Data Flow Analysis

Rupesh Nasre.

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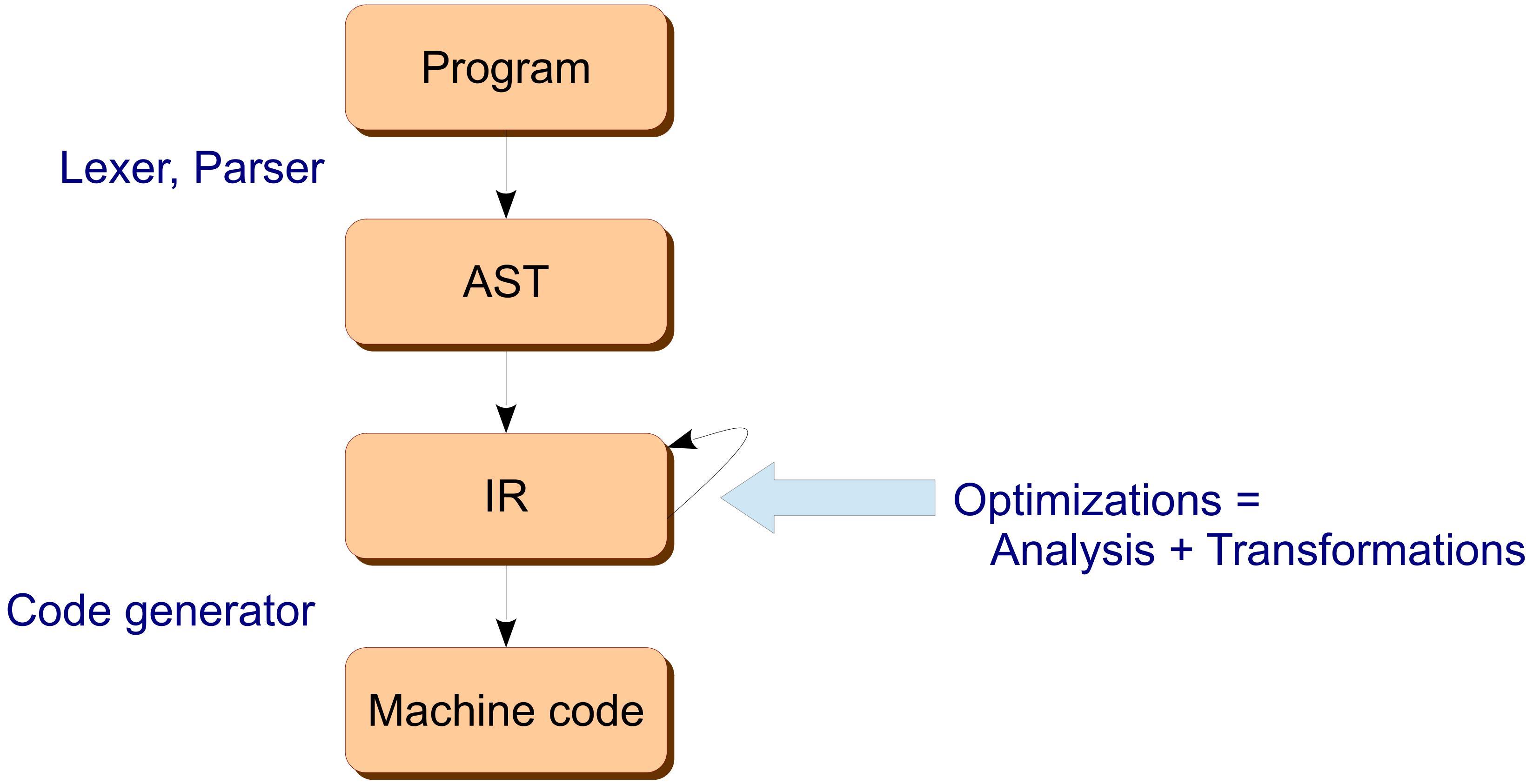
Outline

- What is DFA?
  - Reaching definitions
  - Live variables
- DFA framework
  - Monotonicity
  - Confluence operator
  - MFP/MOP solution
- Analysis dimensions

Compiler Organization



Compiler Basics

- Program as Data
- Control-Flow Graph (CFG)
- Basic Blocks
- Optimizations
  - *gcc -O2 prog.c*

```
int main() {
  int x = 1;
  if (x > 0)
      ++x;
  else
      x = 100;
  printf(“%d\n”, x);
}
```

```
int main() {
  int x = 1;
  if (1 > 0)
      ++x;
  else
      x = 100;
  printf(“%d\n”, x);
}
```

```
int main() {
  int x = 1;
  ++x;
  printf(“%d\n”, x);
}
```

