

# Shape Analysis

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CS6843 Program Analysis  
IIT Madras  
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# Learning Outcomes

- Limitations of pointer analysis
- Identify lists
- Identify trees, DAGs, cyclic graphs
- Identifying rotations
- List reversal and other transformations

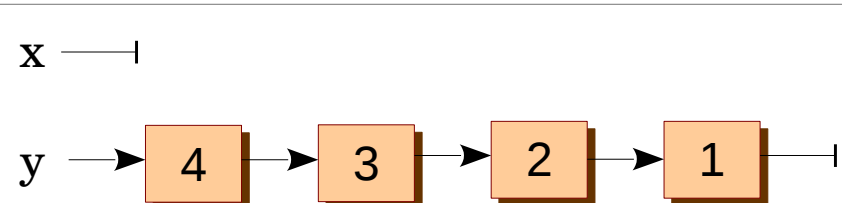
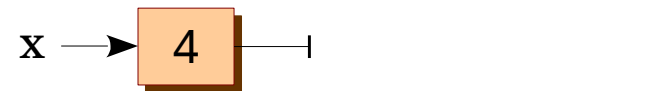
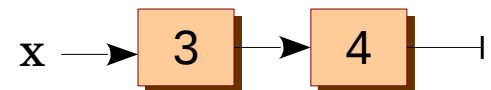
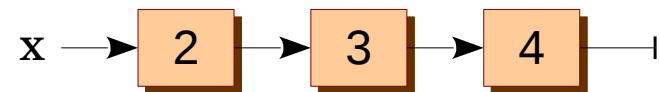
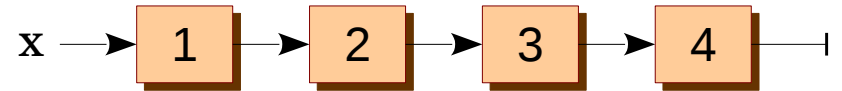
# Limitations of Pointer Analysis

```
listReverse(List x) {  
  assert("x is an acyclic singly linked list");  
  
  for (y = null; x;) {  
    t = y;  
    y = x;  
    x = x→next;  
    y→next = t;  
  }  
  x = y;  
  t = null;  
  y = null;  
}
```

We want to check if x points to a singly linked list at the end of listReverse.

That is,

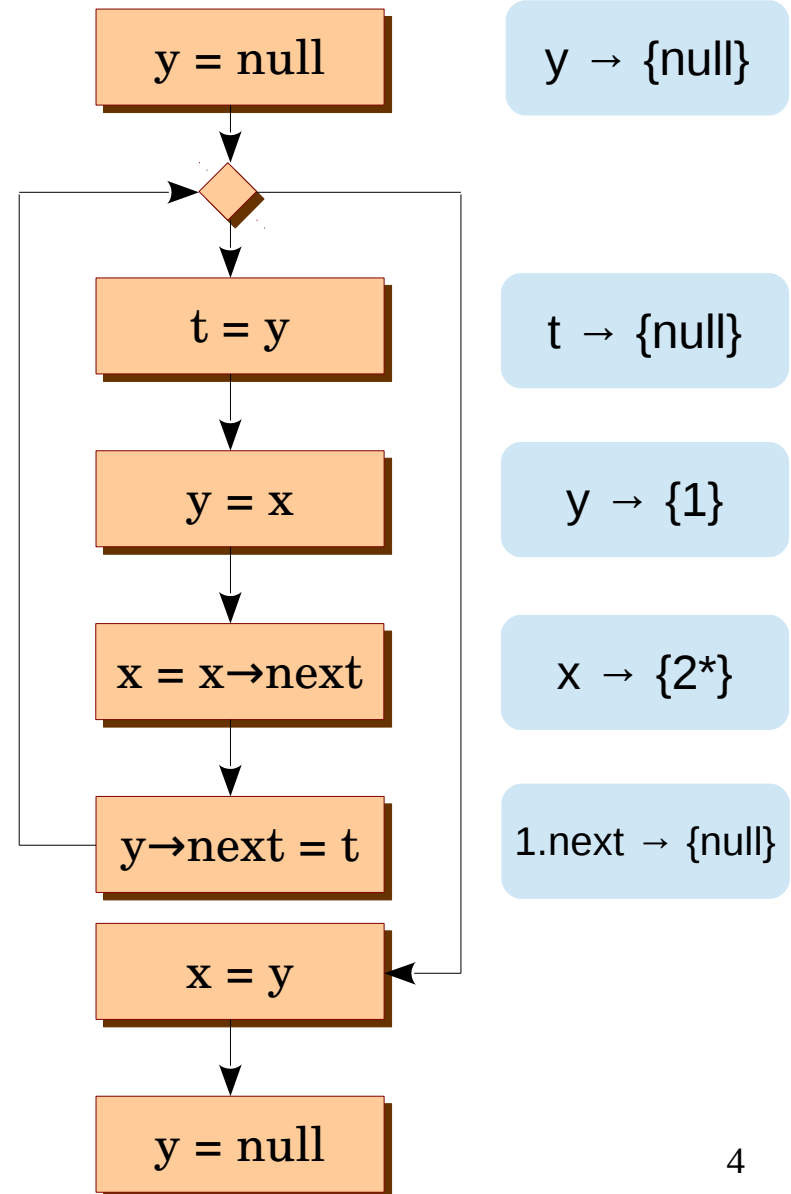
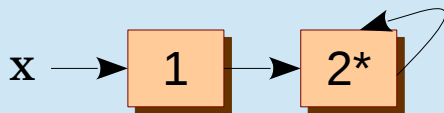
$x \rightarrow \{4\}, 4.\text{next} \rightarrow \{3\}, \dots, 1.\text{next} \rightarrow \{\text{null}\}$



