Mathematical Morphology

Mathematical Morphology is a tool for extracting image components that are useful for representation and description

Morphology can provide boundaries of objects, their skeletons, and their convex hulls. It is also useful for many pre- and post-processing techniques, especially in edge thinning and pruning.

Generally speaking most morphological operations are based on simple expanding and shrinking operations. The primary application of morphology occurs in binary images, though it is also used on grey level images. It can also be useful on range images. (A range image is one where grey levels represent the distance from the sensor to the objects in the scene rather than the intensity of light reflected from them).

MORPHOLOGICAL OPERATION

Morphological operations refer to certain operations where an object is *hit* with a *structuring element* and thereby reduced to a more revealing shape. It is a tool for extracting image components such as boundaries, skeletons and convex hull. These components are useful in the representation and description of region shapes.

Common morphological operations are:

- **Dilation**
- **Erosion**
- **Opening and**
- **Closing**

Set operations

The two basic morphological set transformations are *erosion* and *dilation*

These transformations involve the interaction between an image *A* (the object of interest) and a structuring set *B*, called the *structuring element*.

Typically the structuring element *B* is a circular disc in the plane, but it can be any shape. The image and structuring element sets need not be restricted to sets in the 2D plane, but could be defined in 1, 2, 3 (or higher) dimensions.

Let *A* and *B* be subsets of Z^2. The *translation* of *A* by *x* is denoted A_x and is defined as

$$A_x = \{c : c = a + x, \text{for } a \in A\}.$$

The *reflection* of *B*, denoted , is defined as

$$\hat{B} = \{x : x = -b, \text{for } b \in B\}.$$

The complement of *A* is denoted A^c, and the difference of two sets *A* and *B* is denoted *A* - *B*.

Dilation

Given two sets A and B in R^2 and Φ the empty set, the dilation of A by B, is defined as:

$$A \oplus B = \{ x \mid (B)_x \cap A \neq \Phi \} = \{ x \mid [(B)_x \cap A] \subseteq A \}$$

$$= \{c \in R^2 \mid c = a + b, \text{ for some } a \in A,\ b \in B\}$$

$$= \bigcup_{h \in B} (A)_h$$

Clearly, dilation is a expansion operation. For example, let

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

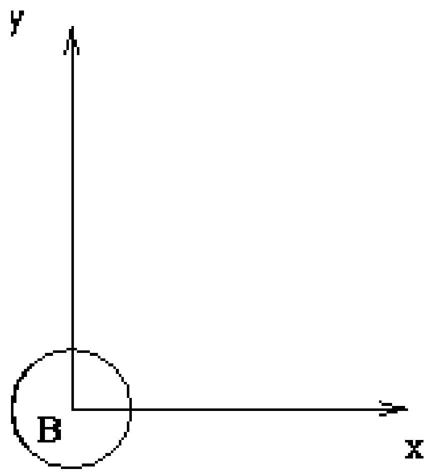
$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \ and \ \ C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

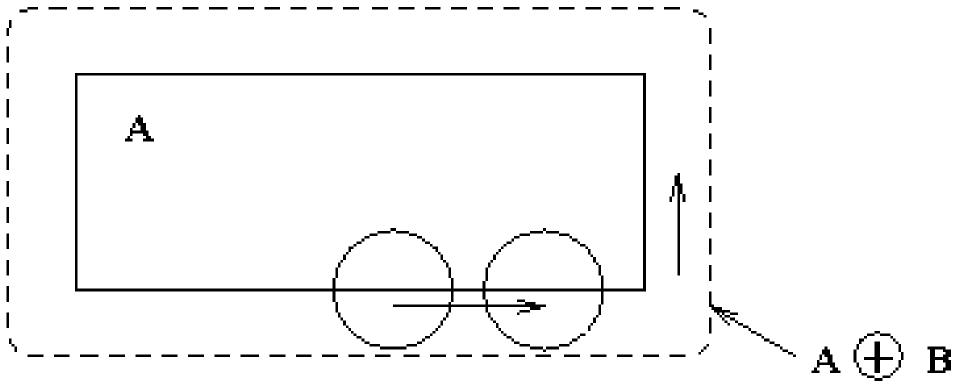
$$Dilation\ of\ A\ by\ B\ will\ be: \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \ and$$

$$Dilation\ of\ A\ by\ C\ will\ be: \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