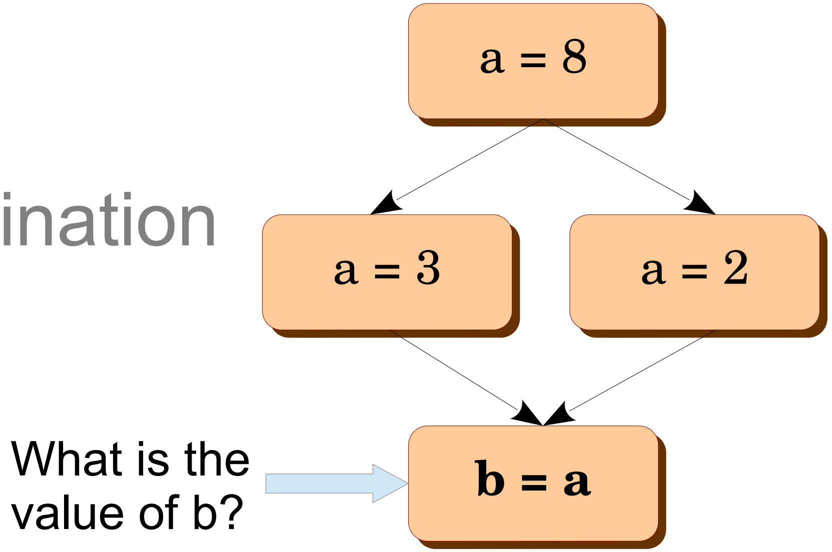
```
int main() {
  printf(“%d\n”, 2);
}
```

Learning Outcomes

- To apply data-flow analysis and its variants on input programs and collect relevant information
  - Given a program, build its control-flow graph
  - Compute gen and kill sets
  - Compute reaching definitions using CFG
  - Compute live variables using CFG
- To design and implement analyses for new problems

Data Flow Analysis

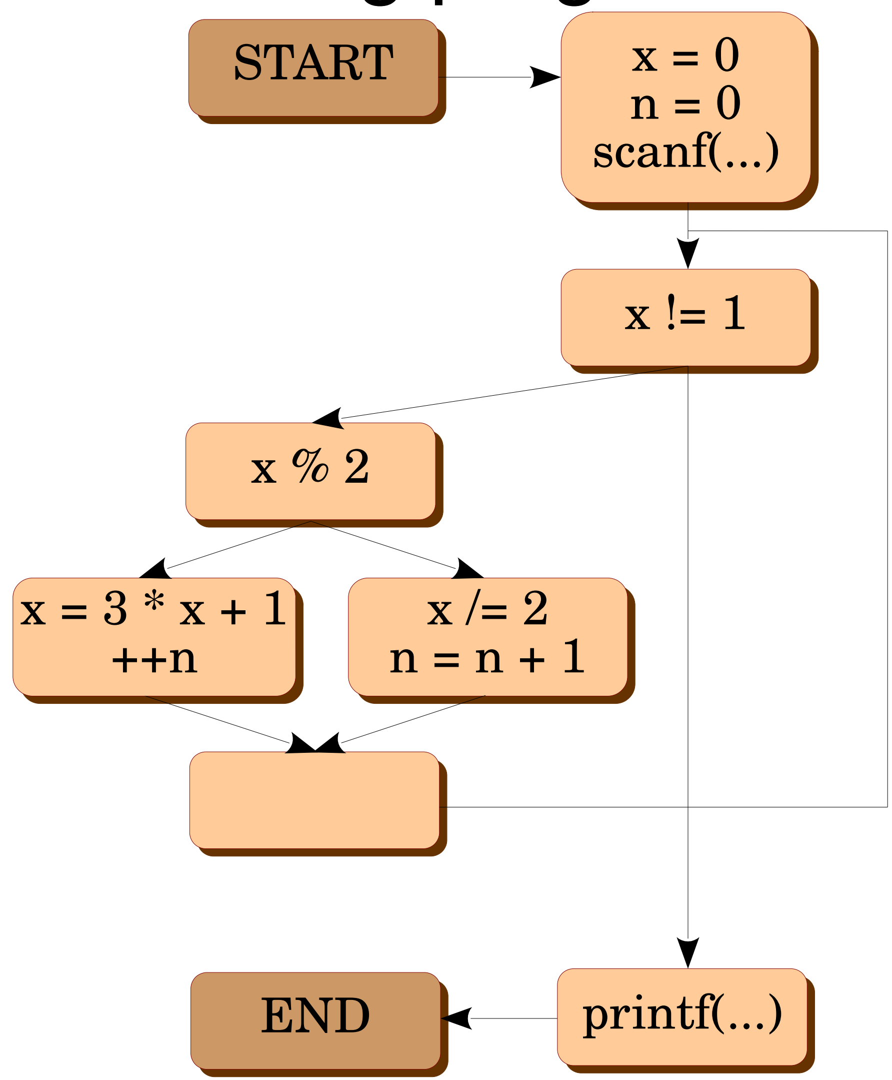
- Flow-sensitive: Considers the control-flow in a function
- Operates on a flow-graph with nodes as basic-blocks and edges as the control-flow
- Examples
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination



Classwork

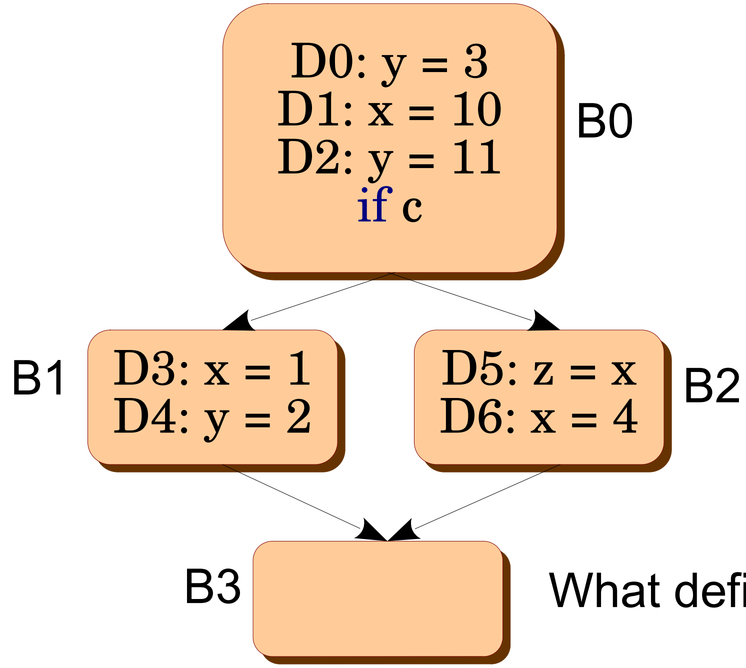
- Draw the CFG for the following program.

```
int main() {
  int x = 0, n = 0;
  scanf("%d", &x);

  while (x != 1) {
    if (x % 2) {
      x = 3 * x + 1;
      ++n;
    } else {
      x /= 2;
      n = n + 1;
    }
  }
  printf("%d\n", n);
}
```



Reaching Definitions

- Every assignment is a definition
- A definition d reaches a program point p if there exists a path from the point immediately following d to p such that d is not killed along the path.



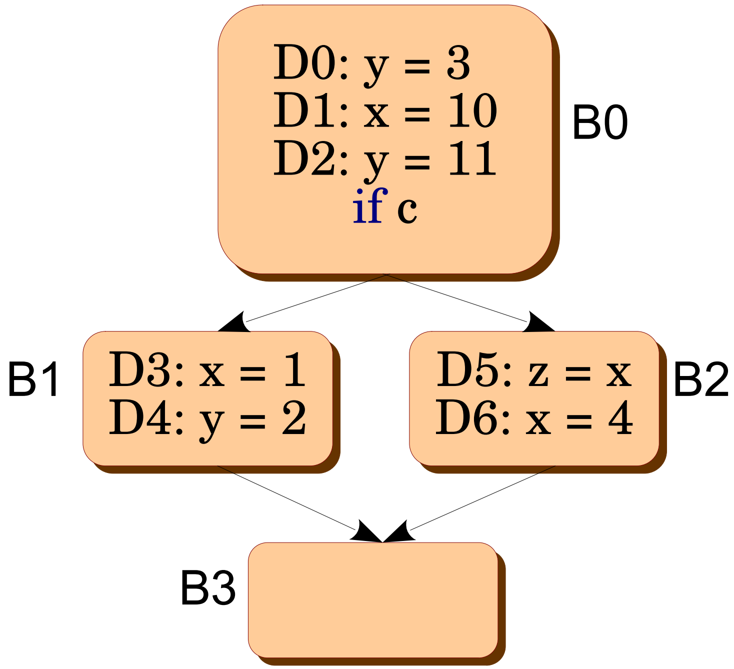
What definitions reach B3?