# Limitations of Pointer Analysis

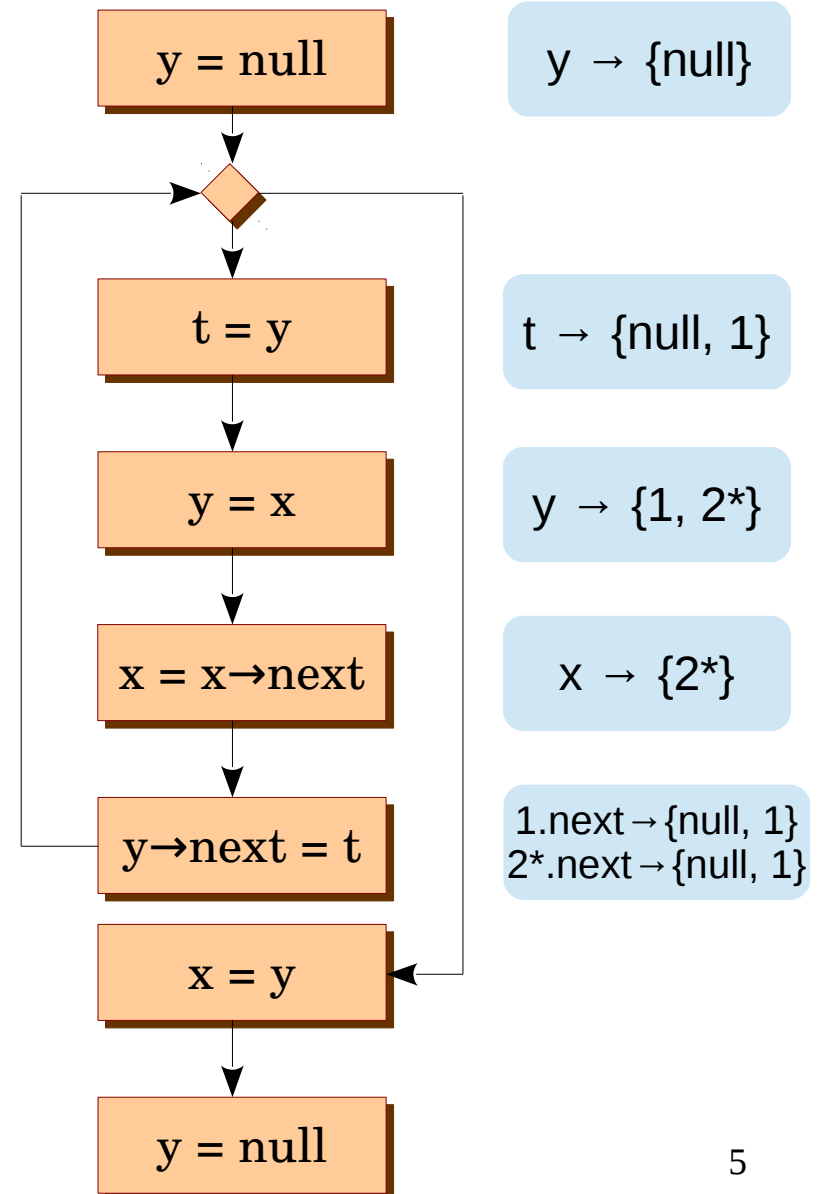
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  }  
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  t = null;  
  y = null;  
}
```

We model the list as a two node structure.



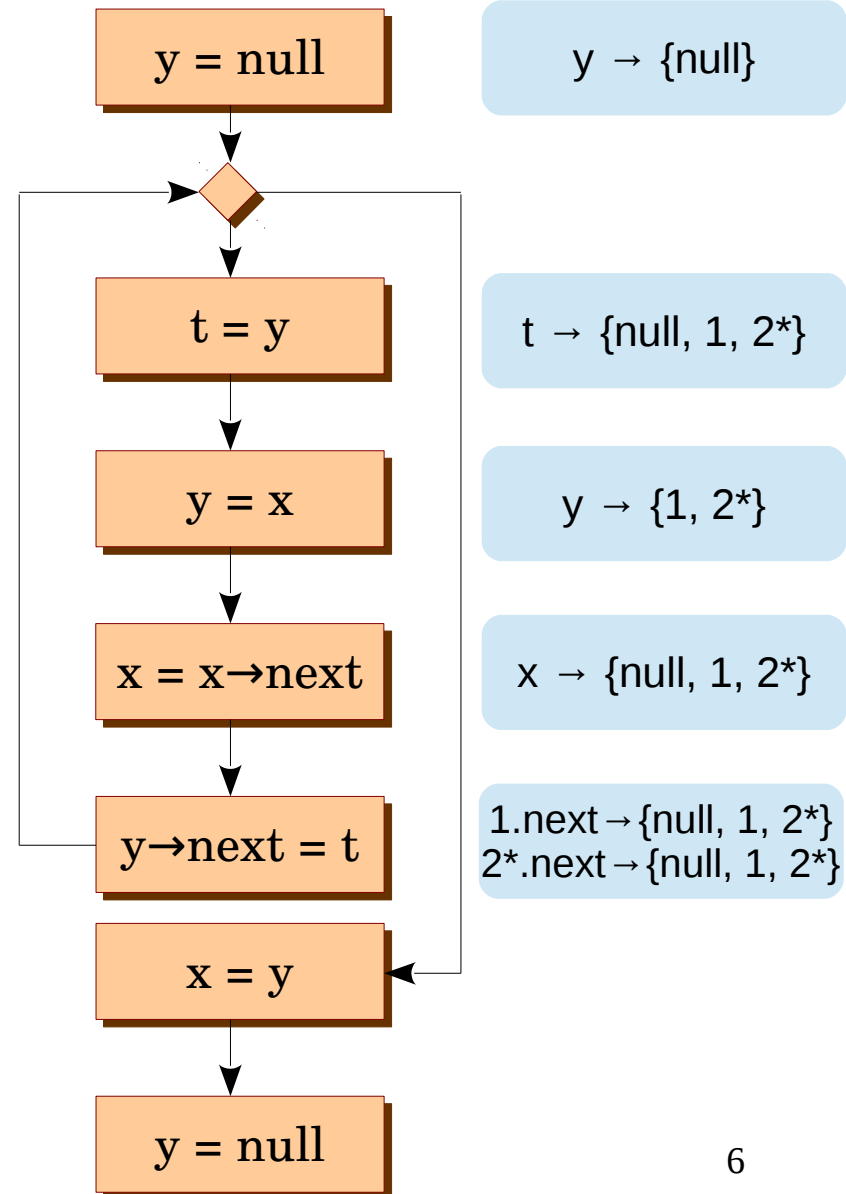
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  y = null;  
}
```



