Shape from Shading

Computer Vision

CS635

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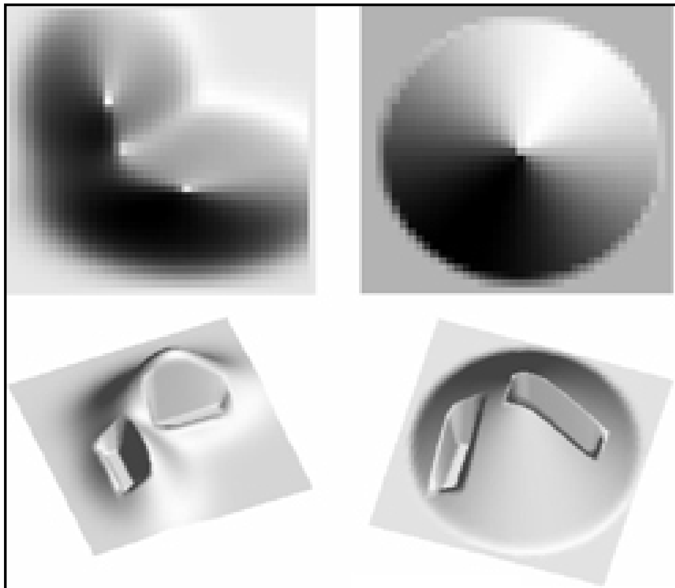
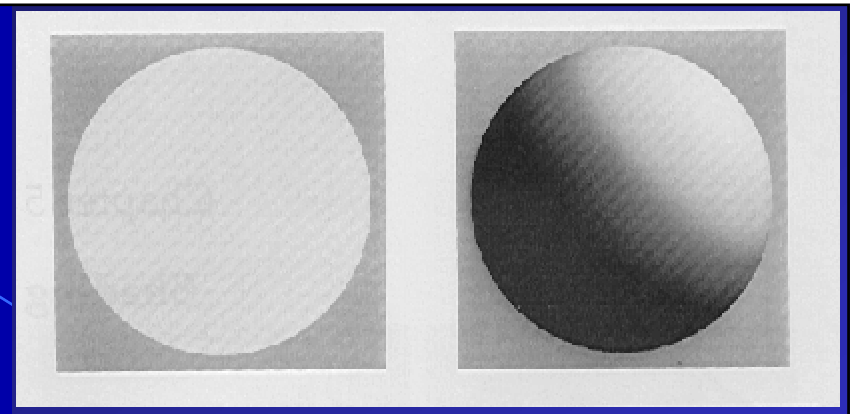
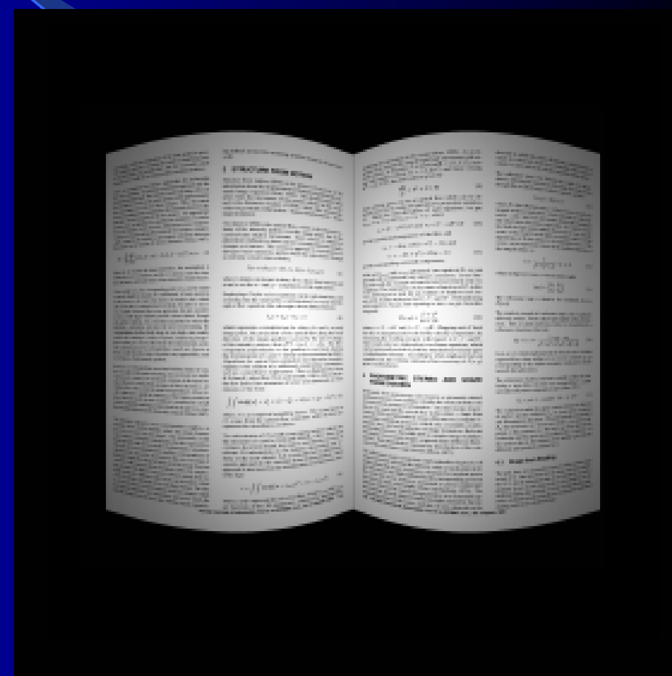
Introduction

- An image is essentially 2D where as the world is 3D
- The human visual system recovers shapes of objects in a 3D scene from a 2D image by a number of cues
  - Motion parallax
  - Binocular disparity
- But even a single image gives a lot of information about shape of an object. Where is the hidden information?
- Some examples to illustrate the point previously mentioned



Any answers?…Yes you are right…..
Its shading on the surface that gives the depth information and hence a cue to shape of the surface

Our visual system tries to interpret the brightness pattern on the retina as shading due to spatial fluctuations of the surface orientation and spatial variations in reflecting properties of the surface.