DFA Equations

- in(B) = set of data flow facts entering block B
- out(B) = …
- gen(B) = set of data flow facts generated in B
- kill(B) = set of data flow facts from the other blocks killed in B

DFA for Reaching Definitions

- in(B) = U out(P) where P is a predecessor of B
- out(B) = gen(B) U (in(B) – kill(B))
- Initially, out(B) = { }



gen(B0) = {D1, D2}
gen(B1) = {D3, D4}
gen(B2) = {D5, D6}
gen(B3) = { }

kill(B0) = {D3, D4, D6}
kill(B1) = {D0, D1, D2, D6}
kill(B2) = {D1, D3}
kill(B3) = { }

	in1	out1	in2	out2	in3	out3
B0	{}	{D1, D2}	{}	{D1, D2}	{}	{D1, D2}
B1	{}	{D3, D4}	{D1, D2}	{D3, D4}	{D1, D2}	{D3, D4}
B2	{}	{D5, D6}	{D1, D2}	{D2, D5, D6}	{D1, D2}	{D2, D5, D6}
B3	{}	{}	{D3, D4, D5, D6}	{D3, D4, D5, D6}	{D2, D3, D4, D5, D6}	{D2, D3, D4, D5, D6}

Algorithm for Reaching Definitions

This general template of fixed-point computation is called Kildall's Algorithm

for each basic block B

compute gen(B) and kill(B)

out(B) = {}

Can you do better?

do {

for each basic block B

in(B) = U out(P) where P \in pred(B)

out(B) = gen(B) U (in(B) - kill(B))

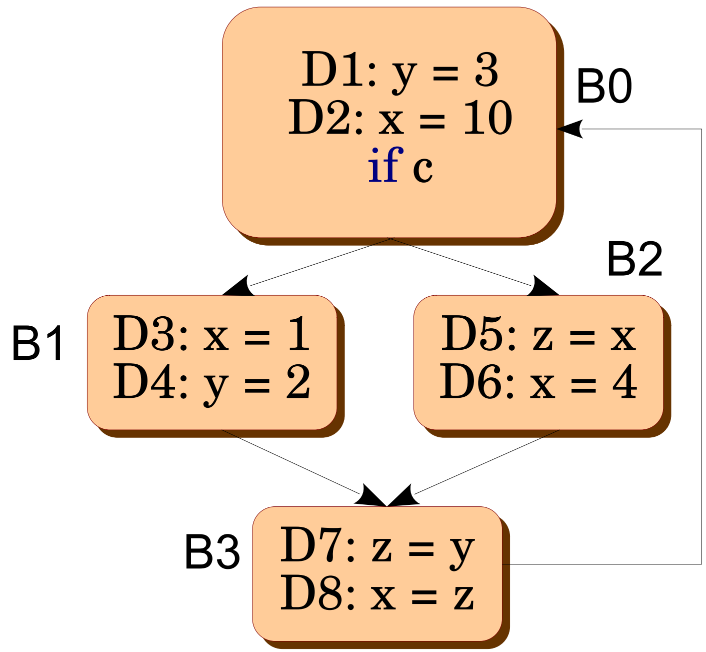
} **while** in(B) changes for any basic block B

Classwork

- in(B) = U out(P) where P is a predecessor of B
- out(B) = gen(B) U (in(B) – kill(B))

- Initially, out(B) = { }

gen(B0) = {D1, D2} kill(B0) = {D3, D4, D6, D8}
gen(B1) = {D3, D4} kill(B1) = {D1, D2, D6, D8}
gen(B2) = {D5, D6} kill(B2) = {D2, D3, D7, D8}
gen(B3) = {D7, D8} kill(B3) = {D2, D3, D5, D6}



	in1	out1	in2	out2	in3	out3	in4	out4
B0	{}	{D1, D2}	{D7, D8}	{D1, D2, D7}	{D4, D7, D8}	{D1, D2, D7}	{D1,4,7, 8}	{D1,2,7}
B1	{}	{D3, D4}	{D1, D2}	{D3, D4}	{D1, D2, D7}	{D3, D4, D7}	{D1,2,7}	{D3,4,7}
B2	{}	{D5, D6}	{D1, D2}	{D1, D5, D6}	{D1, D2, D7}	{D1, D5, D6}	{D1,2,7}	{D1,5,6}
B3	{}	{D7, D8}	{D3,4,5,6}	{D4,7,8}	{D1, D3, D4, D5, D6}	{D1,4, 7, 8}	{D1,3,4,5,6,7}	{D1,4,7,8}