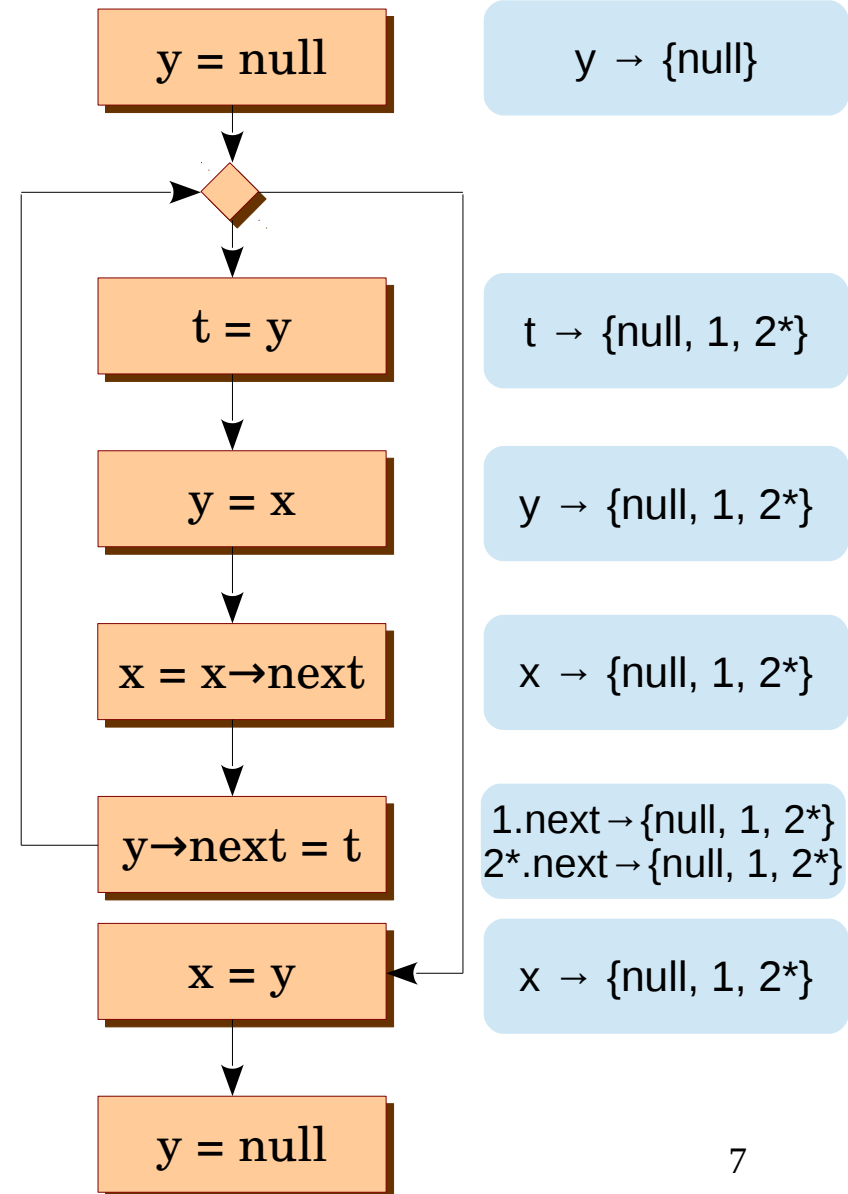
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  }  
  x = y;  
  t = null;  
  y = null;  
}
```



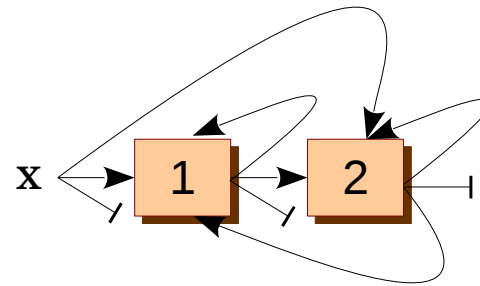
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# Limitations of Pointer Analysis

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    x = x→next;  
    y→next = t;  
  }  
  x = y;  
  t = null;  
  y = null;  
}
```



$y \rightarrow \{\text{null}\}$

$t \rightarrow \{\text{null}, 1, 2^*\}$

$y \rightarrow \{\text{null}, 1, 2^*\}$

$x \rightarrow \{\text{null}, 1, 2^*\}$

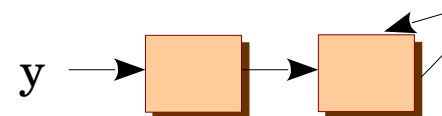
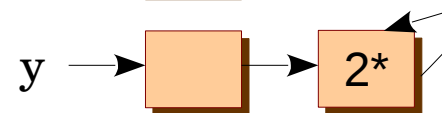
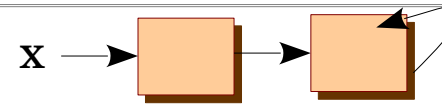
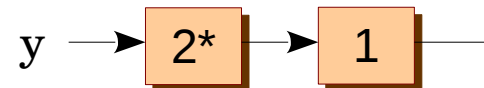
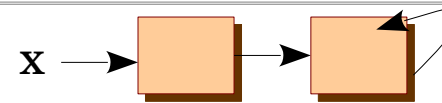
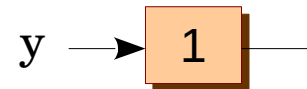
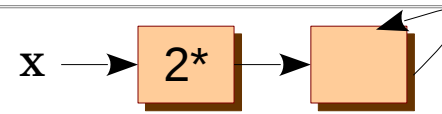
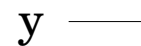
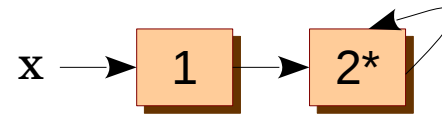
$1.\text{next} \rightarrow \{\text{null}, 1, 2^*\}$   
 $2^*.\text{next} \rightarrow \{\text{null}, 1, 2^*\}$

$x \rightarrow \{\text{null}, 1, 2^*\}$

# Shape Analysis

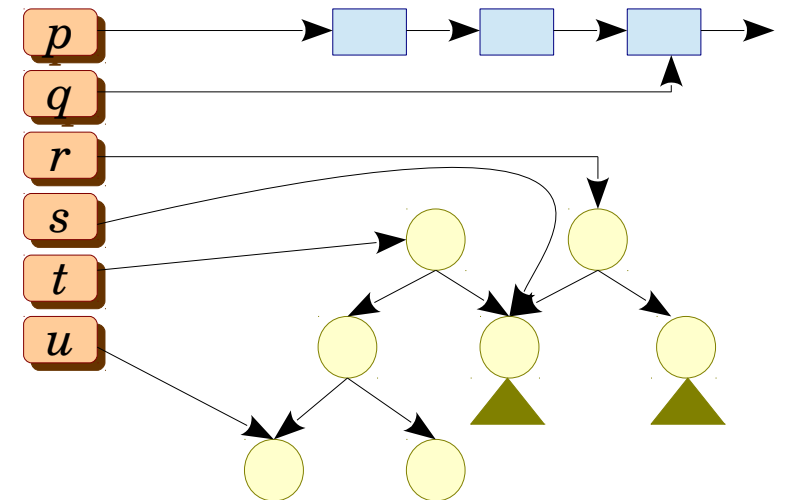
```
listReverse(List x) {  
  assert("x is an acyclic singly linked list");  
  
  for (y = null; x;) {  
    t = y;  
    y = x;  
    x = x→next;  
    y→next = t;  
  }  
  x = y;  
  t = null;  
  y = null;  
}
```

Maintain additional information  
with 2\* that it is acyclic.  
Use the fact that node removal  
maintains acyclicity.