The Reflectance map

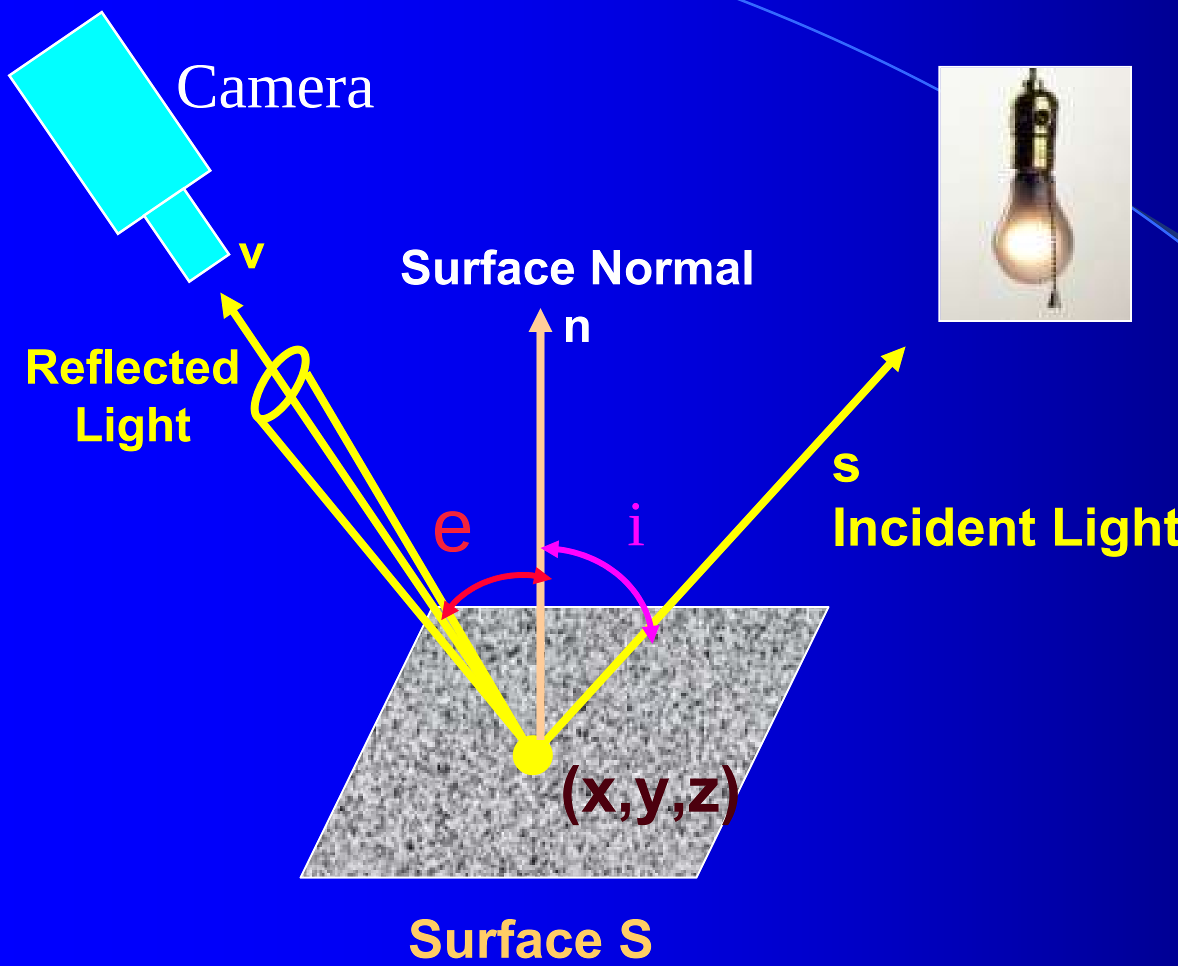


Image intensity can be related to that in the scene by this equation

$$I(x^{'}, y^{'}) = ka(x, y, z)\phi(n, s, v)$$

- ***S*** **– surface**
- ***(x,y,z)*** **– Point on the surface**
- ***n*** **– surface normal**
- ***s*** **– source direction**
- ***v*** **– viewing direction**
- ***i*** **– incident angle**
- ***e*** **– emergent angle**
- ***(x',y')*** **– Points on the image plane**
- ***a(x,y,z)*** **– Incident brightness at each point on the 3D scene**
- ***Φ(n,s,v)*** **– the reflecting properties of the surface in scene**