DFA for Reaching Definitions

Domain	Sets of definitions
Transfer function	in(B) = U out(P) out(B) = gen(B) U (in(B) - kill(B))
Direction	Forward
Meet / confluence operator	U
Initialization	out(B) = { }

Memory Optimization

- Reuse memory / register wherever possible.
- y is *dead* at lines 2, 3, 4.
- It is also *dead* at the start of the else block.
- z and y can reuse memory / register.

```
0  int x = 2, y = 3, z = 1;
1  if (x == 2) {
2     y = z;
3     x = 9;
4     y = 7;
5     x = x – y;
6  } else {
7     y = x + z;
8     ++x;
9  }
10 printf("%d", y);
```

This optimization demands computation of live variables.

DFA for Live Variables

Domain	Sets of variables
Transfer function	in(B) = use(B) U (out(B) - def(B)) out(B) = U in(S) where S is a successor of B
Direction	Backward
Meet / confluence operator	U
Initialization	in(B) = { }

Definition: A variable v is live at a program point p if v is used along some path in the flow graph starting at p. Otherwise, the variable v is dead.

How to compute live variables?

Classwork

- Write an algorithm for Live Variable Analysis

```
for each basic block B
  compute gen(B) and kill(B)
  out(B) = {}

do {
  for each basic block B
      in(B) = U out(P) where P \in pred(B)
      out(B) = gen(B) U (in(B) - kill(B))
} while in(B) changes for any basic block B
```

Algo for **reaching definitions**

Domain	Sets of variables
Transfer function	in(B) = use(B) U (out(B) - def(B)) out(B) = U in(S) where S is a successor of B
Direction	Backward
Meet / confluence operator	U
Initialization	in(B) = { }

Parameters for **live variable analysis**

Direction and Confluence

	Forward	Backward
U	Reaching Definitions	Live Variables
∩	Available Expressions	Very Busy Expressions

An expression is available at a program point P if the expression is computed along each path to P (from START) without getting invalidated.

An expression is very busy at a program point P if along each path from P (to END) the expression is computed without getting invalidated.

Data Flow Framework

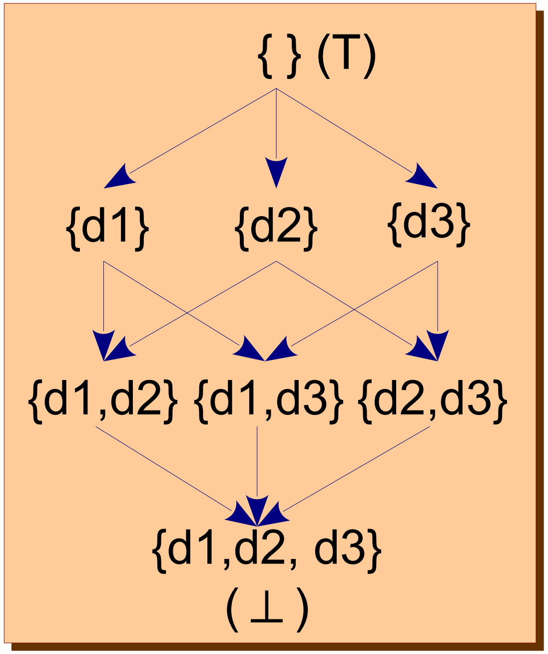
- Point: start or end of a basic block
- Information flow direction: forward / backward
- Transfer functions
- Meet / confluence operator

- One can define a transfer function over a path in the CFG $f_k(f_{k-1}(\ldots f_2(f_1(f_0(T))\ldots))$ // small k(block)
- $MOP(x) = \sqcap f_K(T) \quad K \in Paths(x)$ // capital K (path)

Meet over all paths
Path enumeration is expensive

Structure in Data Flow Framework

- A semilattice $\mathcal{L}$ with a binary meet operator Π, such that $a, b, c \in \mathcal{L}$
  - Idempotency: $a \sqcap a = a$
  - Commutativity: $a \sqcap b = b \sqcap a$
  - Associativity: $a \sqcap (b \sqcap c) = (a \sqcap b) \sqcap c$
- Π imposes an order on $\mathcal{L}$
  - $a >= b \Leftrightarrow a \sqcap b = b$
- $\mathcal{L}$ has a bottom element ⊥, $a \sqcap \bot = \bot$
- $\mathcal{L}$ has a top element T, $a \sqcap T = a$