# Shape Analysis

- Identify structural / topological properties of a data structure under manipulation.
- Usually categorized as slist, tree, DAG or cycle.
- Precision reduces along slist → tree → DAG → cycle.
- **Challenges:**
  - Size of the data structure is statically unknown.
  - Same pointer iterates through various nodes.
  - Shape needs to be determined from program variables.



# Classwork

- Identify shapes of data structures.

```
n = sizeof(node);  
p = malloc(n);  
q = p;  
q→data = 20;  
q→next = NULL;
```

```
n = sizeof(node);  
p = malloc(n);  
q = p;  
q→data = 20;  
q→next = p;
```

```
n = sizeof(node);  
p = createTree(); SarthakPandey();  
q = p;  
q→data = 20;  
q→left = p→right;
```

## Takeaways:

- Analysis is restricted to program-defined pointers.
- We cannot afford to represent the heap!
- Meanings of function or variables names are useless for the analysis.

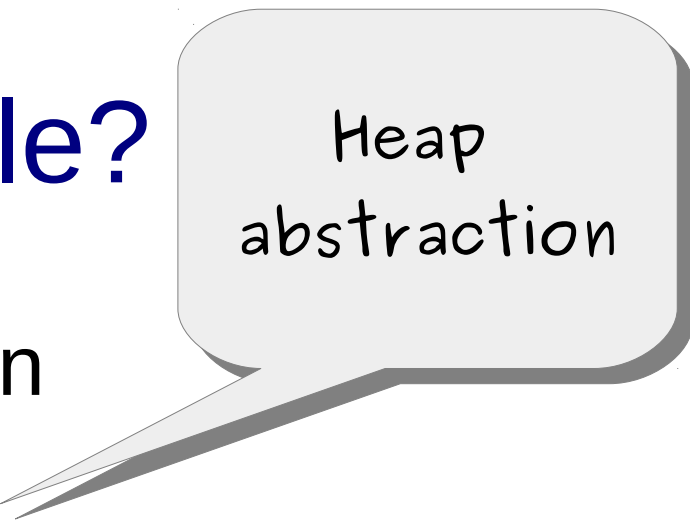
# Classwork

- Identify shapes of data structures.

```
q = malloc(...);  
q→f = malloc(...);  
t = malloc(...);  
t→g = malloc(...);  
x = t→g;  
x→g = q→f;  
p = q→f;  
p→f = t;
```

There could be a long chain of links from  $t$  to  $q \rightarrow f$ .

# Tree, DAG, Cycle?



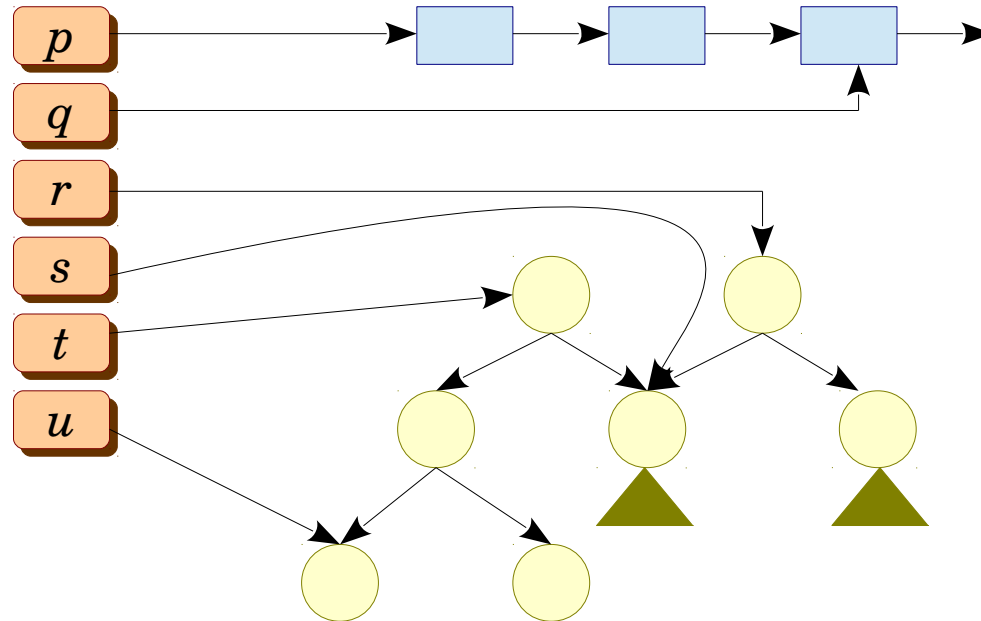
Heap  
abstraction

- Proposed by Ghiya and Hendren
- Maintains three data structures:
  - **Shape** reachable from a pointer.
  - **Interference** matrix: encodes common reachability
  - **Direction** matrix: encodes direct reachability
- Performs iterative **data-flow analysis** to update shape, I and D information

## *Takeaways:*

- *Analysis is restricted to program-defined pointers.*
- *We cannot afford to represent the heap!*
- *Meanings of function or variables names are useless for the analysis.*

