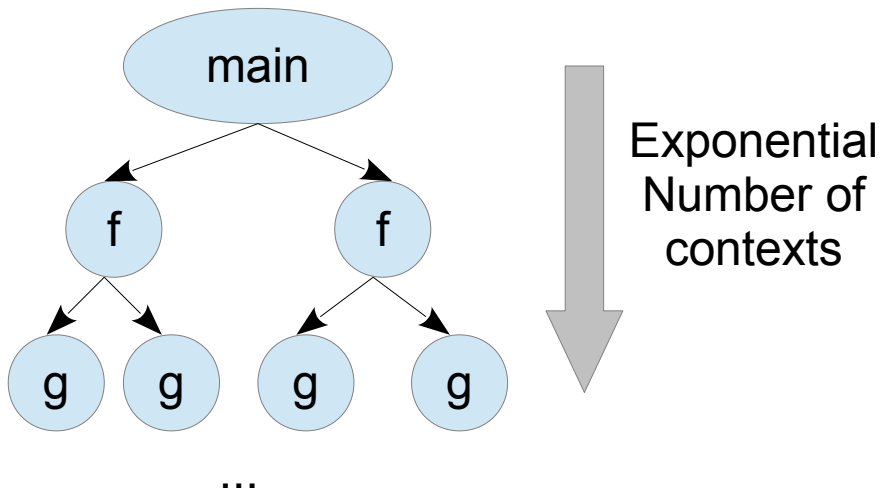
```
fun(int *a) {  
  b = a;  
}
```

Context-sensitive solution:

*b points to x along L0, b points to y along L1*

Context-insensitive solution:

*b's points-to set is {x, y} in the program*



Along main-f1-g1, ...  
Along main-f1-g2, ...  
Along main-f2-g1, ...  
Along main-f2-g2, ...

Exponential time requirement

Exponential storage requirement

# Context-sensitivity

<pre>main() {   L0: fun(&amp;x);   L1: fun(&amp;y); }</pre>	<pre>fun(int *a) {   b = a; }</pre>
---	---

Context-sensitive solution:

*b points to x along L0, b points to y along L1*

Context-insensitive solution:

Inter-procedural —→ *b's points-to set is {x, y} in the program*

intra-procedural —→ *b's points-to set is {all address-taken variables}*

# Path-sensitivity

```
if (a == 0)
    b = &x;
else
    b = &y;
```

Path-sensitive solution:

*b points-to x when a is 0, b points-to y when a is not 0*

Path-insensitive solution:

*b's points-to set is {x, y} in the program*

```
if (c1)
    while (c2) {
        if (c3)
            ...
        else
            for (; c4; )
                ...
    }
else
    ...
```

```
c1 and c2 and c3, ...
c1 and c2 and !c3 and c4, ...
c1 and c2 and !c3 and !c4, ...
c1 and !c2, ...
!c1 ...
...
```

# Field-sensitivity

```
struct T s;
```

```
s.a = &x;
```

```
s.b = &y;
```

Field-sensitive solution:

*s.a points-to x, s.b points-to y*

Field-insensitive solution:

*s's points-to set is {x, y}*

Aggregates are collapsed into a single variable.  
e.g., arrays, structures, unions.

This reduces the number of variables tracked during the analysis and reduces precision.



# Andersen's Analysis

- Inclusion-based / subset-based / constraint-based analysis
- Flow-insensitive analysis

For a statement  $p = q$ ,

create a constraint  $\text{ptsto}(p) \supseteq \text{ptsto}(q)$

where  $p$  is of the form  $*a$ ,  $a$ , and  $q$  is of the form  $*a$ ,  $a$ ,  $\&a$ .

Solving these inclusion constraints results into the points-to solution.

# Andersen's Analysis: Example

## Program

```
a = &x;  
b = &y;  
p = &a;  
c = b;  
*p = c;
```

## Constraints

```
ptsto(a)  $\supseteq$  {x}  
ptsto(b)  $\supseteq$  {y}  
ptsto(p)  $\supseteq$  {a}  
ptsto(c)  $\supseteq$  ptsto(b)  
ptsto(*p)  $\supseteq$  ptsto(c)
```

fixed-point

Pointers	Iteration 0	Iteration 1	Iteration 2
a	{ }	{x, y}	
b	{ }	{y}	
c	{ }	{y}	
p	{ }	{a}	
x	{ }		
y	{ }		

Imprecision

# Andersen's Analysis: Modified Example

## Program

```
a = &x;  
b = &y;  
p = &a;  
*p = c;  
c = b;
```

## Constraints

```
ptsto(a)  $\supseteq$  {x}  
ptsto(b)  $\supseteq$  {y}  
ptsto(p)  $\supseteq$  {a}  
ptsto(*p)  $\supseteq$  ptsto(c)  
ptsto(c)  $\supseteq$  ptsto(b)
```

Order does not matter  
for correctness,  
but it does matter  
for efficiency.

fixed-point

Pointers	Iteration 0	Iteration 1	Iteration 2	Iteration 3
a	{ }	{x}	{x, y}	
b	{ }	{y}		
c	{ }	{y}		
p	{ }	{a}		
x	{ }			
y	{ }			

# Andersen's Analysis: Classwork

## Program

```
*p = c;
b = &y;
b = *p;
p = &a;
a = &x;
*p = c;
c = p;
c = &z;
```

## Constraints

```
ptsto(*p)  $\supseteq$  ptsto(c)
ptsto(b)  $\supseteq$  {y}
ptsto(b)  $\supseteq$  ptsto(*p)
ptsto(p)  $\supseteq$  {a}
ptsto(a)  $\supseteq$  {x}
ptsto(*p)  $\supseteq$  ptsto(c)
ptsto(c)  $\supseteq$  ptsto(p)
ptsto(c)  $\supseteq$  {z}
```

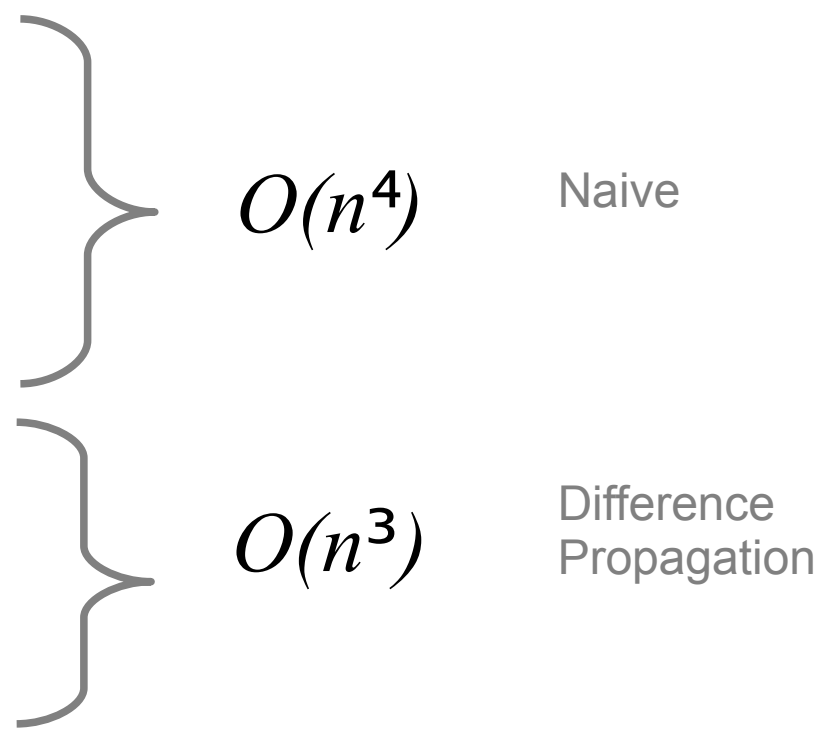
fixed-point

Pointers	Iteration 0	Iteration 1	Iteration 2	Iteration 3
a	{ }	{x}	{a, x, z}	
b	{ }	{y}	{a, x, y, z}	
c	{ }	{a, z}	{a, z}	
p	{ }	{a}	{a}	
x	{ }			
y	{ }			
z				