Reaching Definitions Lattice

Monotone Framework

- A framework $<\mathcal{L}, \sqcap, \mathcal{F}>$ is monotone if $\mathcal{F}$ is monotonic, i.e.,

$$(\forall f \in F)(\forall x, y \in \mathcal{L}), x \geq y \Rightarrow f(x) \geq f(y)$$

- Alternative definition,

$$(\forall f \in F)(\forall x, y \in \mathcal{L}), f(x \sqcap y) \leq f(x) \sqcap f(y)$$

- If a data-flow framework is monotonic, the convergence (termination) is guaranteed for finite height lattices.

Distributive Framework

- A framework <$\mathcal{L}$, ⊓, $\mathcal{F}$> is distributive if $\mathcal{F}$ is distributive, i.e.,

$$(\forall f \in F)(\forall x, y \in \mathcal{L})\ f(x \sqcap y) = f(x) \sqcap f(y)$$

- Maximal fixed point (MFP) solution is obtained with our iterative DFA.
- MFP is unique and order independent.
- The best we can do is MOP (most feasible, but undecidable).
- In general, MFP ≤ MOP ≤ Perfect solution.
- If distributive, MFP = MOP.
- Every distributive function is also monotonic.

Outline

- What is DFA?
  - Reaching definitions
  - Live variables
- DFA framework
  - Monotonicity
  - Confluence operator
  - MFP/MOP solution
- Analysis dimensions

How many ancestor names do you need to almost uniquely identify a student in campus?

Learning Outcomes

- To apply data-flow analysis and its variants on input programs and collect relevant information
  - Define various analysis dimensions
  - Apply the dimensions to compute different information for the same program
  - Perform abstract interpretation
- To design and implement analyses for new problems

Analysis Dimensions

An analysis's precision and efficiency is guided by various design decisions.

- Flow-sensitivity
- Context-sensitivity
- Path-sensitivity
- Field-sensitivity

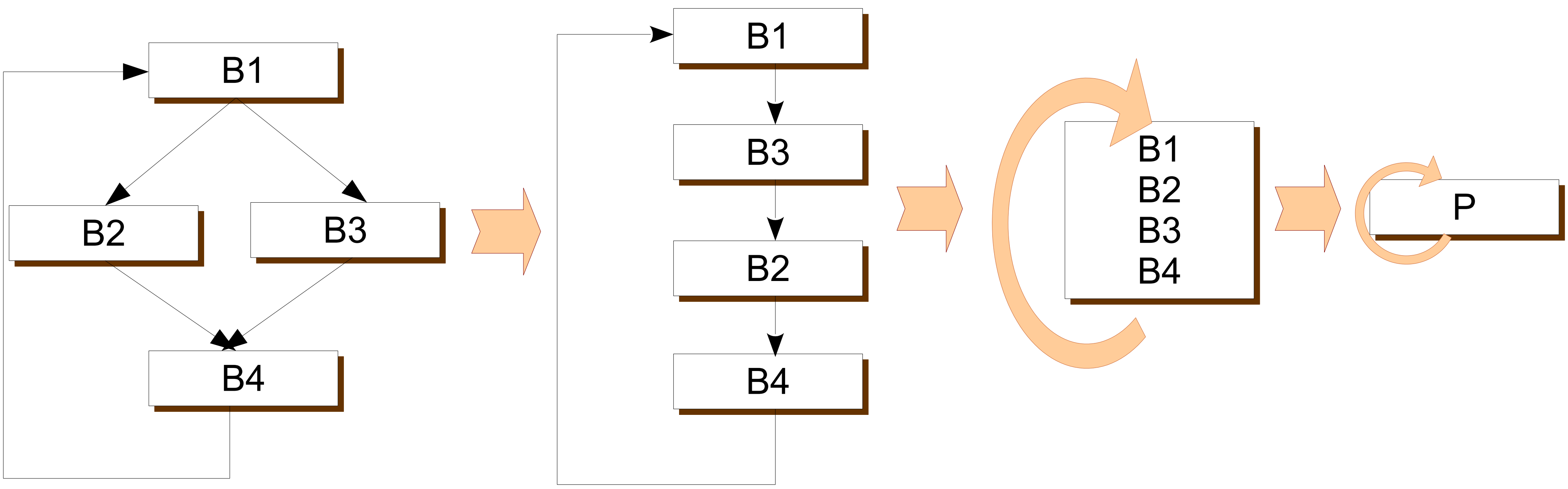
How many hands are required to know the time precisely?