# Tree, DAG, Cycle?



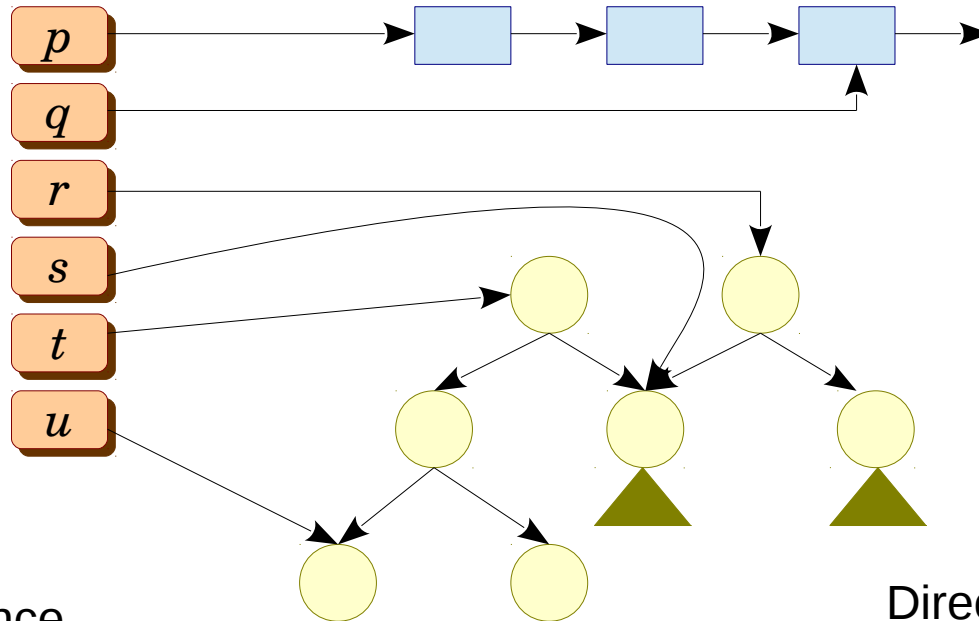
## Interference( $x, y$ )

Whether  $x$  and  $y$  have a common reachable node.

## Direction( $x, y$ )

Whether  $x$  has a path to the node pointed to by  $y$ .

# Tree, DAG, Cycle?



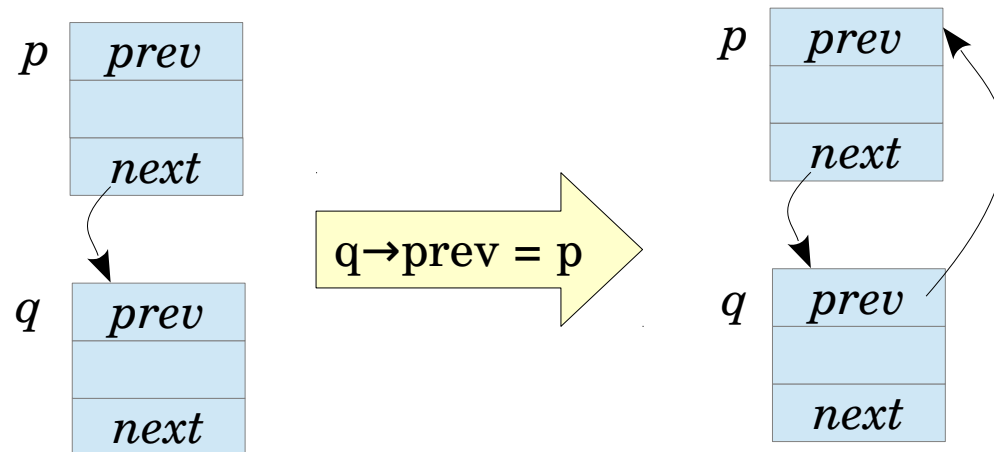
Interference

|     | $p$ | $q$      | $r$ | $s$      | $t$      | $u$      |
|-----|-----|----------|-----|----------|----------|----------|
| $p$ | 1   | <b>1</b> | 0   | 0        | 0        | 0        |
| $q$ |     | 1        | 0   | 0        | 0        | 0        |
| $r$ |     |          | 1   | <b>1</b> | <b>1</b> | 0        |
| $s$ |     |          |     | 1        | <b>1</b> | 0        |
| $t$ |     |          |     |          | 1        | <b>1</b> |
| $u$ |     |          |     |          |          | 1        |

Direction

|     | $p$      | $q$      | $r$      | $s$      | $t$      | $u$      |
|-----|----------|----------|----------|----------|----------|----------|
| $p$ | 1        | <b>1</b> | 0        | 0        | 0        | 0        |
| $q$ | <b>0</b> | 1        | 0        | 0        | 0        | 0        |
| $r$ | 0        | 0        | 1        | <b>1</b> | <b>0</b> | 0        |
| $s$ | 0        | 0        | <b>0</b> | 1        | <b>0</b> | 0        |
| $t$ | 0        | 0        | <b>0</b> | <b>1</b> | 1        | <b>1</b> |
| $u$ | 0        | 0        | 0        | 0        | <b>0</b> | 1        |

# Shape Estimation Example



$D[p][q] = 1, D[q][p] = 0$   
 $p.shape = \text{Tree}$   
 $q.shape = \text{Tree}$

$D[p][q] = 1, D[q][p] = \mathbf{1}$   
 $p.shape = \mathbf{Cycle}$   
 $q.shape = \mathbf{Cycle}$

## Classwork:

- What is the data-flow information?
- Write down the statements of interest for this shape analysis.
- How are  $D$ ,  $I$  and shape initialized?

# Relevant Statements and Inference Rules

`p = malloc(...)`

`p = q`

`p = q → f`

`p = &(q → f)`

`p = q op k`

`p = null`

`p->f = q`

`p->f = null`

Inference rules define local processing; they generate and kill data-flow facts.

What are the gen and kill sets for these relevant statements?

# Inference Rules

**p = malloc(...)**

p = q

p = q → f

p = &(q → f)

p = q op k

p = null

p → f = q

p → f = null

$D\_kill = \{D[p][s] \mid D[p][s] == 1\} \cup \{D[s][p] \mid D[s][p] == 1\}$

$I\_kill = \{I[p][s] \mid I[p][s] == 1\}$

$D\_gen = \{D[p][p]\}$

$I\_gen = \{I[p][p]\}$

p.shape = Tree

# Inference Rules

|                               |                                                                                    |
|-------------------------------|------------------------------------------------------------------------------------|
| <code>p = malloc(...)</code>  | <code>D_kill</code> and <code>I_kill</code> sets same as for allocation statement. |
| <b><code>p = q</code></b>     |                                                                                    |
| <code>p = q → f</code>        | $D\_gen = \{D[s][p] \mid D[s][q] \text{ and } s \neq p\} \cup$                     |
| <code>p = &amp;(q → f)</code> | $\{D[p][s] \mid D[q][s] \text{ and } s \neq p\} \cup$                              |
| <code>p = q op k</code>       | $\{D[p][p] \mid D[q][q]\}$                                                         |
| <code>p = null</code>         | $I\_gen = \{I[p][s] \mid I[q][s] \text{ and } s \neq p\} \cup$                     |
|                               | $\{I[p][p] \mid I[q][q]\}$                                                         |
| <code>p → f = q</code>        |                                                                                    |
| <code>p → f = null</code>     | <code>p.shape = q.shape</code>                                                     |

The implementation should create new D/I matrices from their current copies. In-situ update would lead to unsound or imprecise analysis.