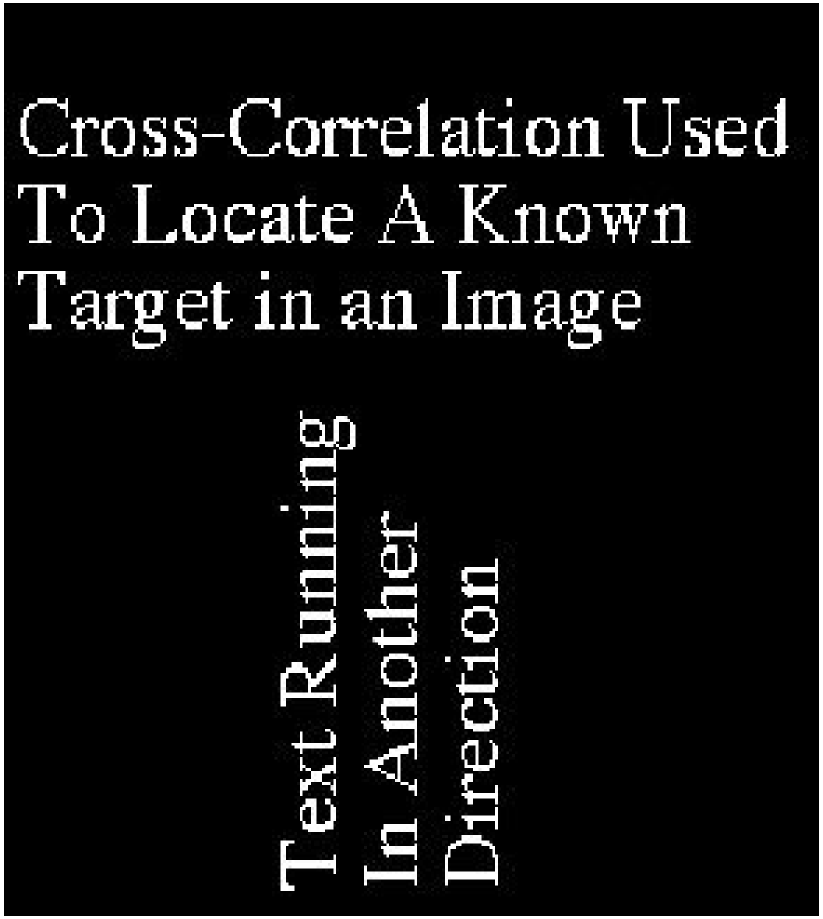
Dilation of the object *A* by the structuring element *B* produces a new set made up of all points generated by obtaining the reflection of *B* about its origin and then shifting this relection by *x*.

Consider the example where *A* is a rectangle and *B* is a disc centred on the origin. (Note that if *B* is not centred on the origin we will get a translation of the object as well.) *B* is a symmetric structuring element in this case.

