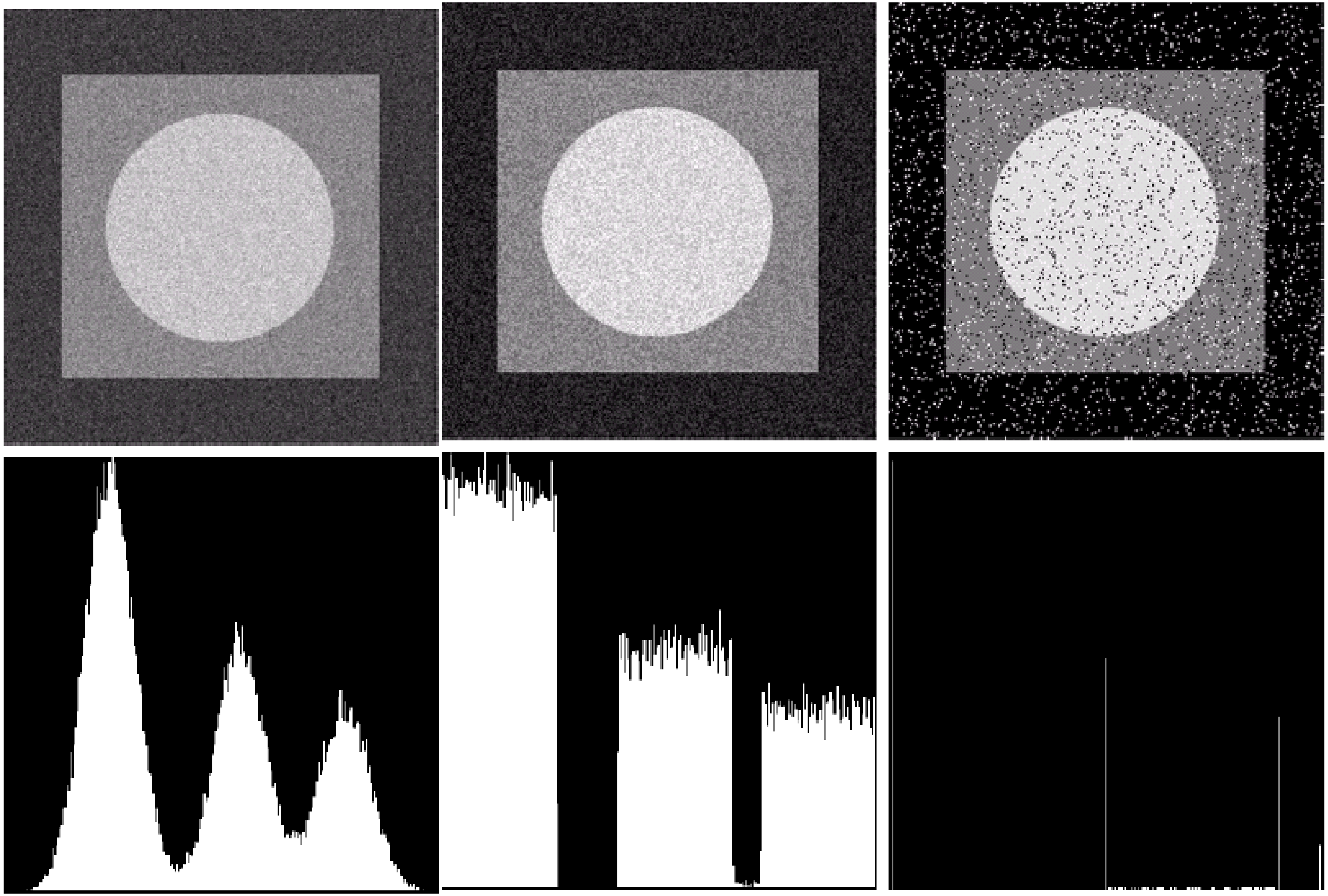
IMAGE RESTORATION –

Some methods and examples



Gaussian Uniform Salt & Pepper

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Courtesy: Jen-Chang Liu

Mean filters

- Arithmetic mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Window centered at (x,y)

- Geometric mean

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{1/mn}$$

Other Means (cont…)

Harmonic Mean:

$$\hat{f}(x, y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise.

Also does well for other kinds of noise such as Gaussian noise.

Other Means (cont…)

Contraharmonic Mean:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q=-1, harmonic

Q=0, airth. mean

Q=+, ?

Q is the *order* of the filter.

Positive values of Q eliminate pepper noise.

Negative values of Q eliminate salt noise.

It cannot eliminate both simultaneously.

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

- Max/min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

Order-statistics filters (cont.)