# Inference Rules

$p = \text{malloc}(\dots)$

$p = q$

**$p = q \rightarrow f$**

$p = \&(q \rightarrow f)$

$p = q \text{ op } k$

$p = \text{null}$

$p \rightarrow f = q$

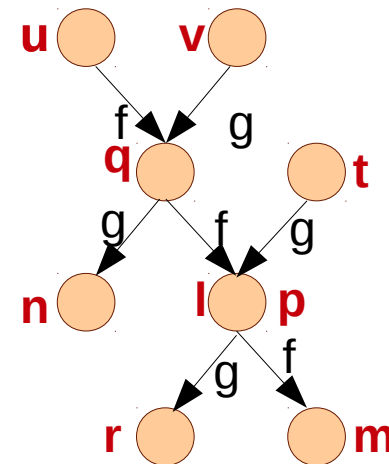
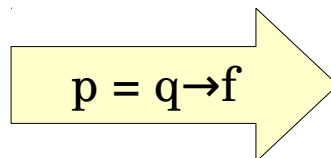
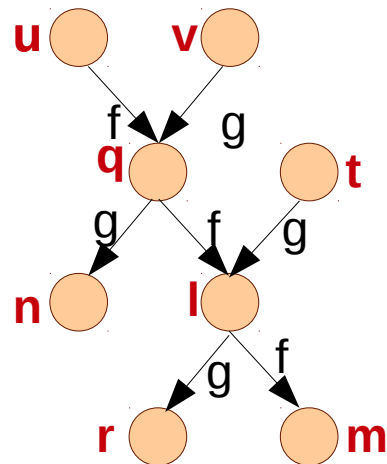
$p \rightarrow f = \text{null}$

$D\_kill$  and  $I\_kill$  sets same as for allocation statement.

$$D\_gen = \{D[s][p] \mid D[s][q] \text{ and } s \neq p\} \cup \\ \{D[p][s] \mid D[q][s] \text{ and } s \neq p \text{ and } s \neq q\} \cup \\ \{D[p][p] \mid D[q][q]\} \cup \\ \{D[p][q] \mid q.\text{shape} == \text{Cycle}\}$$

$$I\_gen = \{I[p][s] \mid I[q][s] \text{ and } s \neq p\} \cup \\ \{I[p][p] \mid I[q][q]\}$$

$p.\text{shape} = q.\text{shape}$



$D[t][p]$  should be 1.

Which  $D\_gen$  rule sets  $D[t][p]$ ?

# Inference Rules

$p = \text{malloc}(\dots)$

$p = q$

**$p = q \rightarrow f$**

$p = \&(q \rightarrow f)$

$p = q \text{ op } k$

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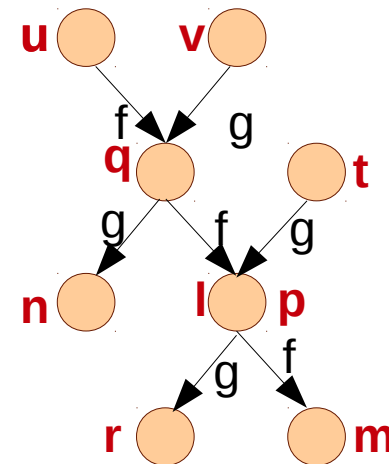
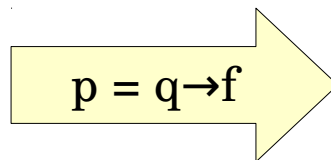
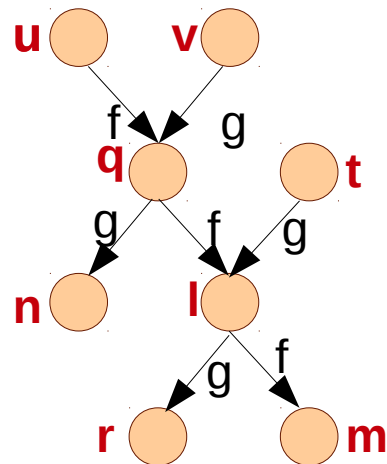
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$D\_kill$  and  $I\_kill$  sets same as for allocation statement.

$$D\_gen = \{D[s][p] \mid \mathbf{I}[s][q] \text{ and } s \neq p\} \cup \\ \{D[p][s] \mid D[q][s] \text{ and } s \neq p \text{ and } s \neq q\} \cup \\ \{D[p][p] \mid D[q][q]\} \cup \\ \{D[p][q] \mid q.\text{shape} == \text{Cycle}\}$$

$$I\_gen = \{I[p][s] \mid I[q][s] \text{ and } s \neq p\} \cup \\ \{I[p][p] \mid I[q][q]\}$$

$p.\text{shape} = q.\text{shape}$



$I[t][q]$  was 1.

Hence,  $D[t][p]$  becomes 1.

# Inference Rules

|                                                                                                                                          |                                                                                                                                                                                                                         |
|------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>p = malloc(...)</code>                                                                                                             | <code>D_kill</code> and <code>I_kill</code> sets same as for allocation statement.                                                                                                                                      |
| <code>p = q</code><br><code><b>p = q → f</b></code><br><code>p = &amp;(q → f)</code><br><code>p = q op k</code><br><code>p = null</code> | $D\_gen = \{D[s][p] \mid \mathbf{I}[s][q] \text{ and } s \neq p\} \cup$ $\{D[p][s] \mid D[q][s] \text{ and } s \neq p \text{ and } s \neq q\} \cup$ $\{D[p][q] \mid q.shape == Cycle\} \cup$ $\{D[p][p] \mid D[q][q]\}$ |
| <code>p → f = q</code><br><code>p → f = null</code>                                                                                      | $I\_gen = \{I[p][s] \mid I[q][s] \text{ and } s \neq p\} \cup$ $\{I[p][p] \mid I[q][q]\}$                                                                                                                               |
|                                                                                                                                          | <code>p.shape = q.shape</code>                                                                                                                                                                                          |

**Classwork:** Find D, I and shape for the following program fragment.

```
q = malloc(...);
p = q;
r = q → f;
```

# Inference Rules

`p = malloc(...)`

`p = q`

`p = q → f`

**`p = &(q → f)`**

**`p = q op k`**

`p = null`

`p → f = q`

`p → f = null`

Processing is the same as for `p = q` statement.  
This means the analysis loses field-sensitivity.