# Andersen's Analysis: Optimizations

- Avoid duplicates
- Reorder constraints
- Process address-of constraints once
- Difference propagation

# Andersen's Analysis: Complexity

- Total information computed (storage) =  $O(n^2)$
  - From each pointer
    - To each other pointer
    - Propagate  $O(n)$  information
    - $O(n)$  times
  - From each pointer
    - To each other pointer
    - Propagate  $O(n)$  information
- 
- The diagram uses curly braces to group the steps for each method. For the Naive method, the steps 'To each other pointer', 'Propagate  $O(n)$  information', and ' $O(n)$  times' are grouped by a large brace pointing to the complexity  $O(n^4)$ . For the Difference Propagation method, the steps 'To each other pointer' and 'Propagate  $O(n)$  information' are grouped by a large brace pointing to the complexity  $O(n^3)$ .
- $O(n^4)$  Naive
- $O(n^3)$  Difference Propagation

*Open:* Can you reduce the gap between storage and time complexities?

# Steensgaard's Analysis

- Unification-based
- Almost linear time  $O(m\alpha(m))$
- More imprecise

For a statement  $p = q$ , merge the points-to sets of  $p$  and  $q$ .

In subset terms,  $\text{ptsto}(p) \supseteq \text{ptsto}(q)$  **and**  $\text{ptsto}(q) \supseteq \text{ptsto}(p)$  with a single representative element.

# Steensgaard's Analysis: Example

Program	Andersen's	Steensgaard's
$a = \&x;$ $b = \&y;$ $p = \&a;$ $c = b;$ $*p = c;$	$a \rightarrow \{x, y\}$ $b \rightarrow \{y\}$ $c \rightarrow \{y\}$ $p \rightarrow \{a\}$	$a \rightarrow \{x, y\}$ $b \rightarrow \{x, y\}$ $c \rightarrow \{x, y\}$ $p \rightarrow \{a\}$

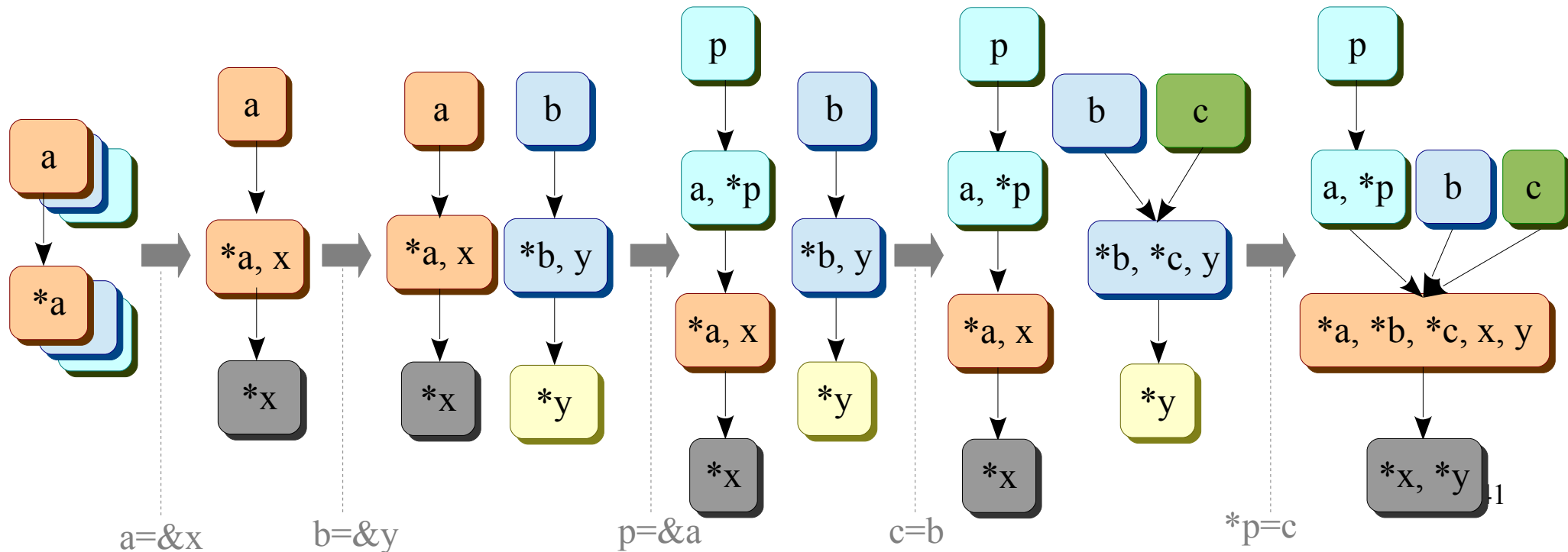
Pointers	Iteration 0	Iteration 1
a	$\{*a\}$	$\{*a, *b, *c, x, y\}$
b	$\{*b\}$	$\{*a, *b, *c, x, y\}$
c	$\{*c\}$	$\{*a, *b, *c, x, y\}$
p	$\{*p\}$	$\{*p, a\}$
x	$\{*x\}$	
y	$\{*y\}$	

Only one iteration



# Steensgaard's Hierarchy

Program	Andersen's	Steensgaard's
<pre> a = &amp;x; b = &amp;y; p = &amp;a; c = b; *p = c; </pre>	<pre> <math>a \rightarrow \{x, y\}</math> <math>b \rightarrow \{y\}</math> <math>c \rightarrow \{y\}</math> <math>p \rightarrow \{a\}</math> </pre>	<pre> <math>a \rightarrow \{x, y\}</math> <math>b \rightarrow \{x, y\}</math> <math>c \rightarrow \{x, y\}</math> <math>p \rightarrow \{a\}</math> </pre>



# Classwork

## Program

```
*p = c;  
b = &y;  
b = *p;  
p = &a;  
a = &x;  
*p = c;  
c = p;  
c = &z;
```

## Andersen's

```
a  $\rightarrow$  {a, x, z}  
b  $\rightarrow$  {a, x, y, z}  
c  $\rightarrow$  {a, z}  
p  $\rightarrow$  {a}
```

## Steensgaard's

# Steensgaard's Hierarchy

- What is its structure?
- How many incoming edges to each node?
- How many outgoing edges from each node?
- Can there be cycles?
- What happens to  $p = \&p$ ?
- What is the precision difference between Andersen's and Steensgaard's analyses?
- If for each  $P = Q$ , we add  $Q = P$  and solve using Andersen's analysis, would it be equivalent to Steensgaard's analysis?

# Unifying Model Two

- Steensgaard's hierarchy is characterized by a single outgoing edge.
- Andersen's points-to graph can have arbitrary number of outgoing edges (maximum  $n$ ).
- Number of edges in between the two provide precision-scalability trade-off.