- Midpoint filter

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}}g_r(s,t)$$

Middle (mn-d) pixels

Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - $g(x,y)$: noisy image pixel value
  - σ^2_η: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L : local variance

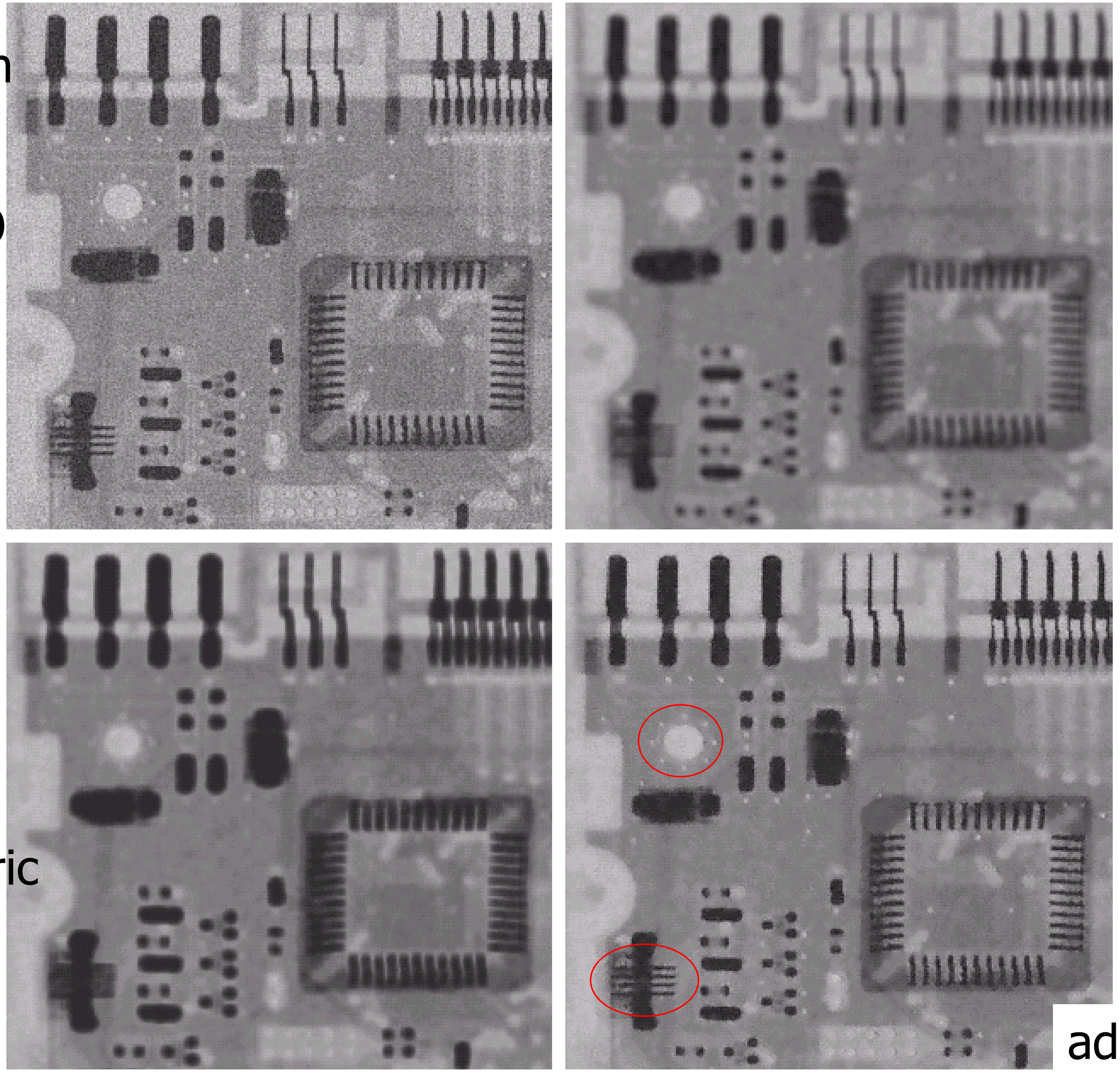
Adaptive local noise reduction filter (cont.)

- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Gaussian noise
$\mu=0$
$\sigma^2=1000$

Arith. mean 7x7



Geometric mean 7x7

adaptive

Periodic noise reduction (cont.)

- Bandreject filters
- Bandpass filters
- Notch filters
- Optimum notch filtering