A former work from IITK (Dasgupta, Karkare, Reddy) addresses this issue.

# Inference Rules

`p = malloc(...)`

`D_kill` and `I_kill` sets same as for allocation statement.

`p = q`

`D_gen = { }`

`p = q → f`

`I_gen = { }`

`p = &(q → f)`

`p = q op k`

`p.shape = Tree`

**`p = null`**

`p → f = q`

`p → f = null`

# Inference Rules

$p = \text{malloc}(\dots)$

$p = q$

$p = q \rightarrow f$

$p = \&(q \rightarrow f)$

$p = q \text{ op } k$

$p = \text{null}$

**$p \rightarrow f = q$**

$p \rightarrow f = \text{null}$

$D\_kill = \{ \}, I\_kill = \{ \}$

$D\_gen = \{ D[r][s] \mid D[r][p] \text{ and } D[q][s] \}$

$I\_gen = \{ I[r][s] \mid I[r][p] \text{ and } I[q][s] \}$

$D[q][p] \text{ and } D[s][q] \Rightarrow s.\text{shape} = \text{Cycle}$

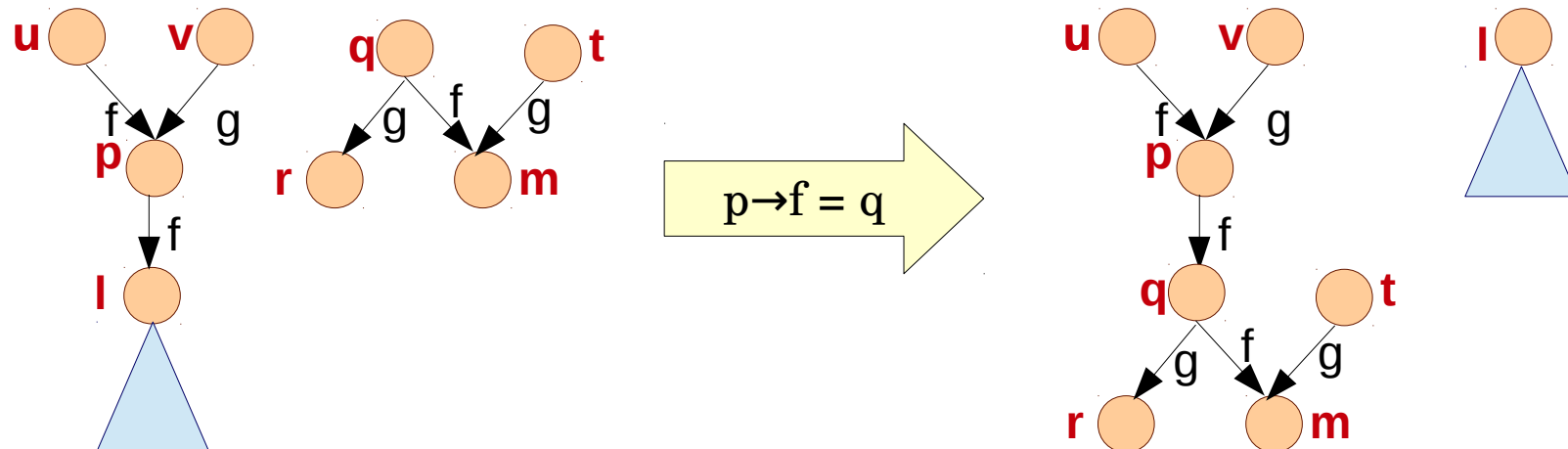
$D[q][p] \text{ and } D[s][p] \Rightarrow s.\text{shape} = \text{Cycle}$

$\neg D[q][p] \text{ and } D[s][p] \text{ and } I[s][q] \text{ and } q.\text{shape} == \text{Tree}$

$\Rightarrow s.\text{shape} = \max(s.\text{shape}, \text{DAG})$

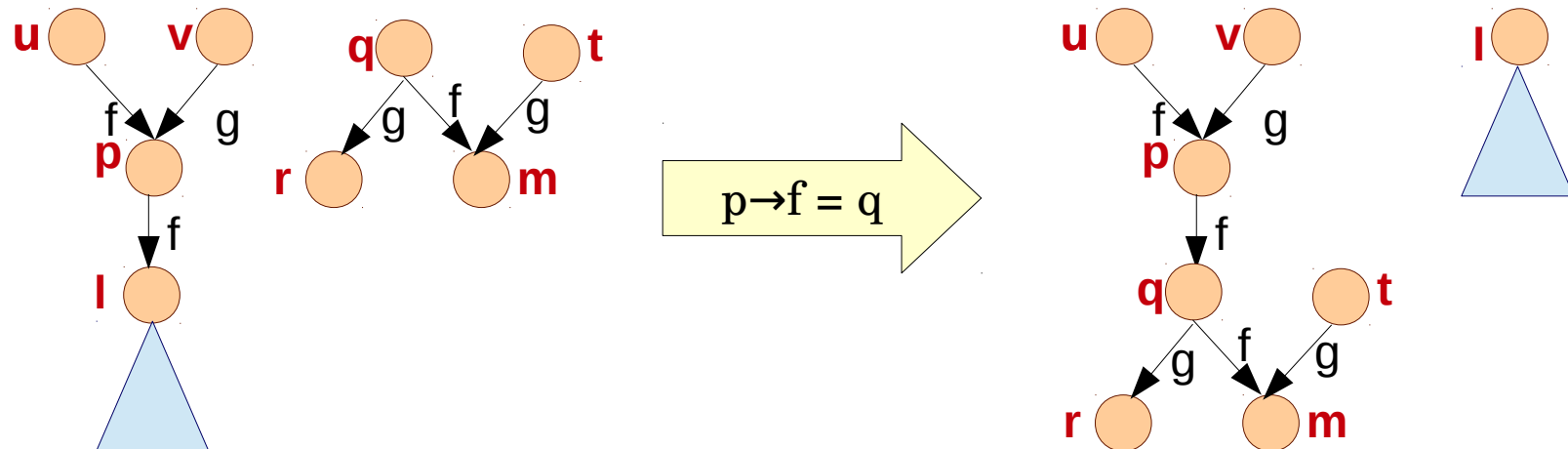
$\neg D[q][p] \text{ and } D[s][p] \text{ and } q.\text{shape} \neq \text{Tree}$