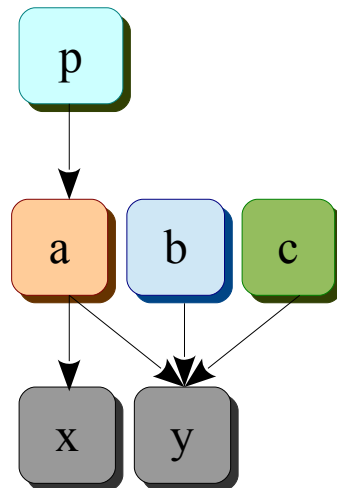
# Unifying Model Two

Program

```
a = &x;  
b = &y;  
p = &a;  
c = b;  
*p = c;
```

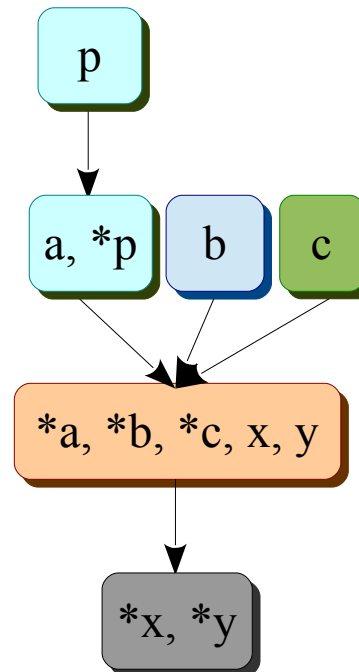
Andersen's

```
a → {x, y}  
b → {y}  
c → {y}  
p → {a}
```



Steensgaard's

```
a → {x, y}  
b → {x, y}  
c → {x, y}  
p → {a}
```



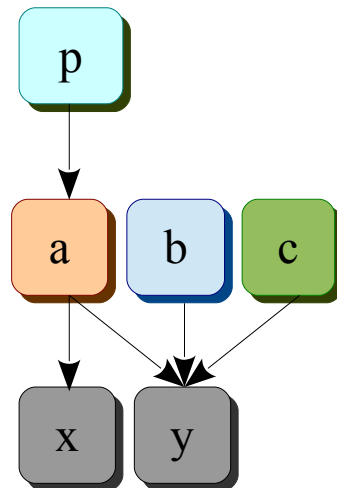
# Unifying Model Two

Program

```
a = &x;  
b = &y;  
p = &a;  
c = b;  
*p = c;
```

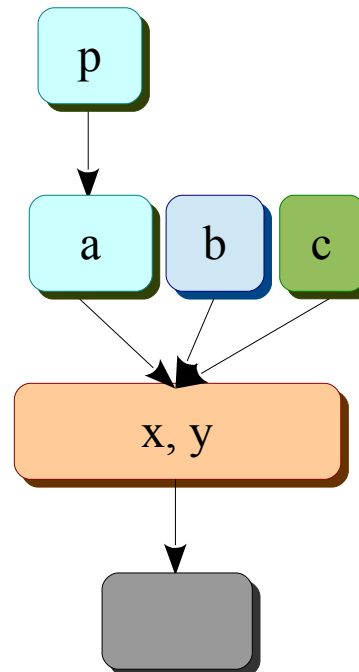
Andersen's

```
a → {x, y}  
b → {y}  
c → {y}  
p → {a}
```



Steensgaard's

```
a → {x, y}  
b → {x, y}  
c → {x, y}  
p → {a}
```



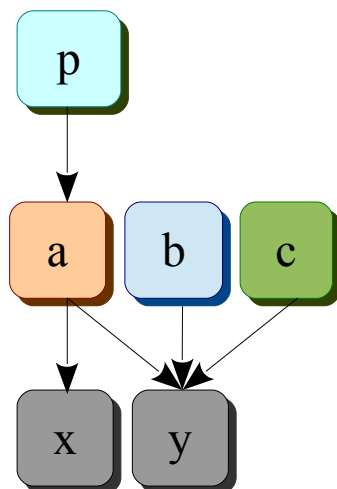
# Unifying Model Two

Program

```
a = &x;  
b = &y;  
p = &a;  
c = b;  
*p = c;
```

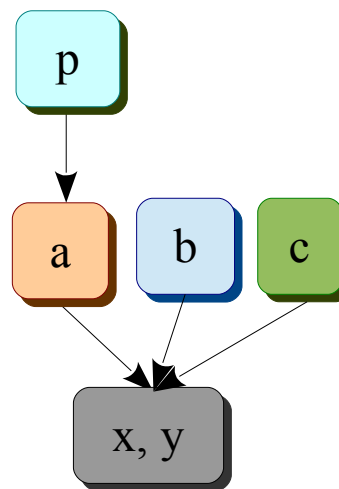
Andersen's

```
a → {x, y}  
b → {y}  
c → {y}  
p → {a}
```



Steensgaard's

```
a → {x, y}  
b → {x, y}  
c → {x, y}  
p → {a}
```



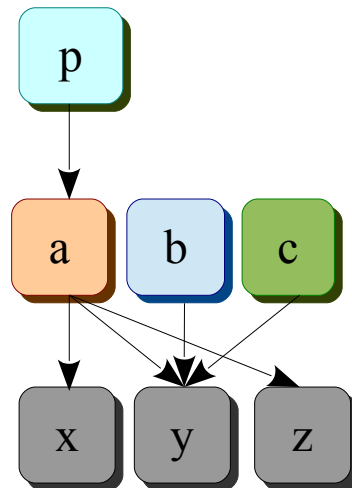
# Unifying Model Two

Program

```
a = &x;  
b = &y;  
p = &a;  
c = b;  
*p = c;  
a = &z;
```

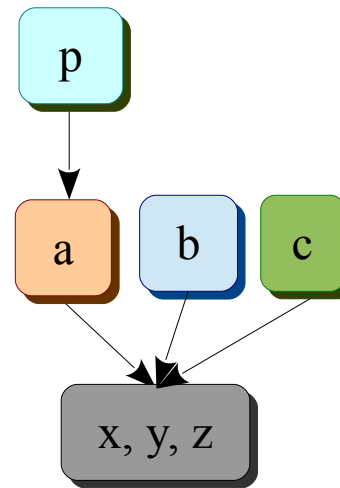
Andersen's

```
a → {x, y, z}  
b → {y}  
c → {y}  
p → {a}
```



Steensgaard's

```
a → {x, y, z}  
b → {x, y, z}  
c → {x, y, z}  
p → {a}
```





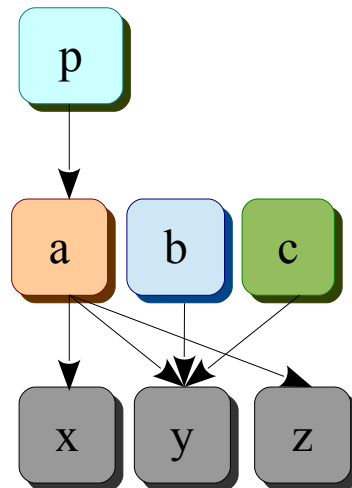
# Unifying Model Two

Program

```
a = &x;
b = &y;
p = &a;
c = b;
*p = c;
a = &z;
```

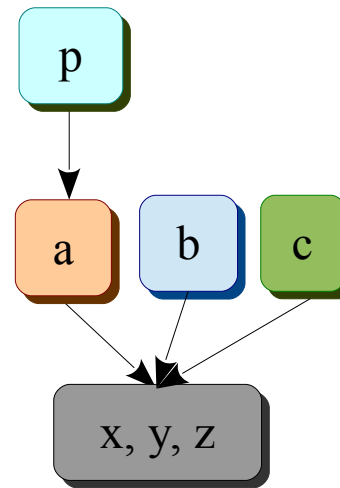
Andersen's

```
a → {x, y, z}
b → {y}
c → {y}
p → {a}
```