Bandreject filters

* Reject an isotropic frequency

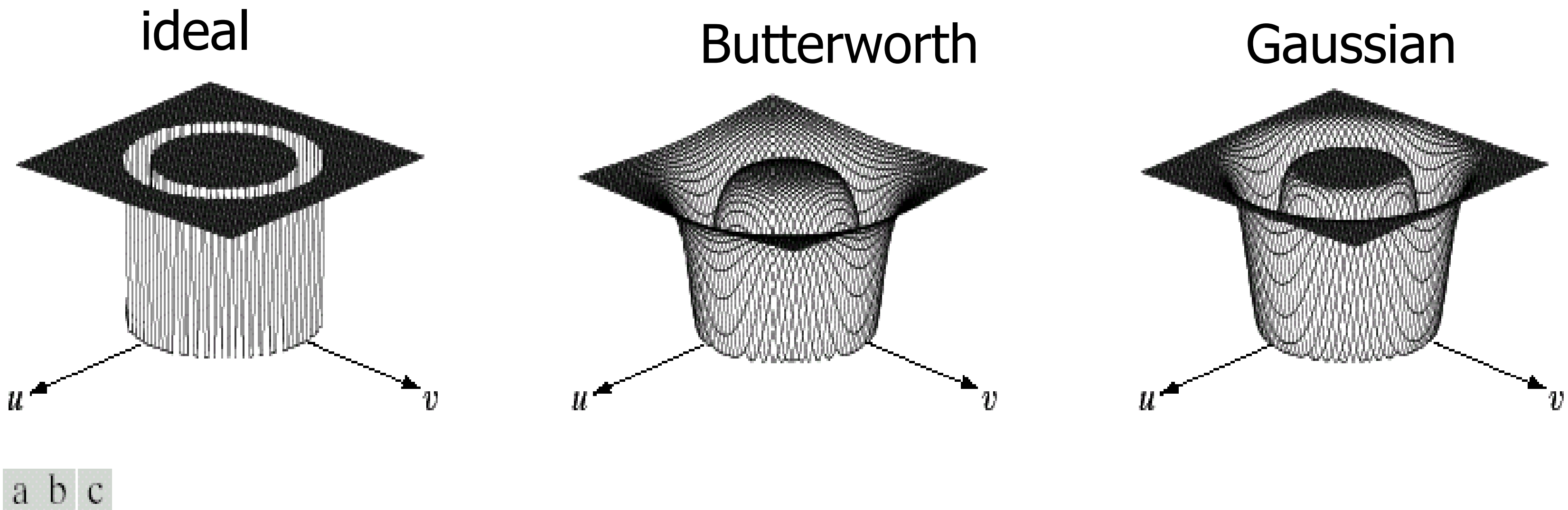