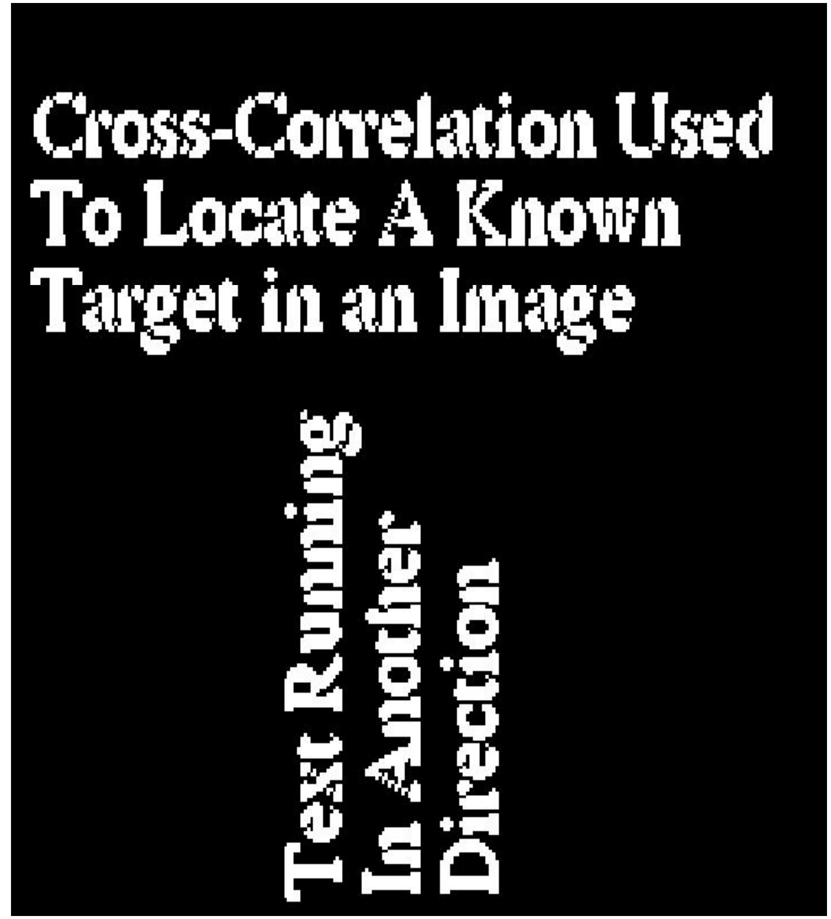




This definition becomes very intuitive when the structuring element *B* is viewed as a convolution mask.



Input Image



Dilated Output

Erosion

For the sets A and B in R^2, the erosion of A by B is defined as

$$A - B = \{ x \mid (B)_x \subseteq A \}$$

$$= \{ c \in R^2 \mid c + b \in a, \text{ for every } b \in B \}$$

$$= \bigcup_{h \in B} (A)_{-h}$$

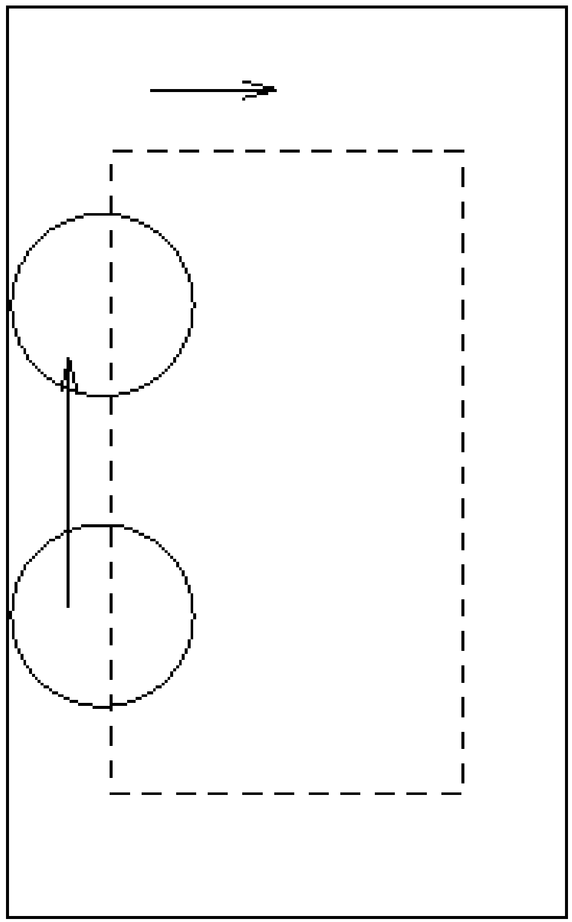
The erosion of A by B is defined as the set of all points x such that B_x is included in A. Clearly, erosion is a shrinking operation. For example, let