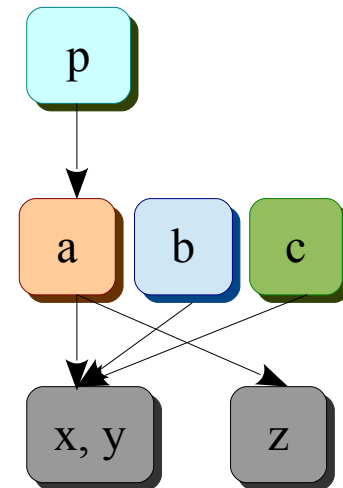
Steensgaard's

```
a → {x, y, z}
b → {x, y, z}
c → {x, y, z}
p → {a}
```



In between

```
a → {x, y, z}
b → {x, y}
c → {x, y}
p → {a}
```



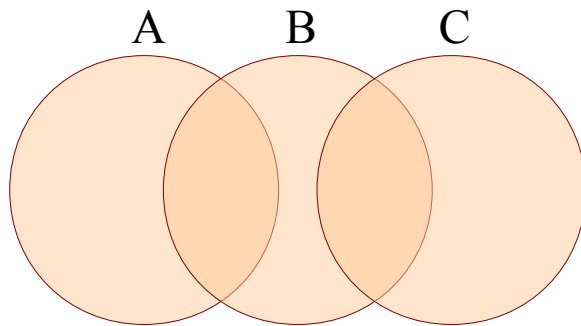
What if x and z are merged?

# Unifying Model One

- Steensgaard's unification can be viewed as equality of points-to sets.
- Thus, if  $a = b$  merges their points-to sets and  $b = c$  merges their points-to sets, then  $a$  and  $c$  become aliases!
- Remember: aliasing is not transitive.
- So, unification adds transitivity to the aliasing relation.

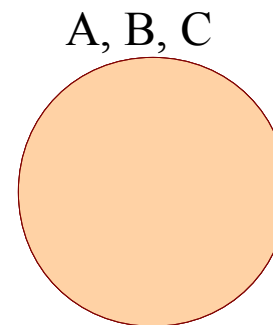
# Unifying Model One

Andersen's



Aliasing is non-transitive

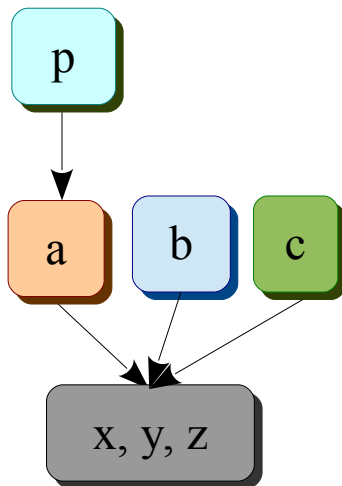
Steensgaard's



Aliasing becomes transitive

# Back to Steensgaard's

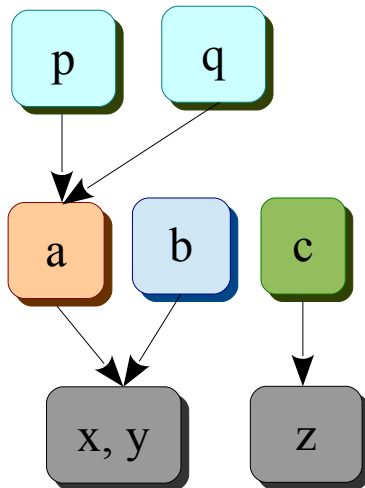
- Aliasing relation is transitive.
- We know that it is also reflexive and symmetric.
- This means aliasing becomes an equivalence relation.
- Steensgaard's unification partitions pointers into equivalent sets.



All predecessors of a node form a partition.  
The equivalence sets are  $\{p\}$ ,  $\{a, b, c\}$ ,  $\{x, y, z\}$ .

# Back to Steensgaard's

- Aliasing relation is transitive.
- We know that it is also reflexive and symmetric.
- This means aliasing becomes an equivalence relation.
- Steensgaard's unification partitions pointers into equivalent sets.



All predecessors of a node form a partition.  
The equivalence sets are  $\{p, q\}$ ,  $\{a, b\}$ ,  $\{c\}$ ,  $\{x, y\}$ ,  $\{z\}$ .

# Realizable Facts

Statements	Andersen's points-to
$a = \&c$	$a \rightarrow \{b, c\}$
$b = \&a$	$b \rightarrow \{a, b, c\}$
$c = \&b$	$c \rightarrow \{b\}$
$b = a$	$d \rightarrow \{a, b, c\}$
$*b = c$	
$d = *a$	

A **realizability sequence** is a sequence of statements such that a given points-to fact is satisfied.

The realizability sequence for  $b \rightarrow c$  is  $a = \&c, b = a$ .

The realizability sequence for  $a \rightarrow b$  is  $c = \&b, b = \&a, *b = c$ .

**Classwork:** What is the realizability sequence for  $d \rightarrow a$ ?

**Classwork:** What is the realizability sequence for  $d \rightarrow c$ ?

$a \rightarrow b$  and  $b \rightarrow c$  are realizable individually, but not simultaneously.

```

int *fun(int *a, int *b) {
    int *c;
    if (*a == *b) {
        c = b;
    } else {
        c = a;
    }
    return c;
}
int *g;
void main() {
    int *x, *y, *z, **w;
    int m = 0, n = 1;
    char *str;
    x = &m;
    y = &n;
    str = (char *)malloc(30);
    w = (int *)&str;
    if (m < n) {
        strcpy(str, "m is smaller\n");
        z = fun(y, x);
    } else {
        printf("m is >= n\n");
        w = &x;
        *w = fun(x, y);
    }
    printf("**w=%d\n", **w);
}

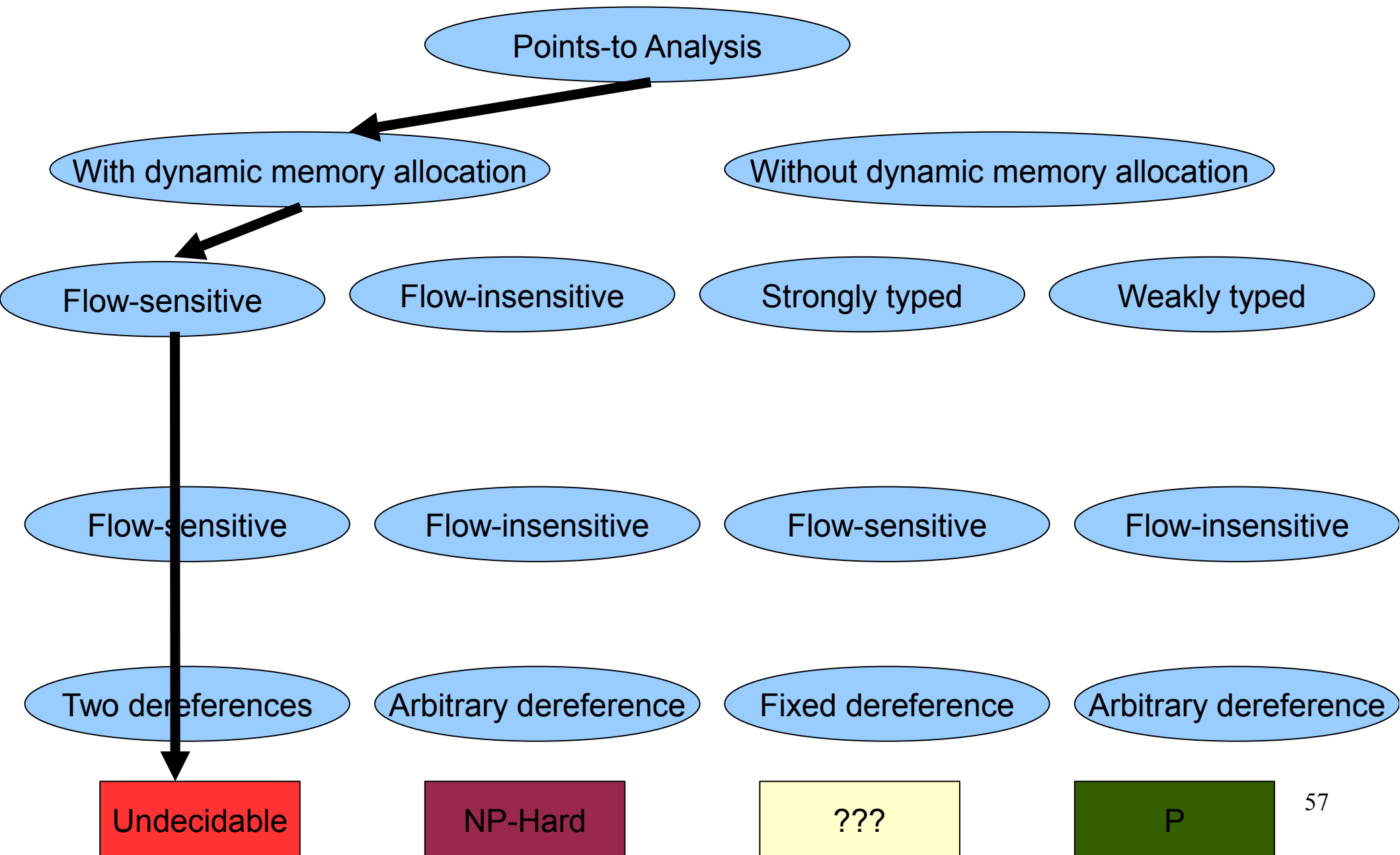
```