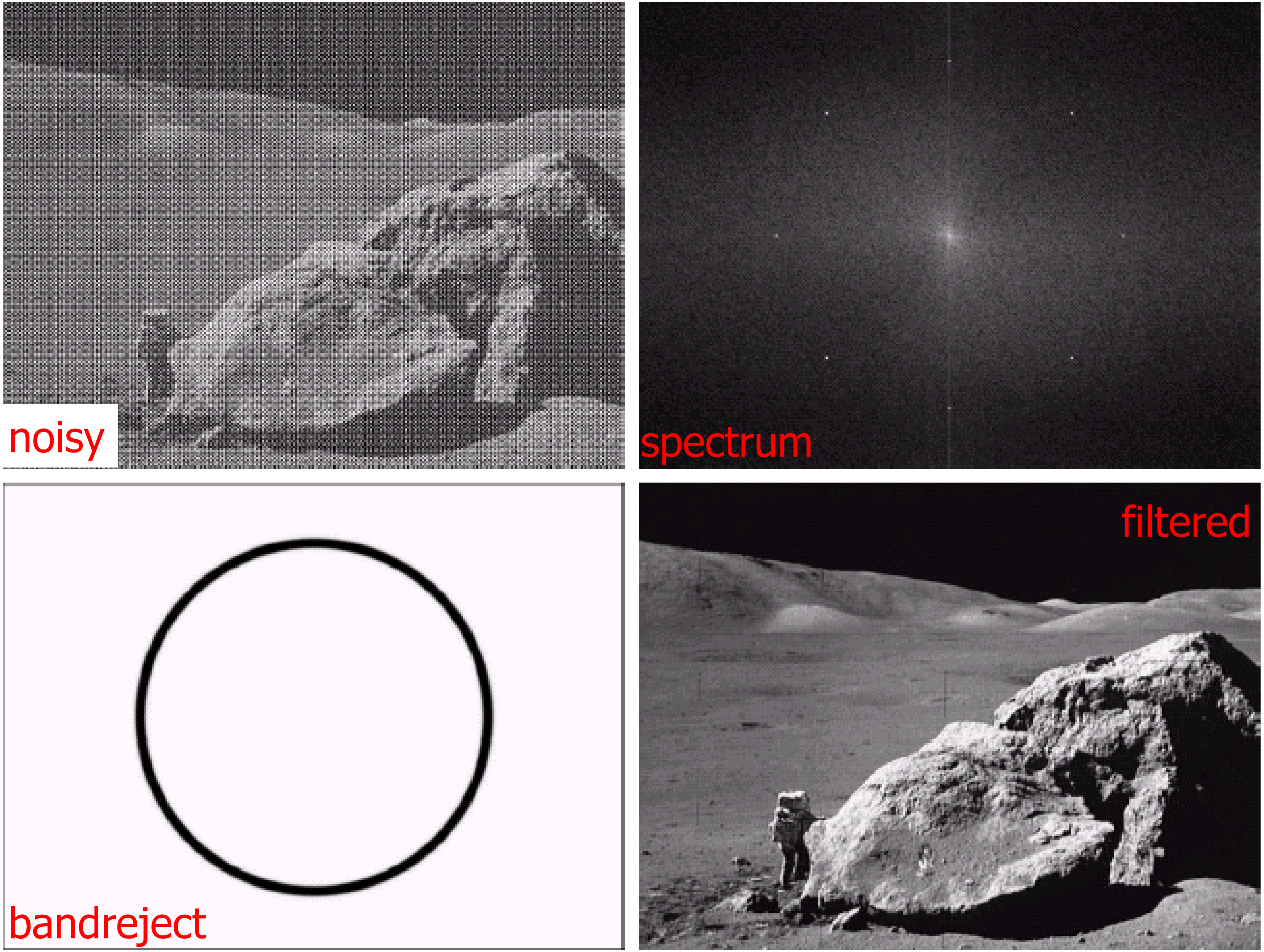
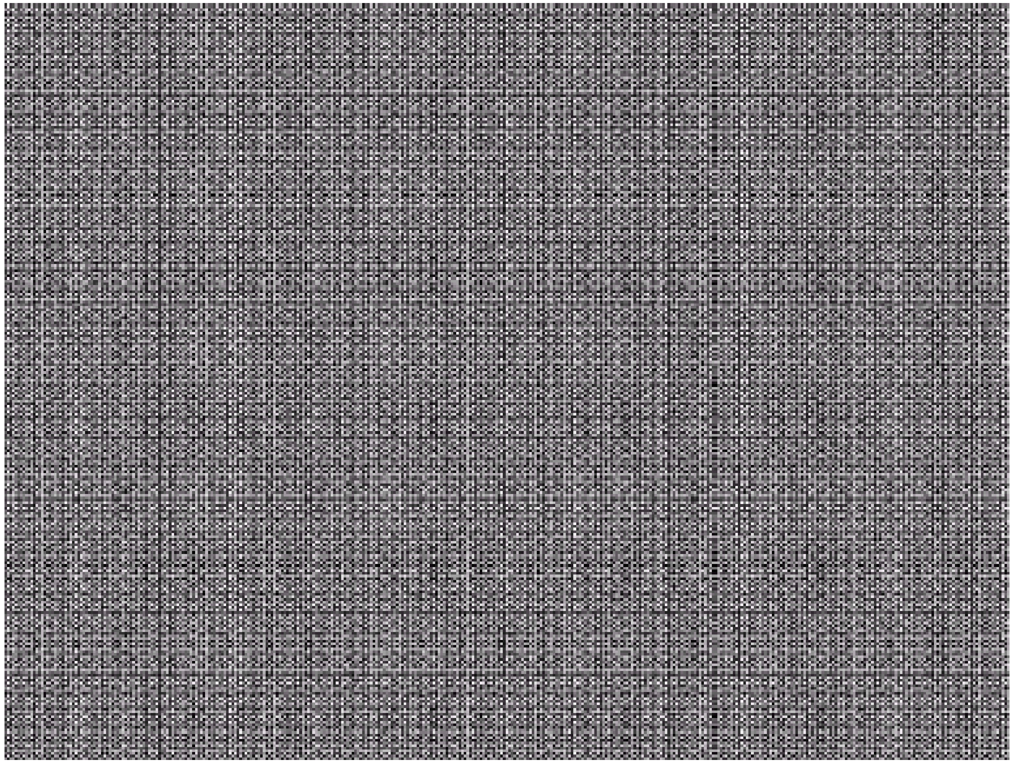