Reflectance functions

Ambient light: **I=k_i , where i indexes into the objects in the scene.**

Diffused reflection

$$\phi(n, s, v) = \rho \cos i$$

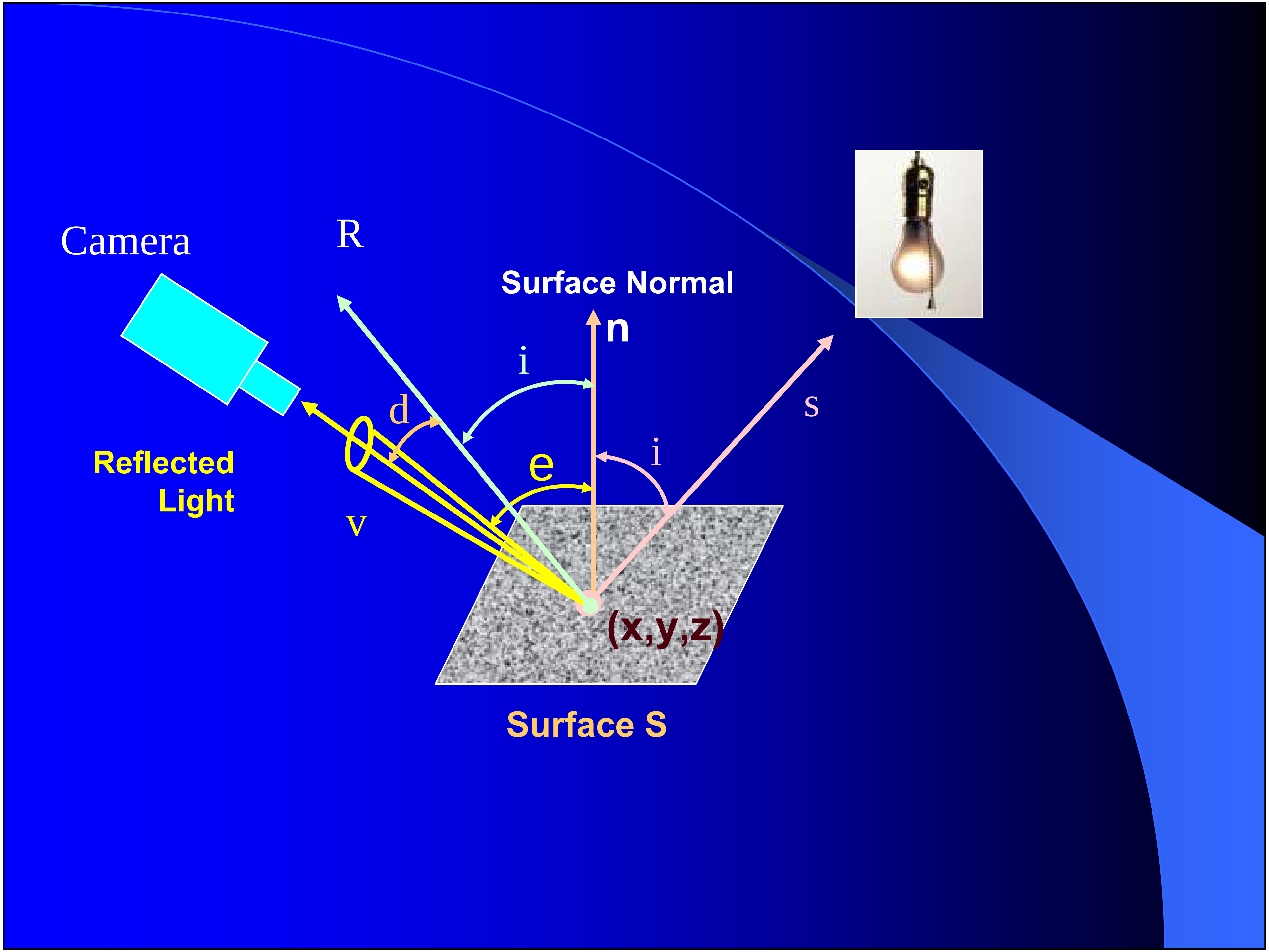
Specular reflection

$$\phi(n, s, v) = \begin{cases} 1 & s + v = n \\ 0 & else \end{cases}$$

Phong model

$$\phi(n, s, v) = \rho_1 \cos i + \rho_2 \cos^m d$$

Some examples to follow


Camera
R
Surface Normal
n
i
d
s
i
e
Reflected
Light
v
(x,y,z)
Surface S