- How do we take care of malloc?
- How do we take care of type-casts?
- Find the set of normalized statements for intra-procedural pointer analysis.
- Perform intra-procedural Andersen's analysis.
- How do we take care of strcpy and printf? How about the global g?
- Perform inter-procedural context-insensitive Andersen's analysis.
- Perform Steensgaard's analysis.

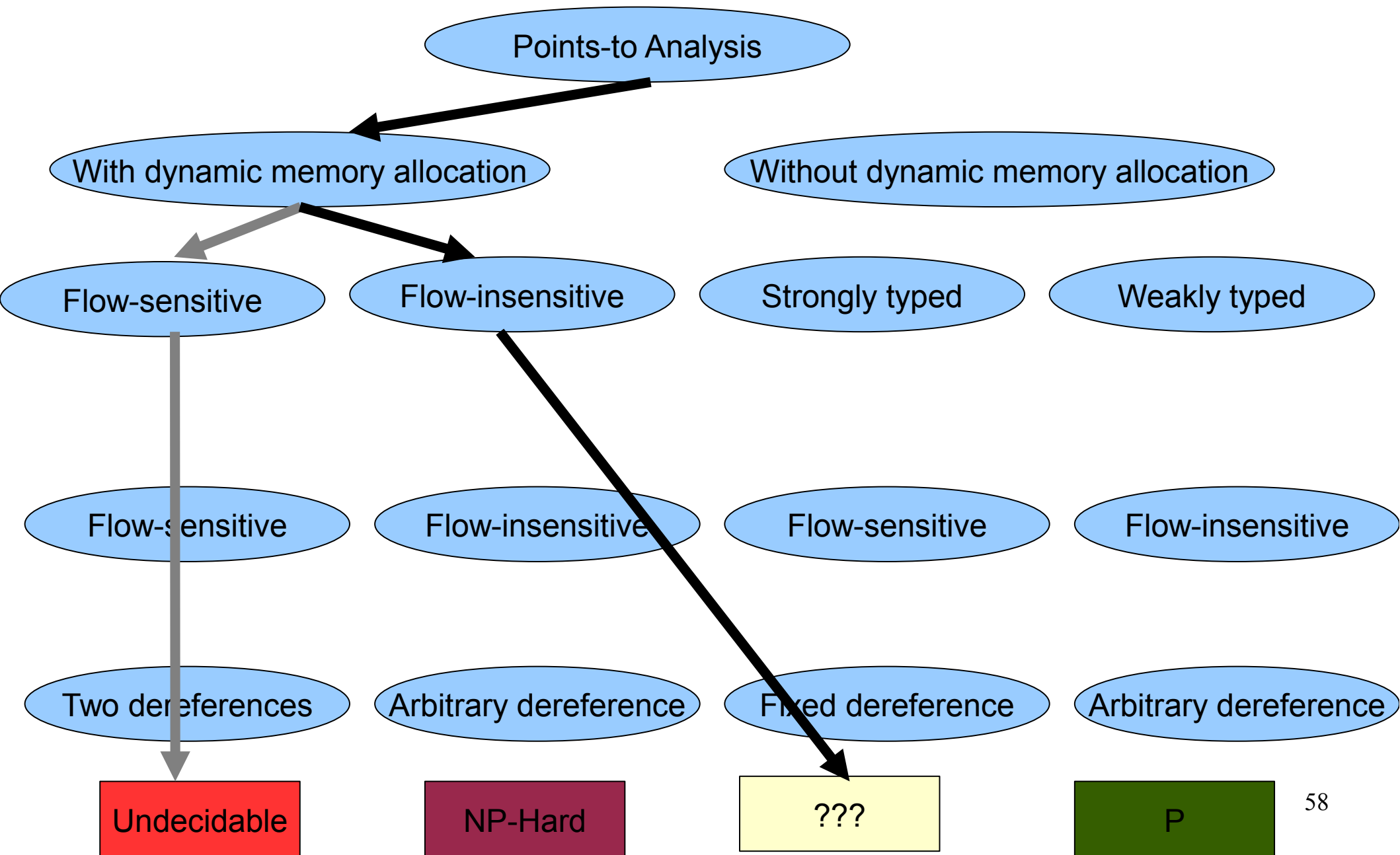
# Extra



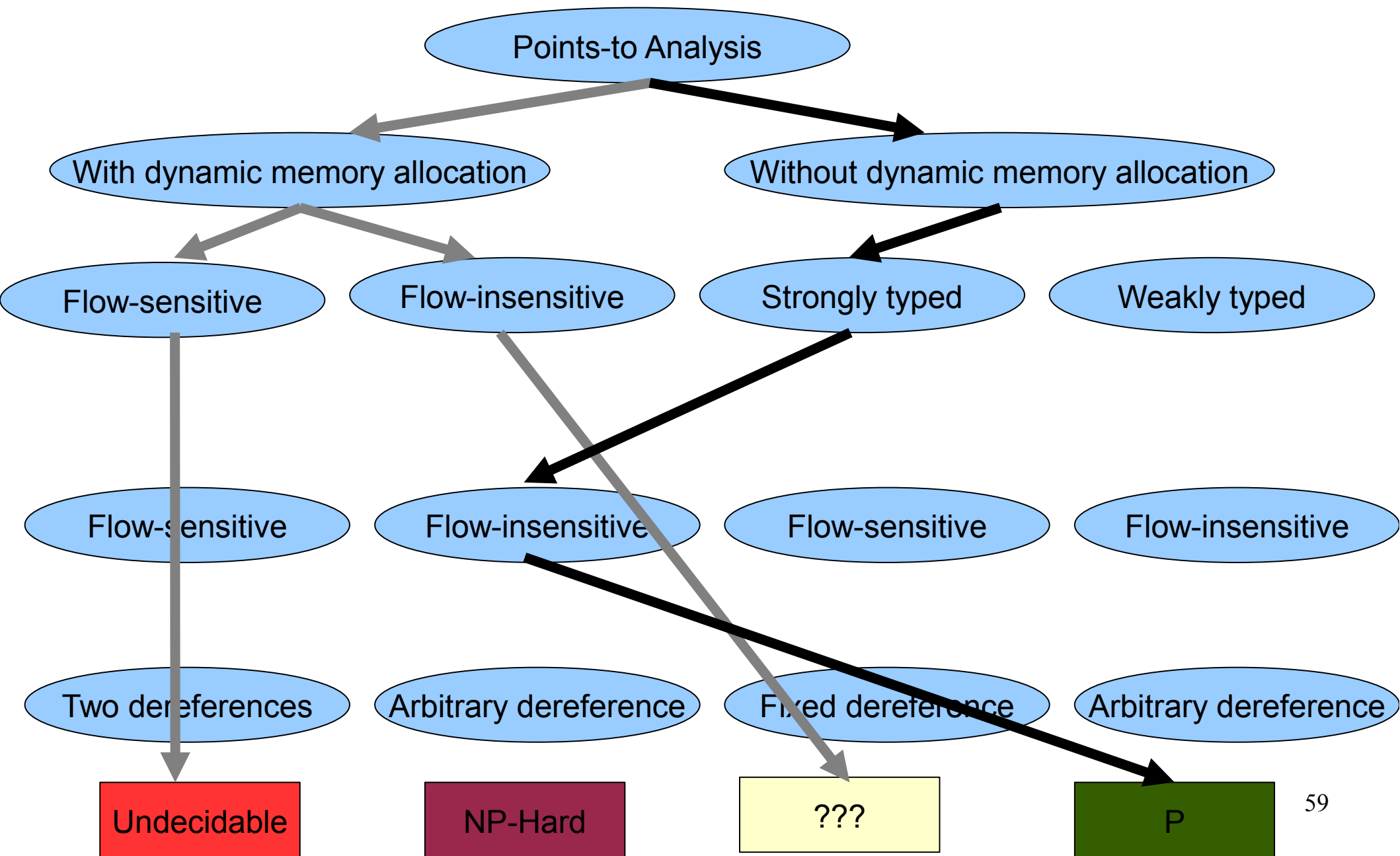
# Complexity of Points-to Analysis



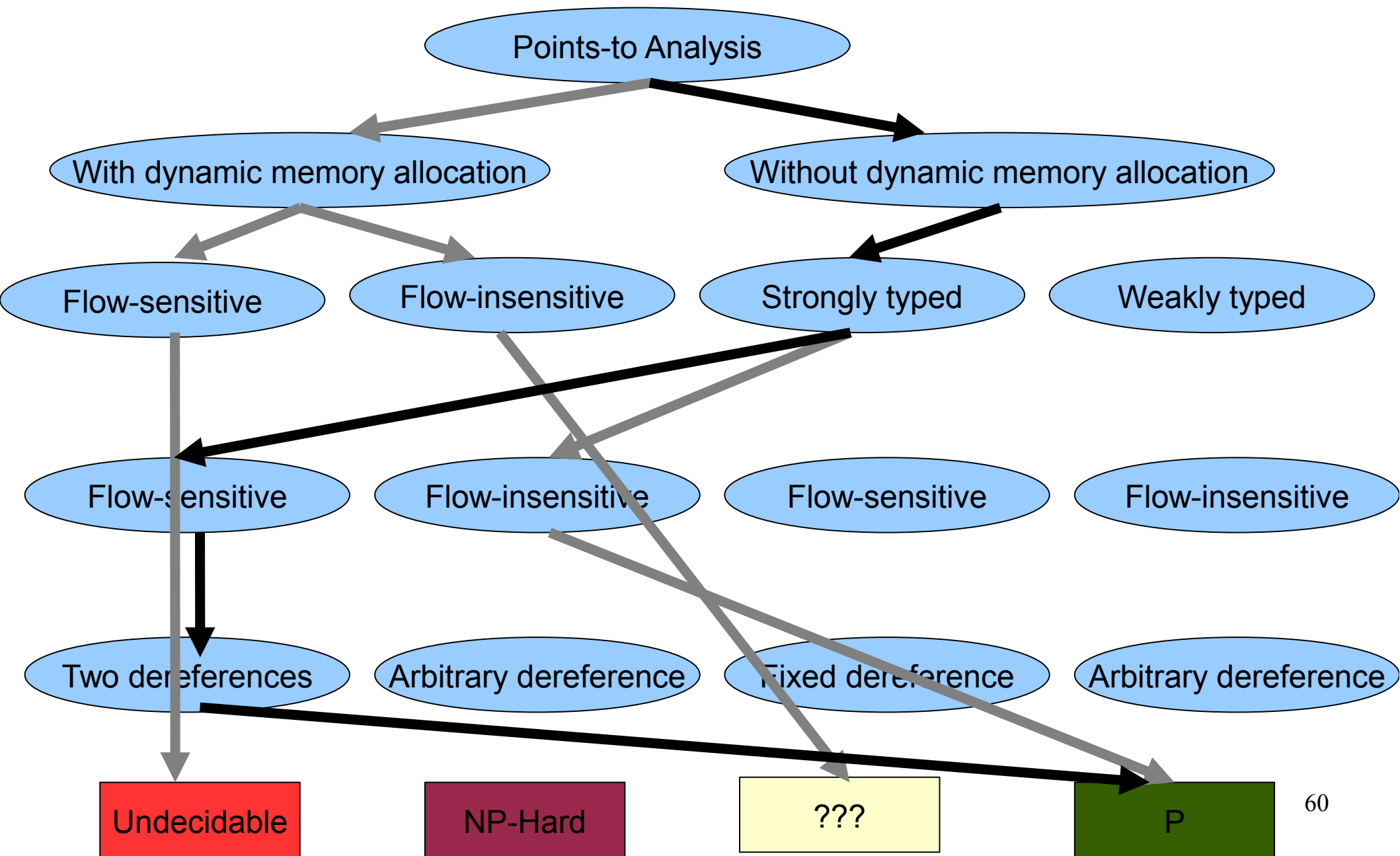
# Complexity of Points-to Analysis



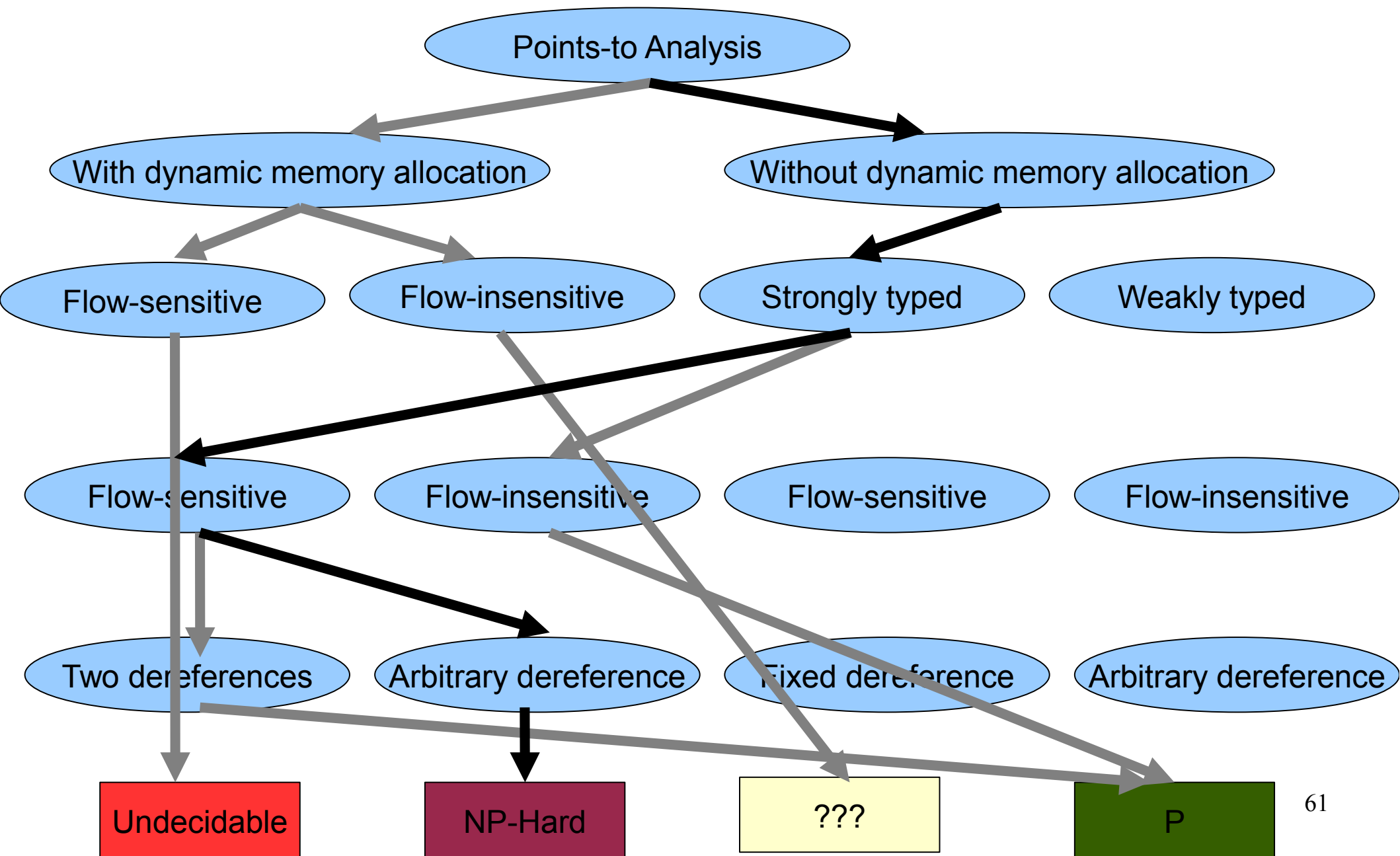
# Complexity of Points-to Analysis



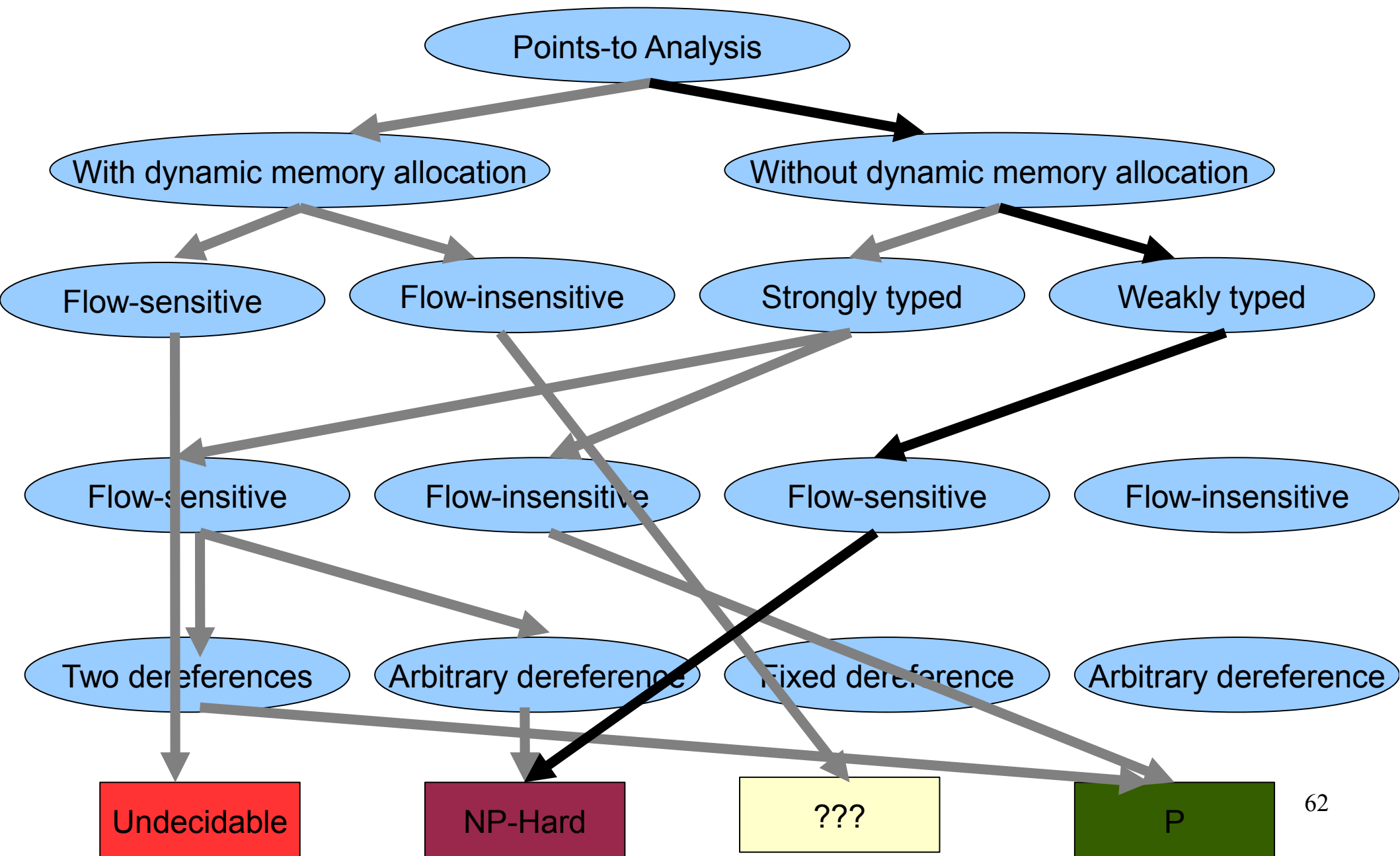