Flow-sensitivity

L0: a = 0;
L1: a = 1;
L2: ...

Flow-sensitive solution: *at L1 a is 0, at L2 a is 1*
Flow-insensitive solution: *in the program a is in {0, 1}*

Flow-insensitive analyses ignore the control-flow in the program.

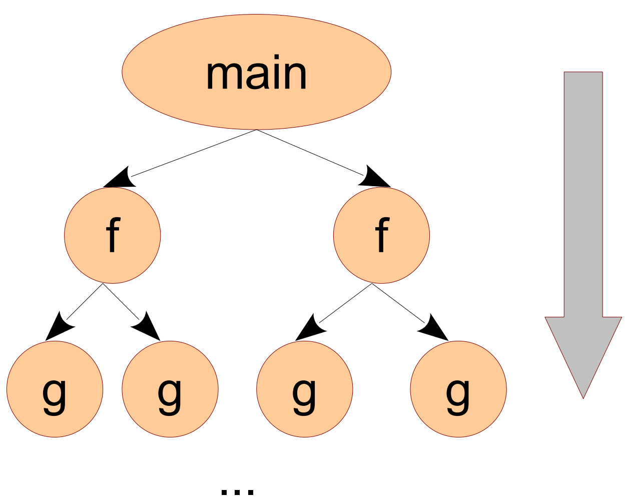


Context-sensitivity

```
main() {                fun(int x) {
      L0:  fun(0);            y = x;
      L1:  fun(1);      }
}
```

Context-sensitive solution:
y is 0 along L0, y is 1 along L1

Context-insensitive solution:
y is in {0, 1} in the program



Exponential
Number of
contexts

Along main-f1-g1, …
Along main-f1-g2, …
Along main-f2-g1, …
Along main-f2-g2, ...

Exponential time requirement

Exponential storage requirement

Context-sensitivity

main() {	fun(int x) {
L0: fun(0);	y = x;
L1: fun(1);	}
}	

Context-sensitive solution:
y is 0 along L0, y is 1 along L1

Context-insensitive solution:

Inter-procedural → *y is in {0, 1} in the program*

intra-procedural → *y is in {-∞, +∞} in the program*

Path-sensitivity

```
if (a == 0)
      b = 1;
else
      b = 2;
```

Path-sensitive solution:
  b is 1 when a is 0, b is 2 when a is not 0

Path-insensitive solution:
  b is in {1, 2} in the program

```
if (c1)
  while (c2) {
    if (c3)
      …
    else
      for (; c4; )
        ...
  }
else
   ...
```

c1 and c2 and c3, …
c1 and c2 and !c3 and c4, …
c1 and c2 and !c3 and !c4, …
c1 and !c2, …
!c1 ...
...

Field-sensitivity

```
struct T s;

s.a = 0;
s.b = 1;
```

Field-sensitive solution:
s.a is 0, s.b is 1

Field-insensitive solution:
s is in {0, 1}

Aggregates are collapsed into a single variable.
e.g., arrays, structures, unions.

This reduces the number of variables tracked during the analysis and reduces precision.

Classwork

- Find the values of variables in
  - context + flow-sensitive analysis
  - interprocedural context-insensitive but flow-sensitive analysis
  - intraprocedural flow-insensitive analysis

```
int g = 0;
void fun(int n) {
    g = n;
}
void main() {
    int a = 1;
    a = 2;
    fun(a);  // L1
    a = 3;
    fun(a);  // L2
}
```

Concrete versus Abstract Interpretation

- Concrete: runtime, actual values
- Abstract: approximate, typically compile-time

Maintain one bit for x == 0
Initialized to F (false)
$\perp$ is F

Concrete Interpretation	Abstract Interpretation
x is undefined	F
x = 0;	x = 0;
x is 0	T
++x;	++x;
x is 1	F
--x;	--x;
x is 0	?