REFLECTANCE MODELS

LAMBERTIAN MODEL

$$\phi(\mathbf{n},\mathbf{s},\mathbf{v}) = \rho \cos \mathbf{i}$$

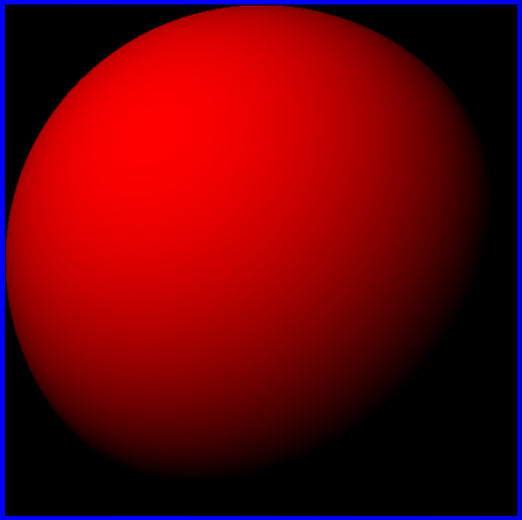
albedo

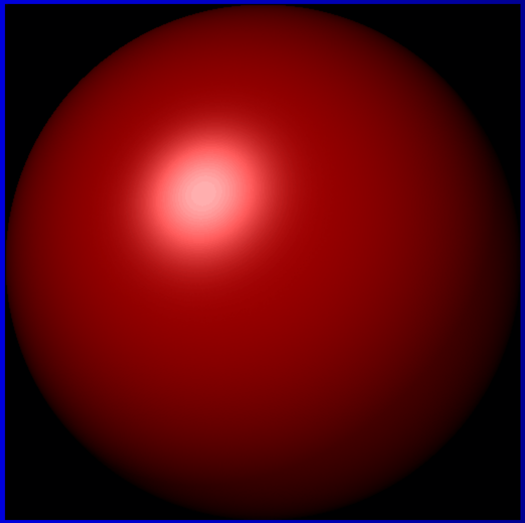
PHONG MODEL

$$\phi(\mathbf{n},\mathbf{s},\mathbf{v}) = \rho_1 \cos \mathbf{i} + \rho_2 \cos^m \mathbf{d}$$

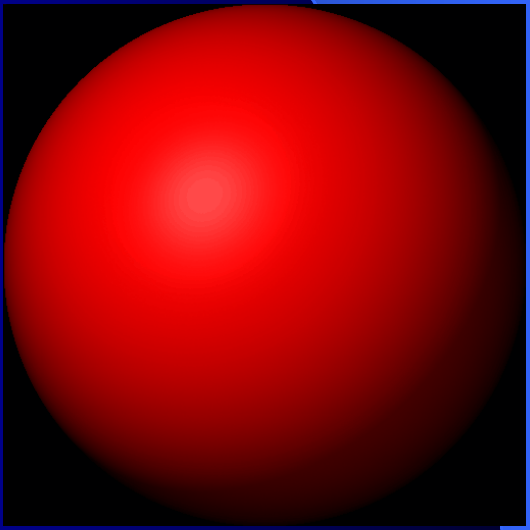
Diffuse albedo

Specular albedo





ρ_1=0.3, ρ_2 =0.7,m=2



ρ_1 =0.7, ρ_2 =0.3, m=0.5

Reflectance map (Contd..)