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Bandpass filters

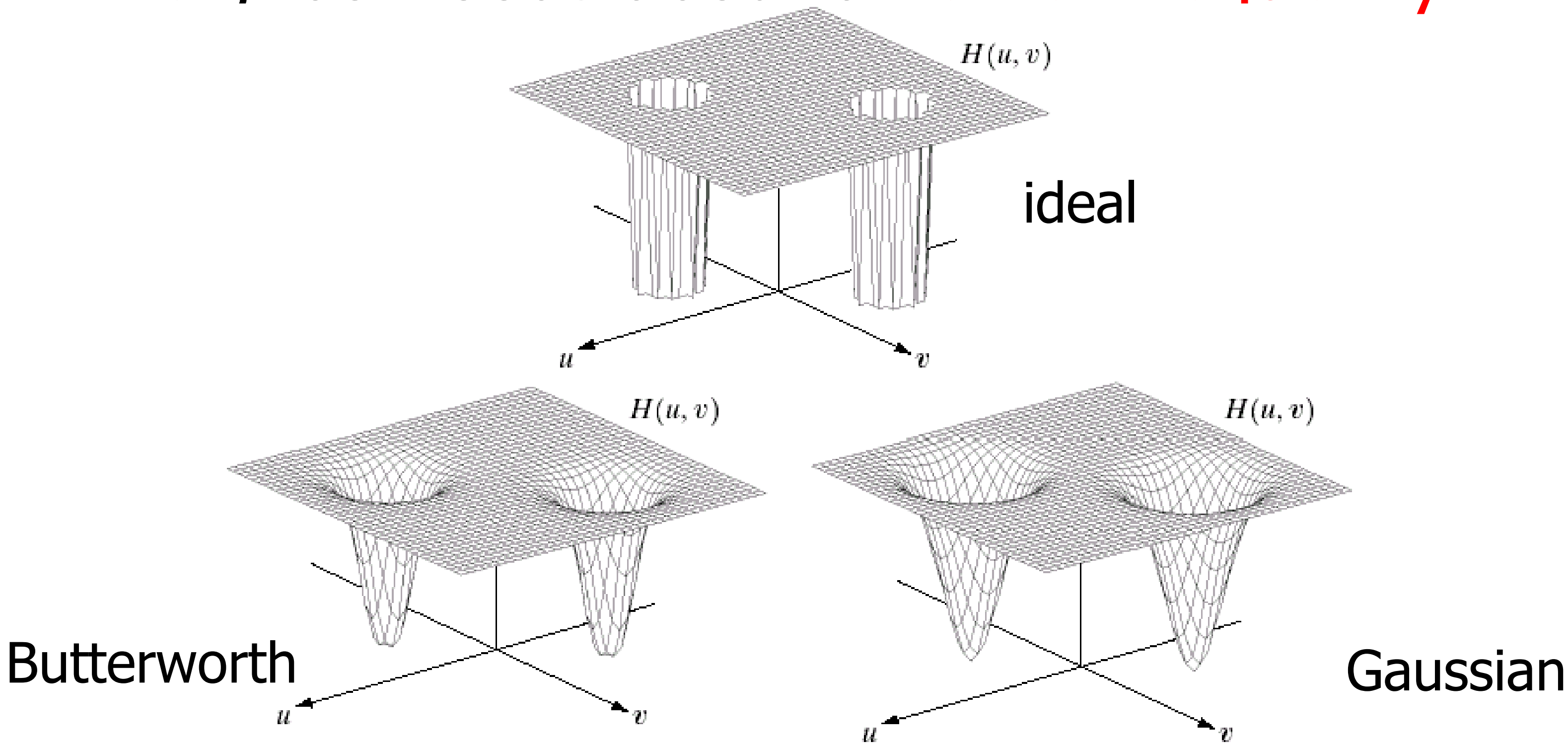
- $H_{bp}(u,v) = 1 - H_{br}(u,v)$

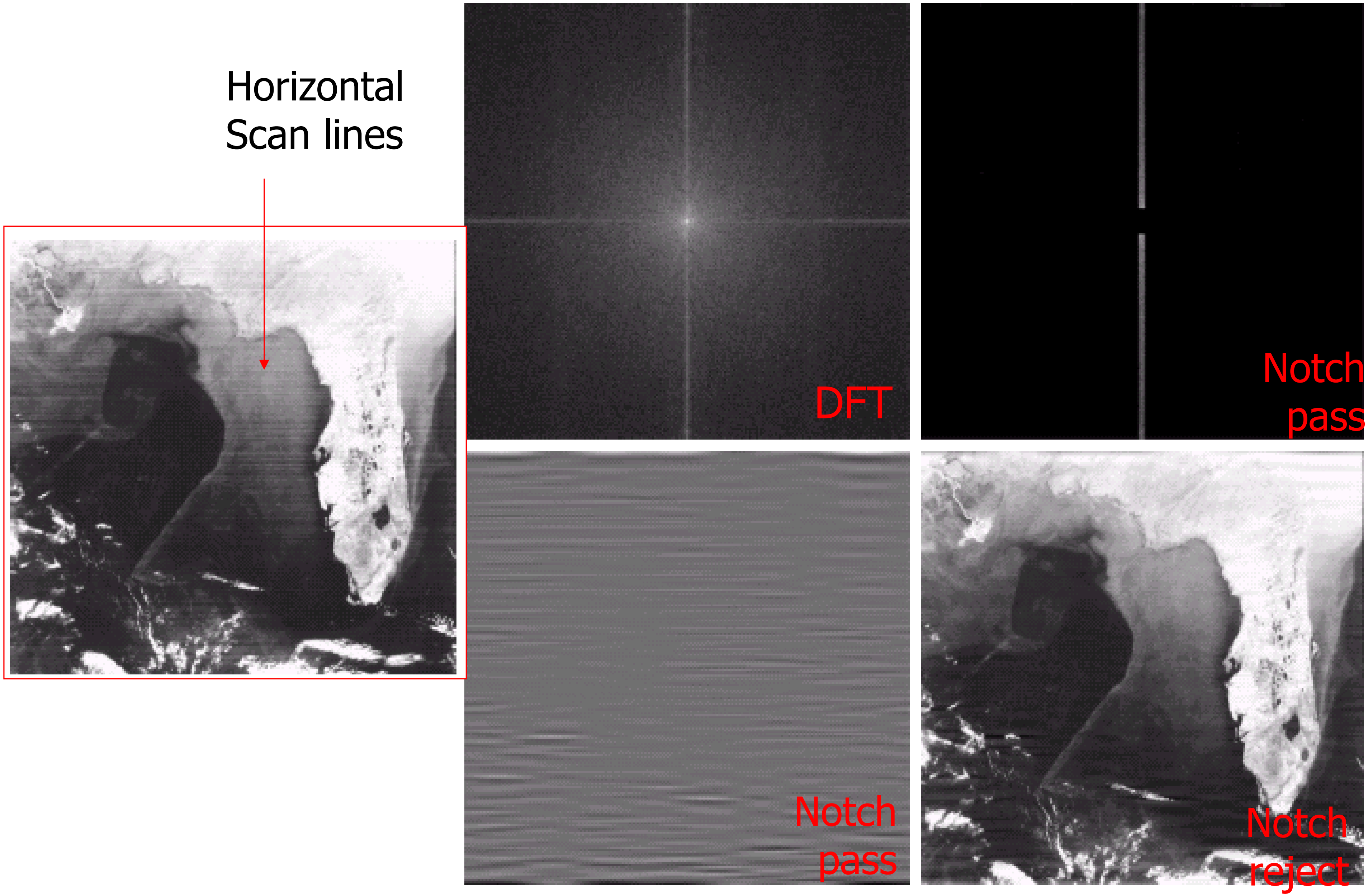


$$\Im^{-1}\left\{G(u,v)H_{bp}(u,v)\right\}$$

Notch filters

- Reject(or pass) frequencies in predefined neighborhoods about a center frequency




Horizontal
Scan lines
DFT
Notch
pass
Notch
pass
Notch
reject

Estimation by modeling (1)

- Ex. Atmospheric model $H(u,v) = e^{-k(u^2+v^2)^{5/6}}$

original

k=0.0025

k=0.001

k=0.00025

Estimation by modeling (2)