# Complexity of Points-to Analysis



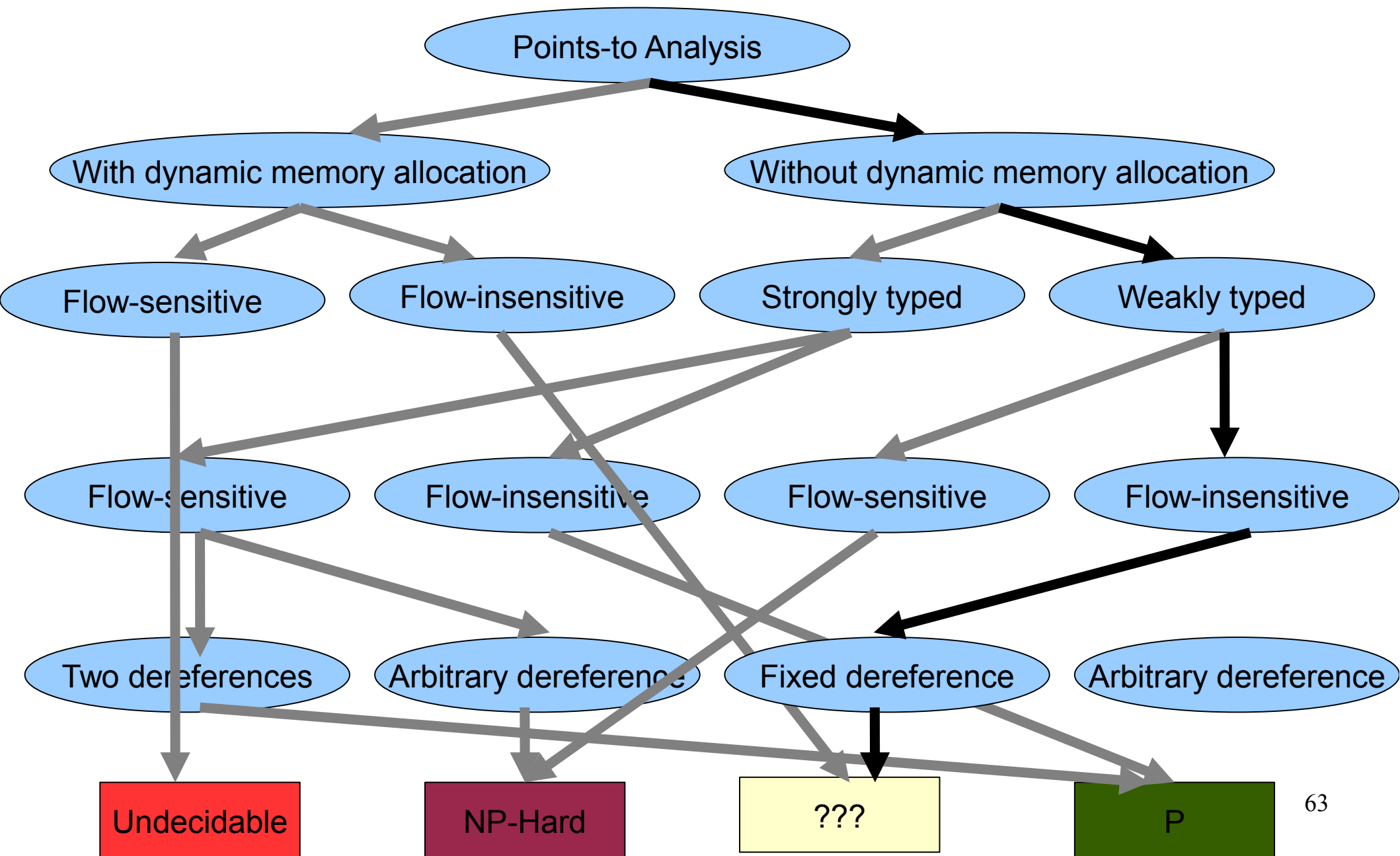
# Complexity of Points-to Analysis



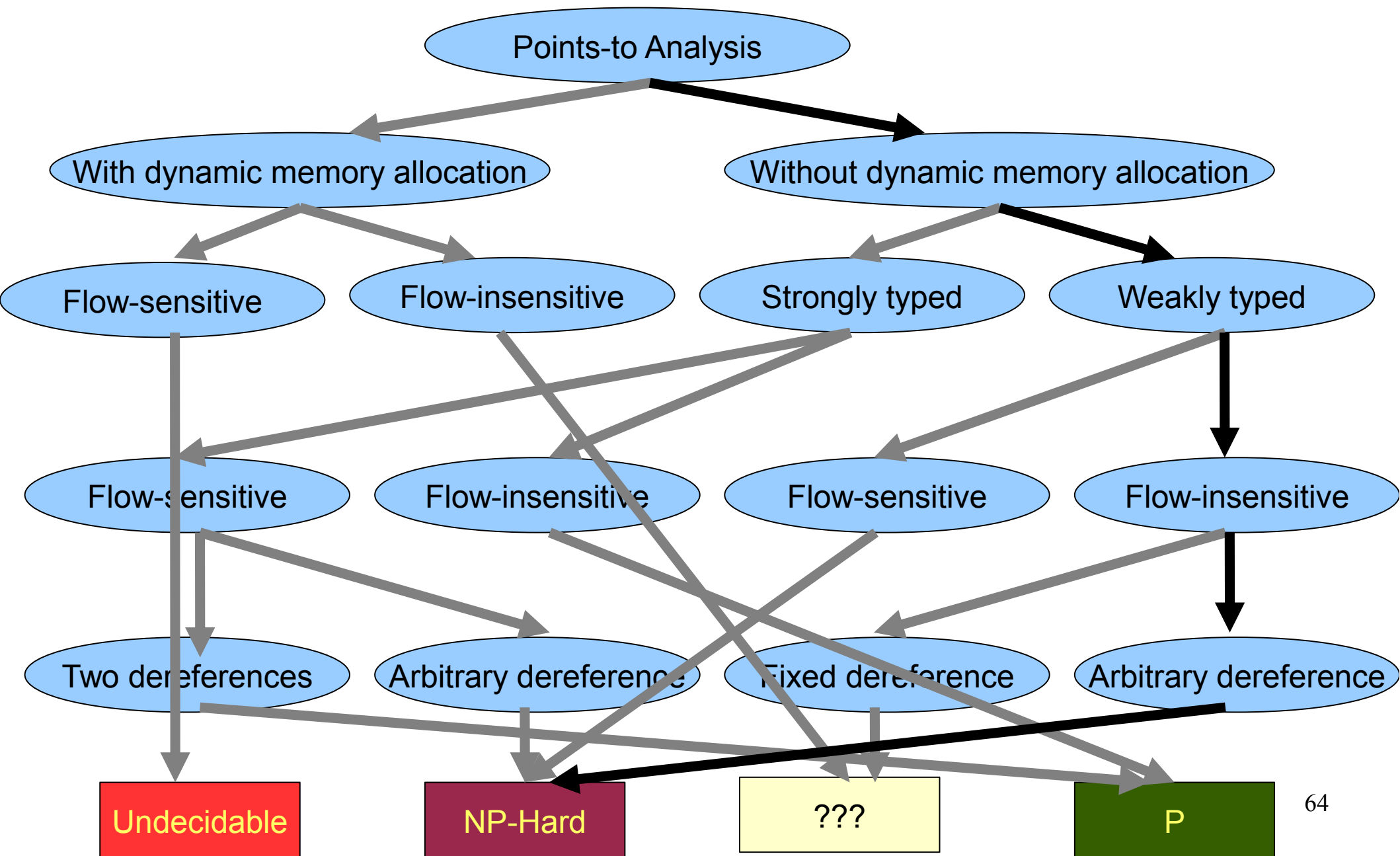
# Complexity of Points-to Analysis



# Complexity of Points-to Analysis



# Complexity of Points-to Analysis





# Related Work

		Precision ←	
		Context-Sensitive	Context-Insensitive
Precision ↑	Flow-Sensitive	Landi, Ryder 92 Choi et al. 93 Emami et al. 94 Reps et al. 95 Hind et al. 99 Kahlon 08	Zheng 98 Hardekopf, Lin 09
	Flow-insensitive	Liang, Harrold 99 Whaley, Lam 04 Zhu, Calman 04 Lattner et al. 07	Andersen 94 Steensgaard 96 Shapiro, Horwitz 97 Fahndrich et al. 98 Das 00 Rountev, Chandra 00 Berndl et al. 03 Hardekopf, Lin 07 Pereira, Berlin 09 Mendez-Lojo 10
	Surveys	Hind, Pioli 00 Qiang, Wu 06	