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

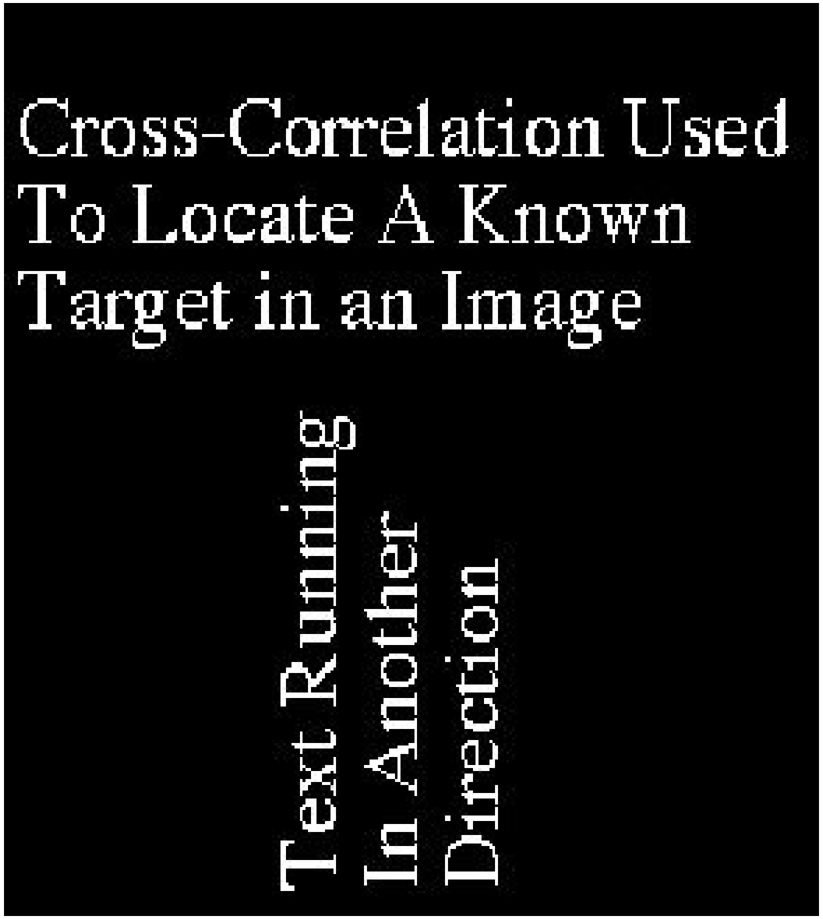
$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Dilation of } A \text{ by } B \text{ will be}: \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

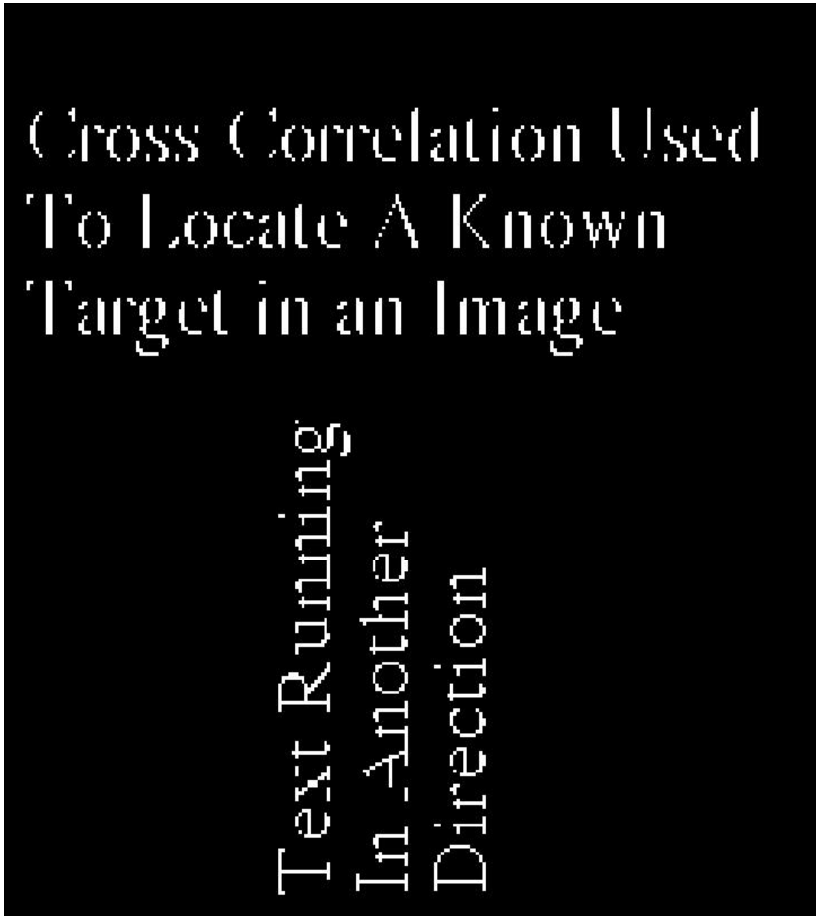
$$\text{Dilation of } A \text{ by } C \text{ will be}: \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



