CS1100 – Introduction to Programming

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Lecture 22

Recursion

Drawing Hands by M. C. Escher.

New Idea - Recursive Function Calls

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- We have not written `fact` function yet, but we want to write it using itself.

Here is the idea :
Write the function `fact` in such a way that :

- it returns 1, if the argument is 1.
- else it returns n times the result of itself when called with argument `n-1`.

fact function : Iterative vs Recursive

```
int fact(int n){
   int i;
   int result;
   result = 1;
   for (i = 1; i <= n; i++)
     result = result * i;
   return result;
}
```

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int fact(int n){
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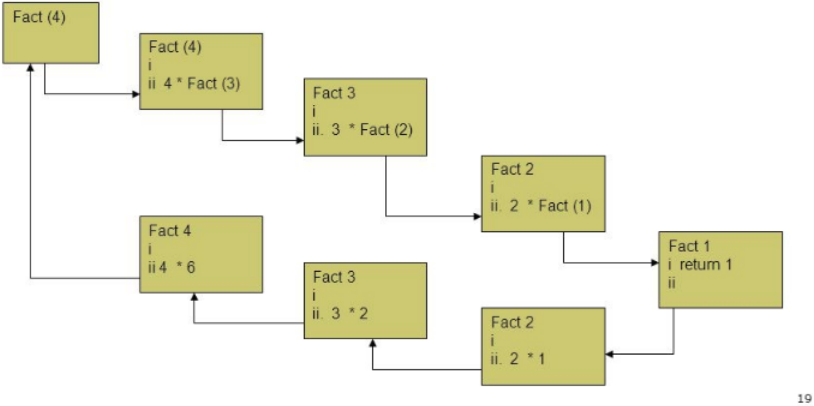
- `(n == 1)` case is called the **base case**. If it not provided, it will turn out to be an infinite recursion.

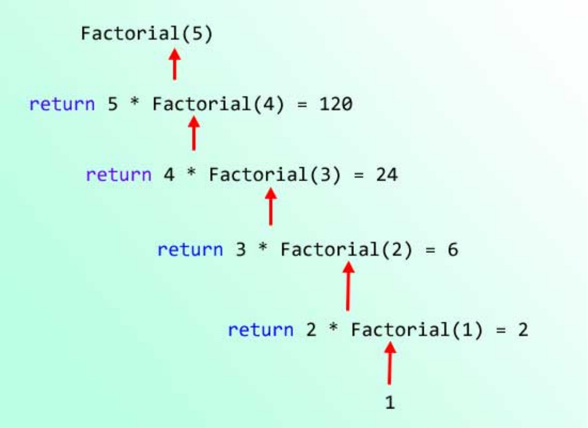
A Graphical Demo of `fact(4)` : Control Flow

Assume that we invoked `fact` with argument as the number 4.

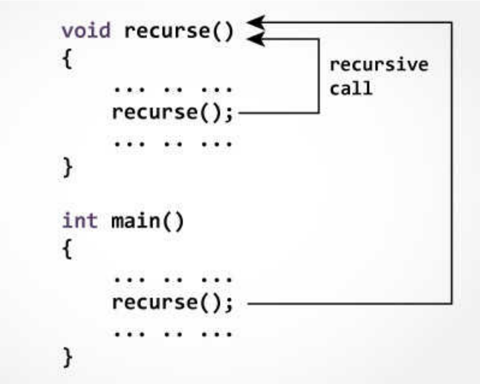


A Graphical Demo of `fact(5)` : Return Values

Assume that we invoked `fact` with argument as the number 5.



Recursion : Control Flow



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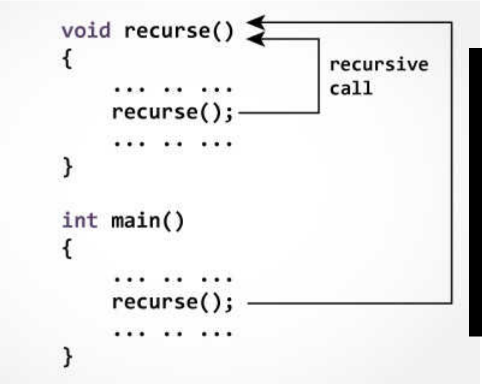

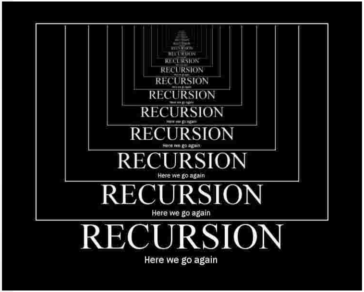

Recursion: Summary

- The function knows how to solve the *simplest* case. This is also called the base case.
- If the function is invoked using a *complex* case, it breaks the input into
  (i) a part that the function knows how to do.
  (ii) a part that it does not know how to do.
- For (ii) it again invokes the same function – this step is called the recursive step.

Multiplication using repeated addition

Mathematical formulation

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```
int mult(int x, int y) {
    if (x == 1)
        return 1;
    else
        return ( y + mult(x-1, y));
}
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Is the program correct?

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int mult(int x, int y) {
    if (x == 1)
        return y;                    //notice the change.
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- Note the usage of `str+1`.
- Why is accessing str+1 valid?
  we know that str[0] != 0, hence we will find at least one more character after str[0].

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Conceptual formulation

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- if $s1$ is of length 1 it is a palindrome.
- if $s1$ is of length 2 and $s1[start] == s1[end]$, it is a palindrome.
- if $s1[start] == s1[end]$ and $s1[start + 1, end - 1]$ is a palindrome, then $s1$ is a palindrome.

Largest Element in an Array

- Till now - we computed only functions which were taught to us or known to us recursively.
- We can solve problems that have a recursive structure using recursive programming. That is more fun !.
- **Key Part :** Formulate the problem recursively.

Example Task : Finding the largest element in an array.

Largest Element in an Array

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{
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  l = largest(i+1,n);
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```
int largest(int i, int j)
{
  if (i == j) return arr[i];

  int l1,l2;
  l1 = largest(i,(i+j)/2);
  l2 = largest((i+j)/2+1,j);
  if (l1 > l2)
       return l1;
  else return l2;
}
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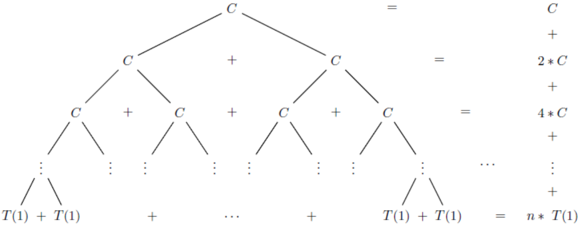

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- So total time is roughly the same. But depth of recursion tree is larger in one case.
- The *space* complexity of recursive algorithm is proportinal to maximum depth of recursion tree generated. So this is how the second algorithm is better than the first.