- Derive a mathematical model
- Ex. Motion of image

$$g(x,y)=\int_0^T f(x-x_0(t),y-y_0(t))dt$$

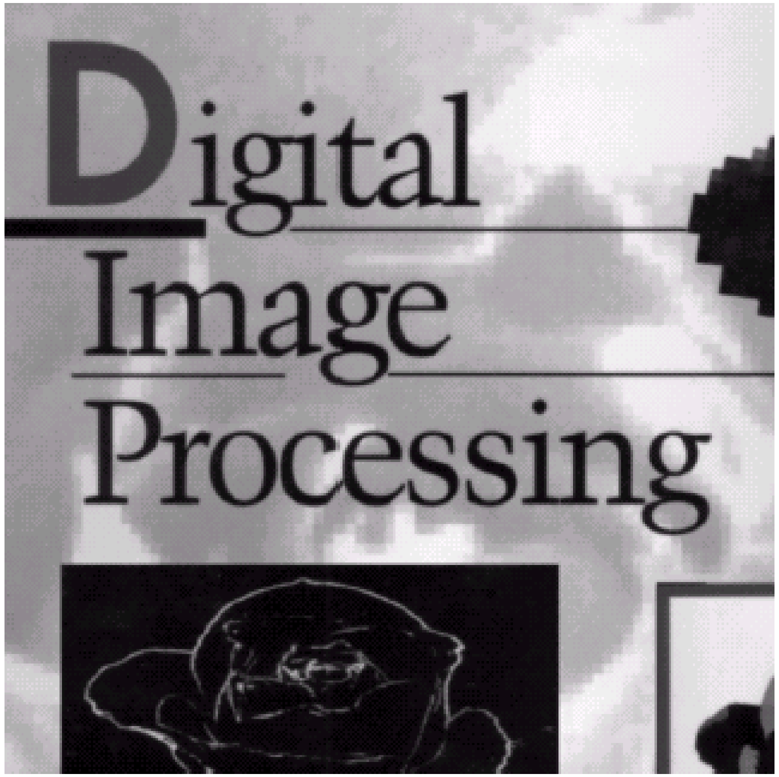
Planar motion

Fourier transform

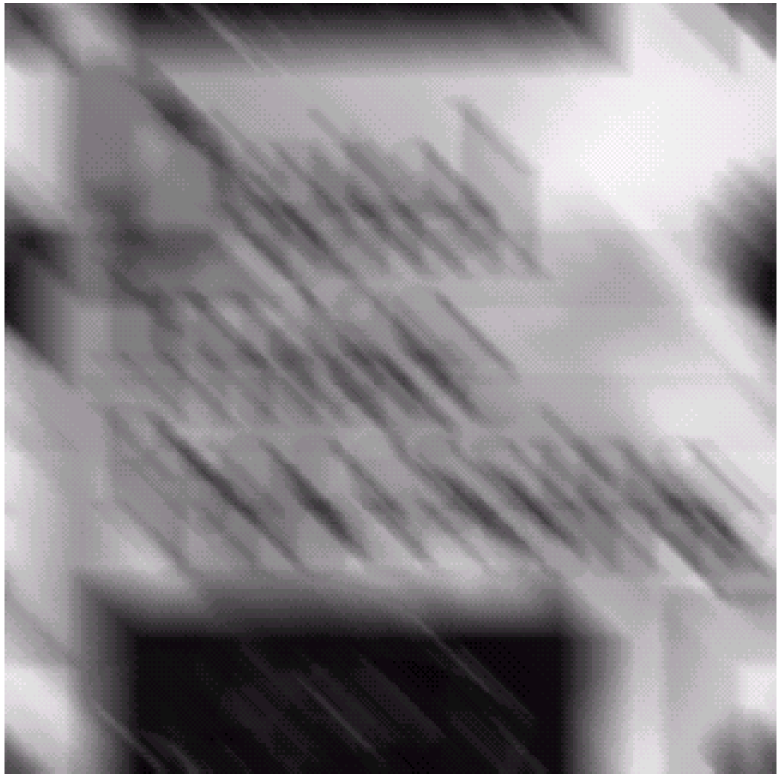
$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by modeling: example

original



Apply motion model



Inverse filtering

- With the estimated degradation function H(u,v)

Unknown noise

G(u,v)=F(u,v)H(u,v)+N(u,v)

$$=> \quad \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Estimate of original image

Problem: 0 or small values

Sol: limit the frequency around the origin

Example Wiener filter

Original

Noise added

Pseudo-inverse

Wiener filter

Wiener Filter: Adaptive Inverse Filter

Purpose: To Remove noise and/or bluriness in the image.

Estimate the local mean and variance in the neighborhood around each pixel

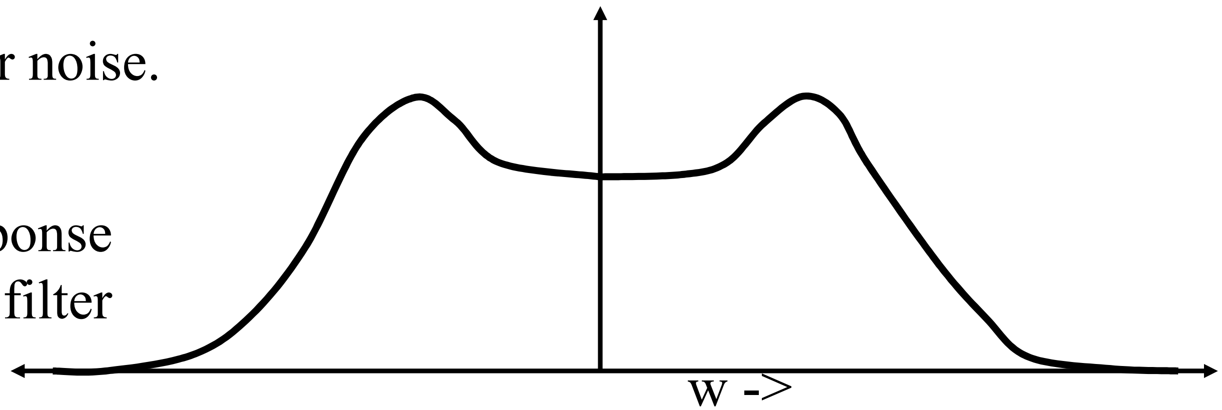
$$\mu = (1/MN)\sum f(x,y) \quad \sigma^2 = (1/MN)\sum [f(x,y)-\mu]^2$$

Wiener filter formulation, for no blur:

$$w(x,y) = \mu + \frac{\sigma^2 - n_v^2}{\sigma^2}(f(x,y)-\mu)$$

Where n_v is the standard deviation for noise.

Typical response
For Wiener filter



Wiener Filter Formulation

- Least Mean Square Filter

$$G(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + [S_n(u, v)/S_x(u, v)]}$$

- In practice

$$G(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K}$$

Wiener filtering

Using notation:
$G(u,v) = H(u,v)F(u,v)$

minimum mean square error: $e^2 = E\{ (f-f^c)^2\}$

$F^c(u,v) = [1/H(u,v)] \, [\, |H(u,v|^2 / (|H(u,v|^2 + S_\eta(u,v)/S_f(u,v))] \, G(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ power spectrum of noise

Approximations of $S_\eta(u,v)/S_f(u,v)$:

- K (constant)
- $\gamma \, |P(u,v)|^2$ (power spectrum of Laplacian)
  - γ found by iterative method to minimize e^2
    - (constrained least squares filtering)