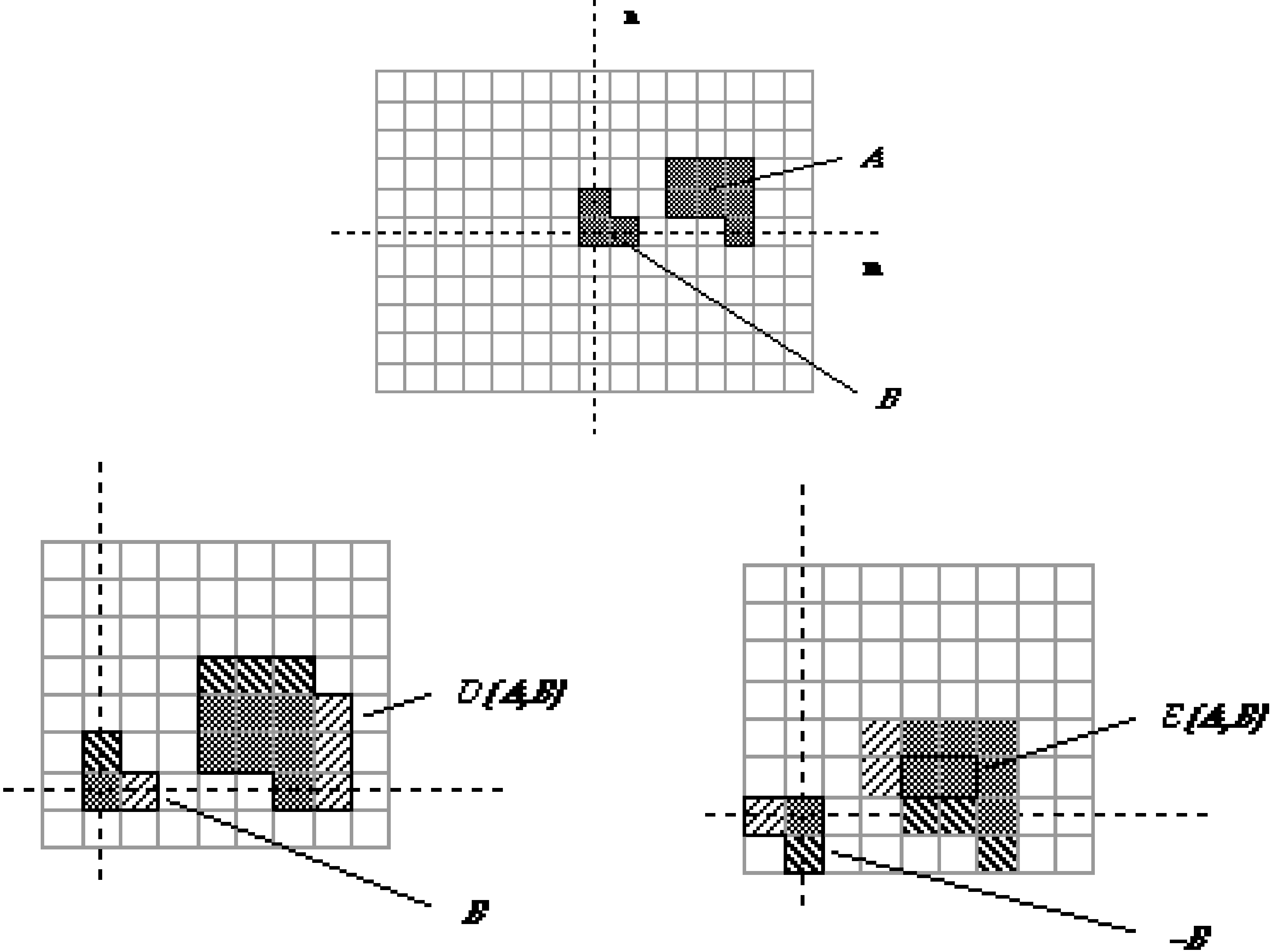
A **is eroded by the structuring element** ***B*** **to give the internal dashed shape**