$\Rightarrow s.\text{shape} = \max(s.\text{shape}, q.\text{shape})$



# Inference Rules

|                                         |                                                                                                                                                                        |
|-----------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $p = \text{malloc}(\dots)$              | $D\_kill = \{ \}, I\_kill = \{ \}$                                                                                                                                     |
| $p = q$                                 | $D\_gen = \{ D[r][s] \mid D[r][p] \text{ and } D[q][s] \}$                                                                                                             |
| $p = q \rightarrow f$                   | $I\_gen = \{ I[r][s] \mid \mathbf{D}[r][p] \text{ and } I[q][s] \}$                                                                                                    |
| $p = \&(q \rightarrow f)$               |                                                                                                                                                                        |
| $p = q \text{ op } k$                   | $D[q][p] \text{ and } D[s][q] \Rightarrow s.\text{shape} = \text{Cycle}$                                                                                               |
| $p = \text{null}$                       | $D[q][p] \text{ and } D[s][p] \Rightarrow s.\text{shape} = \text{Cycle}$                                                                                               |
| <b><math>p \rightarrow f = q</math></b> | $\neg D[q][p] \text{ and } D[s][p] \text{ and } I[s][q] \text{ and } q.\text{shape} == \text{Tree}$<br>$\Rightarrow s.\text{shape} = \max(s.\text{shape}, \text{DAG})$ |
| $p \rightarrow f = \text{null}$         | $\neg D[q][p] \text{ and } D[s][p] \text{ and } q.\text{shape} != \text{Tree}$<br>$\Rightarrow s.\text{shape} = \max(s.\text{shape}, q.\text{shape})$                  |



For max() consider  $D[v][r] == 1$ .

# Inference Rules

$p = \text{malloc}(\dots)$

$D\_kill = \{ \}, I\_kill = \{ \}$

$p = q$

$D\_gen = \{ D[r][s] \mid D[r][p] \text{ and } D[q][s] \}$

$p = q \rightarrow f$

$I\_gen = \{ I[r][s] \mid D[r][p] \text{ and } I[q][s] \}$

$p = \&(q \rightarrow f)$

$p = q \text{ op } k$

$p = \text{null}$

$D[q][p] \text{ and } D[s][q] \Rightarrow s.\text{shape} = \text{Cycle}$

$D[q][p] \text{ and } D[s][p] \Rightarrow s.\text{shape} = \text{Cycle}$

$\neg D[q][p] \text{ and } D[s][p] \text{ and } I[s][q] \text{ and } q.\text{shape} == \text{Tree}$

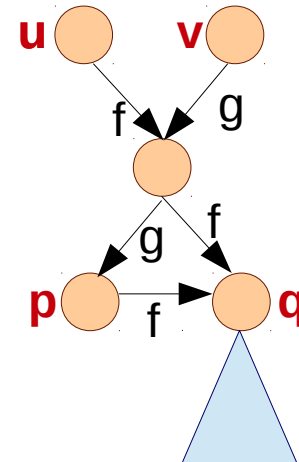
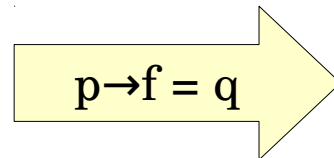
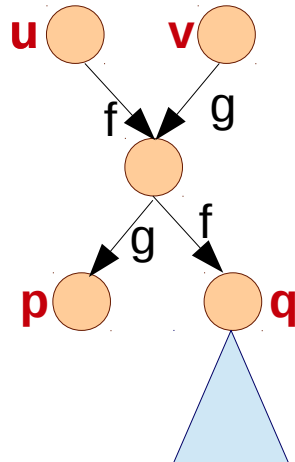
$\Rightarrow s.\text{shape} = \max(s.\text{shape}, \text{DAG})$

$\neg D[q][p] \text{ and } D[s][p] \text{ and } q.\text{shape} \neq \text{Tree}$

$\Rightarrow s.\text{shape} = \max(s.\text{shape}, q.\text{shape})$

**$p \rightarrow f = q$**

$p \rightarrow f = \text{null}$



# Inference Rules

|                                         |                                                                                                                                                                        |
|-----------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $p = \text{malloc}(\dots)$              | $D\_kill = \{ \}, I\_kill = \{ \}$                                                                                                                                     |
| $p = q$                                 | $D\_gen = \{D[r][s] \mid D[r][p] \text{ and } D[q][s]\}$                                                                                                               |
| $p = q \rightarrow f$                   | $I\_gen = \{I[r][s] \mid D[r][p] \text{ and } I[q][s]\}$                                                                                                               |
| $p = \&(q \rightarrow f)$               |                                                                                                                                                                        |
| $p = q \text{ op } k$                   | $D[q][p] \text{ and } D[s][q] \Rightarrow s.\text{shape} = \text{Cycle}$                                                                                               |
| $p = \text{null}$                       | $D[q][p] \text{ and } D[s][p] \Rightarrow s.\text{shape} = \text{Cycle}$                                                                                               |
| <b><math>p \rightarrow f = q</math></b> | $\neg D[q][p] \text{ and } D[s][p] \text{ and } I[s][q] \text{ and } q.\text{shape} == \text{Tree}$<br>$\Rightarrow s.\text{shape} = \max(s.\text{shape}, \text{DAG})$ |
| $p \rightarrow f = \text{null}$         | $\neg D[q][p] \text{ and } D[s][p] \text{ and } q.\text{shape} != \text{Tree}$<br>$\Rightarrow s.\text{shape} = \max(s.\text{shape}, q.\text{shape})$                  |

|       | Tree  | DAG   | Cycle |
|-------|-------|-------|-------|
| Tree  | Tree  | DAG   | Cycle |
| DAG   | DAG   | DAG   | Cycle |
| Cycle | Cycle | Cycle | Cycle |

$\max(\text{shape1}, \text{shape2})$

# Inference Rules

`p = malloc(...)`

`D_kill = { }, I_kill = { }`

`p = q`

`D_gen = { }`

`p = q→f`

`I_gen = { }`

`p = &(q→f)`

`p = q op k`

No changes to the shape of p.

`p = null`

`p->f = q`

**`p->f = null`**

# Classwork

```
r = malloc(10);  
p = malloc(10);  
p->f1 = null;  
q = p->f2;  
q = &(r->f2);  
q->f2 = p;
```

# Improvements

- Field-sensitivity
- Heap modeling
- Path-sensitivity

# Summary

- Shape analysis helps several transforms.
- Existing techniques often trade off precision for efficiency.
- We are still far away from a precise and scalable analysis.