Inverse Filter

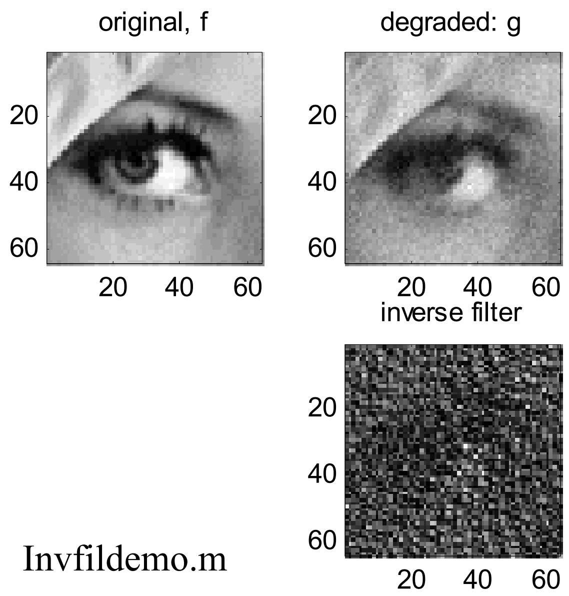
- Recall the degradation model:

Given H(u,v), one may directly estimate the original image by

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

At (u,v) where $H(u,v) \approx 0$, the noise N(u,v) term will be amplified!

$$\hat{F}(u,v) = G(u,v) / H(u,v)$$
$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

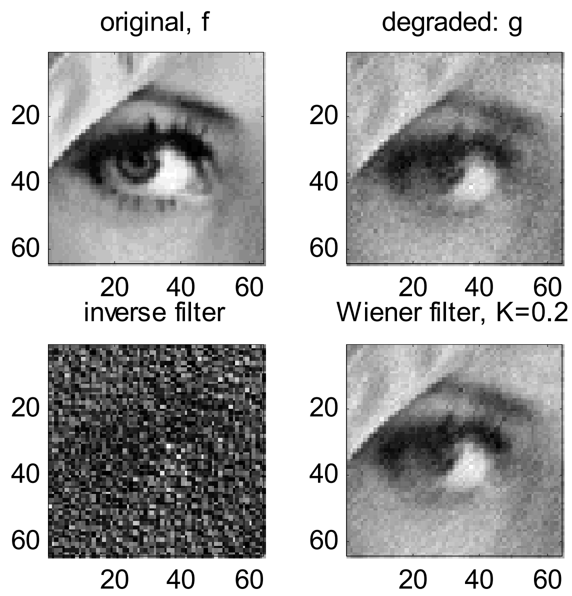


Invfildemo.m

Wiener Filtering

- Minimum mean-square error filter
  - Assume f and η are both 2D random sequences, uncorrelated to each other.
  - Goal: to minimize $E\left\{\left|f-\hat{f}\right|^2\right\}$
  - Solution: Frequency selective scaling of inverse filter solution!

$$\hat{F}(u,v)=\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2+S_\eta(u,v)/S_f(u,v)}\cdot\frac{G(u,v)}{H(u,v)}$$



  - White noise, unknown $S_f(u,v)$:

$$\hat{F}(u,v)=\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2+K}\cdot\frac{G(u,v)}{H(u,v)}$$

Derivation of Wiener Filters

- Given the degraded image g, the Wiener filter is an optimal filter h_{win} such that $E\{\|f-h_{win}g\|^2\}$ is minimized.
- Assume that f and η are uncorrelated zero mean stationary 2D random sequences with known power spectrum S_f *and* S_n. Thus,

$$E\left\{\|F(u,v)\|^2\right\} = S_f(u,v)$$

$$E\left\{\|N(u,v)\|^2\right\} = S_n(u,v)$$

$$E\left\{F(u,v)N^H(u,v)\right\}$$
$$= E\left\{F^H(u,v)N(u,v)\right\} = 0$$

$$C = E\left\{\|f - h_{win}g\|^2\right\} = E\left\{\|F(u,v) - H_{win}(u,v)G(u,v)\|^2\right\}$$
$$= E\left\{\|F(u,v)\|^2\right\} - H_{win}(u,v)\cdot E\left\{F^H(u,v)G(u,v)\right\}$$
$$- H_{win}^H(u,v)\cdot E\left\{F(u,v)G^H(u,v)\right\} + \|H_{win}(u,v)\|^2 \cdot E\left\{\|G(u,v)\|^2\right\}$$
$$= S_f(u,v) + \|H_{win}(u,v)\|^2 \cdot \left(\|H(u,v)\|^2 \cdot S_f(u,v) + S_n(u,v)\right)$$
$$- H_{win}(u,v)\cdot H(u,v)\cdot S_f(u,v) - H_{win}^H(u,v)\cdot H^H(u,v)\cdot S_f(u,v)$$

Set $\partial C / \partial H_{win}(u,v) = 0 \Rightarrow$

$$H_{win}(u,v) = \frac{H^*(u,v)S_f(u,v)}{\|H(u,v)\|^2 S_f(u,v) + S_n(u,v)}$$