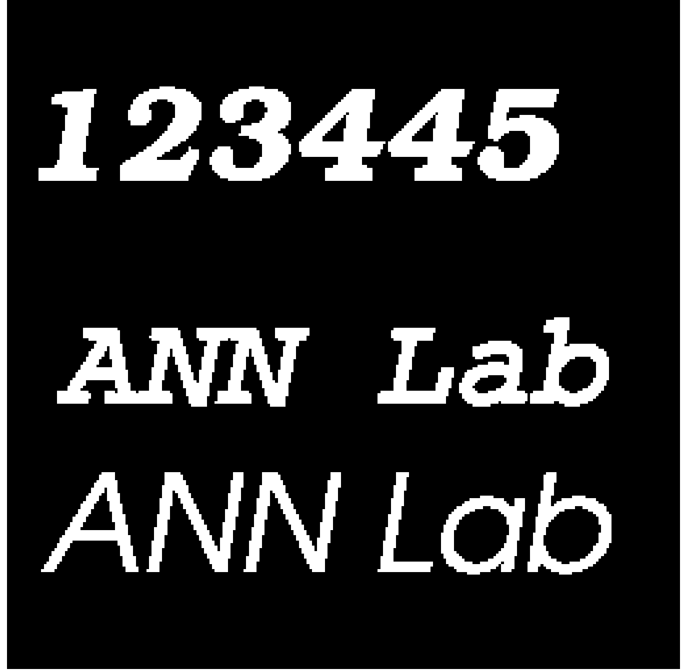


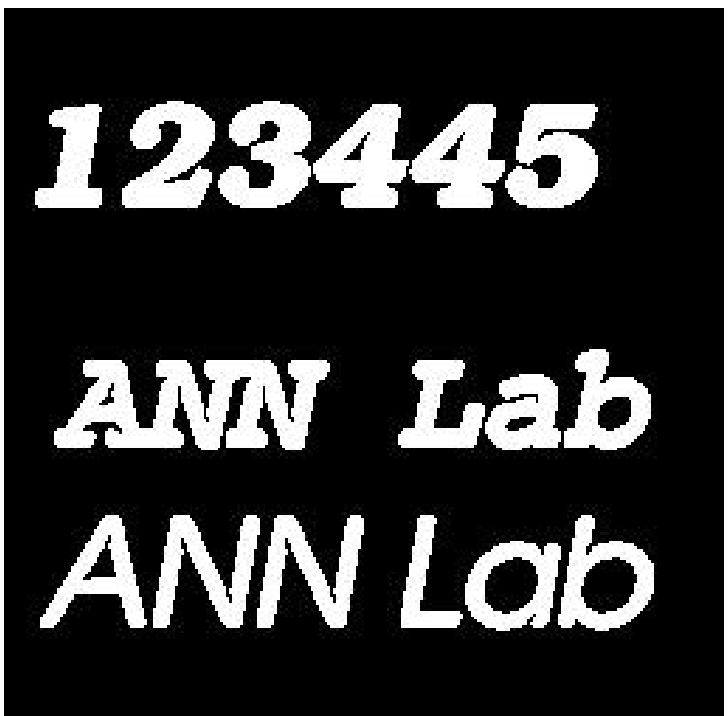
Input Image



Eroded Output


A
B
D(A,B)
B
E(A,B)
−B


123445
ANN Lab
ANN Lab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Two very important transformations are *opening* and *closing*. Now intuitively, dilation expands an image object and erosion shrinks it.