A Note on Choosing Abstraction

Maintain one bit for x == 0
Initialized to F (false)
⊥ is F

```
F
x = 0;
T
++x;
F
--x;
?
```

Maintain two bits for value of x
Initialized to 00
⊥ is 00

```
00
x = 0;
00
++x;
00
--x;
00
```

Maintain one bit for x == 0
Another bit for x < 2
Initialized to and ⊥ is 00

```
00
x = 0;
11
++x;
01
--x;
11
```

If type information available, then {01} --x {11} possible.
Otherwise, {01} --x {01}

Abstraction Storage

- Saturating counters
- Number of values stored faithfully with log(n) bits is (n-2).
- Additional information may help increase the range, e.g., type information as unsigned.

Example

Abstractly interpret the program where abstraction maintains values for integers and points-to information for pointers.

Now change the abstraction for pointers to maintain NULLity.

```
#include <stdio.h>

int main() {
        int x = 3, y = 4;
        int flag = 0;
        int *ptr = &x;

        if (flag == 0) ptr = &y;

        if (*ptr == 4) printf("if\n");
        else printf("else\n");

        return 0;
}
```

Classwork

Abstractly interpret the program where abstraction is maintained as two bits for: x == 1 and y == 1.

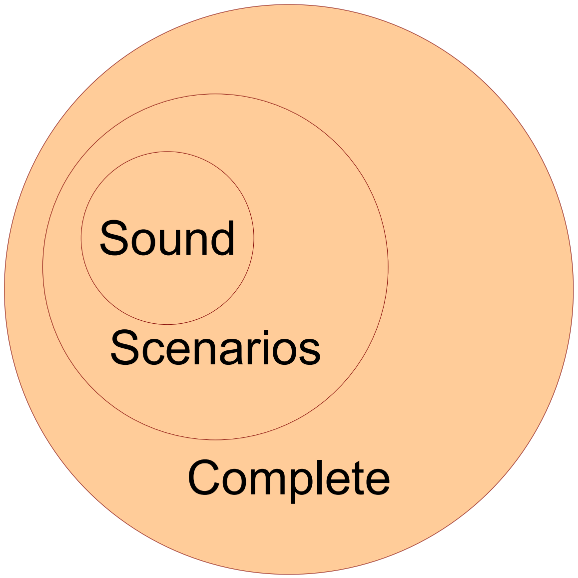
```
x = 1;
y = x;

if (x != y)
    y++;
else
    --x;

x++;
if (x == y)
    ++x;
else
   --y;
```

Conservative Analysis

- Being safe versus being precise
  - Relation with lattice
  - Initializations and confluence
  - Constructive versus destructive operators
- Safety versus liveness property
  - Absence of bugs versus presence of a bug
- May versus Must analysis
  - Reaching definitions, live variables, constant propagation?

Soundness and Completeness

- Analyses enable optimizations.
- An optimization is sound if it maintains the functionality of the original code.
- A program may be optimized in certain scenarios.
- An analysis is sound if it leads to sound optimization.
  - The analysis does not enable optimization outside the above set of scenarios.
- An analysis is complete if it does not disable optimization for any possible scenario.



On Soundness

- Usually, multiple optimizations expect same information-theoretic behavior from analyses.
  - If more information means analysis A1 is less precise according to optimization O1, often optimization O2 also sees A1 that way.
  - This allows us to argue about analysis soundness *without talking about optimizations*.
- But this is not always true.
  - Soundness depends upon optimization enabling.
  - And two opposite optimizations may see the information from the same analysis in opposing ways.

Optimization-specific Soundness

- Consider O1 that changes *p to x if p points to only x.
- Consider O2 that makes p volatile if p points to multiple variables at different program points.
- Analysis A computes points-to information p → {x, y}
  - If A computes more information p → {x, y, z}, O1 is suppressed but O2 is enabled.
  - If A computes less information p → {x}, O1 is enabled and O2 is suppressed.
  - Thus, conservative for one is precise for another.
  - And sound for one is unsound for another.

Optimization-specific Soundness

- Consider O1 that converts multiplication by 2 to a left-bit-shift operation (x * 2 to x << 1).
- Consider O2 that uses a special circuit (fast operation) when there is a sum of reciprocals of powers of 2 ($1 + \frac{1}{2} + \frac{1}{4} + \ldots$)
- Analysis A is used to compute values of arithmetic expressions.
  - Converting 1.98 to 2 enables O1, disables O2.
  - Converting 1.98 to 1.96875 enables O2, disables O1.
  - Precise for one is imprecise for another.
  - Sound for one is unsound for another.

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- To apply data-flow analysis and its variants on input programs and collect relevant information
  - Given a program, build its control-flow graph
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Acknowledgements

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