Opening generally smoothes a contour in an image, breaking narrow isthmuses and eliminating thin protrusions.

Closing tends to narrow smooth sections of contours, fusing narrow breaks and long thin gulfs, eliminating small holes, and filling gaps in contours.

The opening of *A* by *B*, is given by the erosion by *B*, followed by the dilation by *B*.

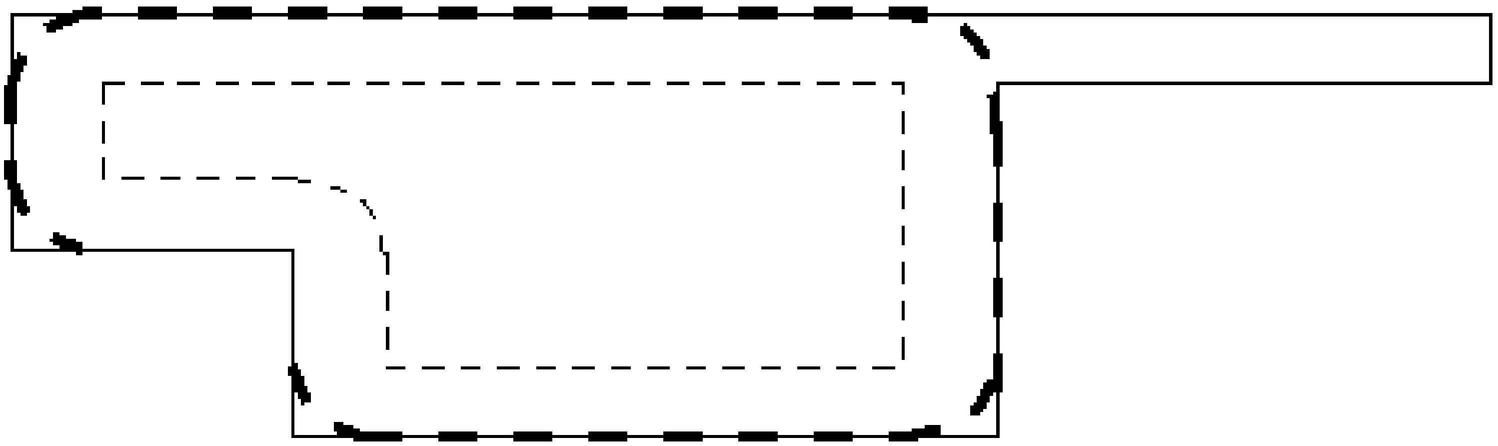
Opening of a set A by B is defined as:

$$A \circ B = (A - B) \oplus B$$

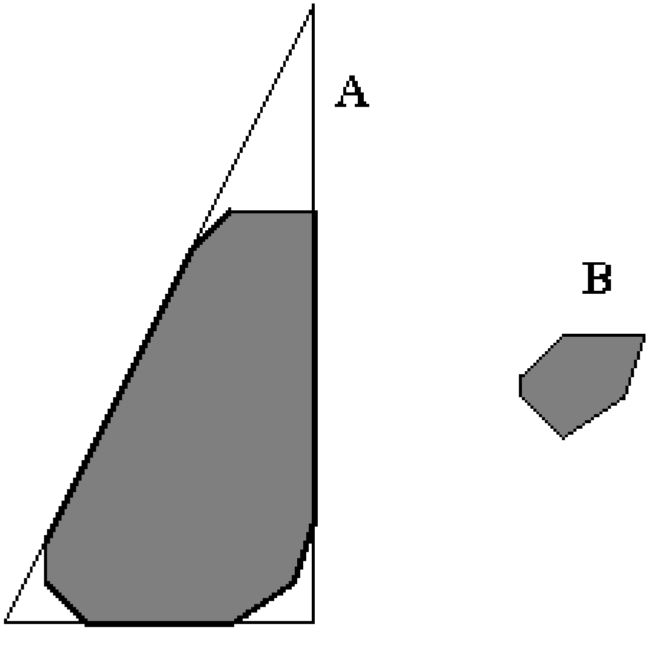
Closing is the dual operation of opening. It is produced by the dilation of *A* by *B*, followed by the erosion by *B*.

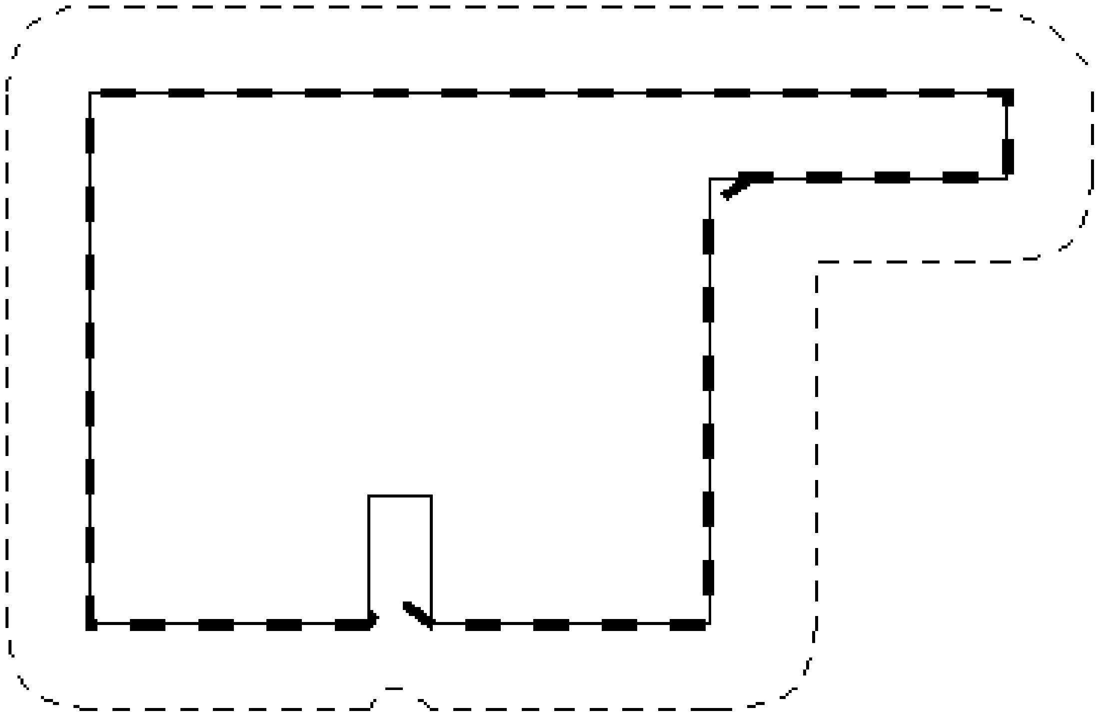
Closing of a set A by B is defined as:

$$A \bullet B = (A \oplus B) - B$$



The opening (given by the dark dashed lines) of *A* (given by the solid lines. The structuring element *B* is a disc. The internal dashed structure is *A* eroded by *B*.





The closing of *A* by the structuring element *B*.

This is like `smoothing from the outside'. Holes are filled in and narrow valleys are `closed'

A=

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Dilation

Erosion

Opening

Closing

Input Image

Dilation

Erosion