- **Assuming light source is at a distance, incident light at every point is assumed to be constant = a**

$$I(x^{'}, y^{'}) = ka\phi(n, s, v)$$

- **Reflectance at each point on surface depends on the surface properties and hence varies with a function Ø(n,s,v) which is directly proportional to the image intensity I(x',y')**
- **Since s and v are constants, Ø(n,s,v) is dependent on n alone**
- **Surface Normal n can be represented in gradient space (p-q space) yielding R(p,q), called the Reflectance map.**
- **What is the gradient space representation?.... What are the other ways to represent the surface orientation....**

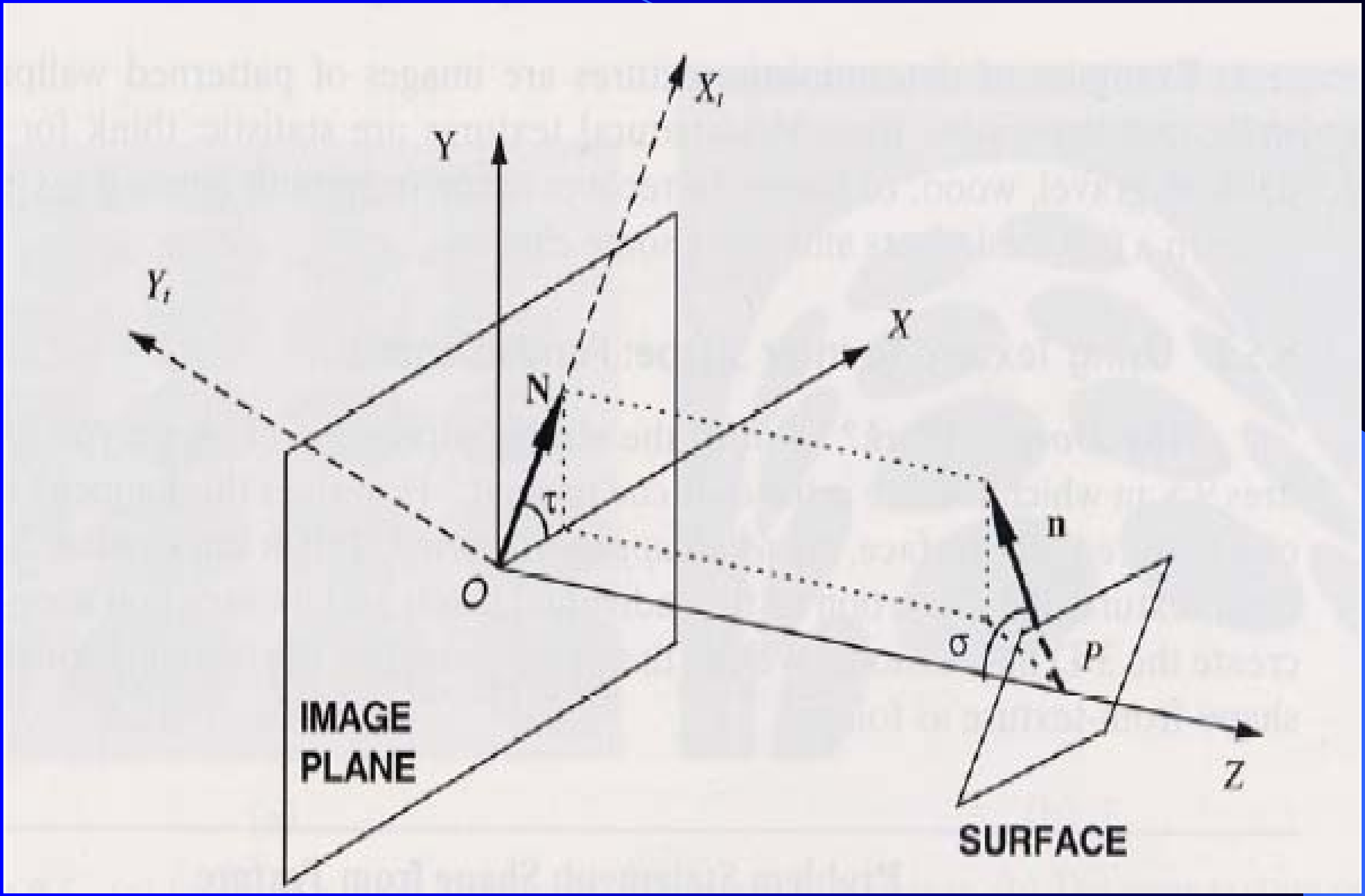
Representation of surface orientation

- **Surface normal : *n* =(n_1,n_2,n_3)**
- **Surface gradient: p-q space**
  - **Given the equation of a surface in 3D world as : z=f(x,y)**
  - **The surface gradient is defined as**

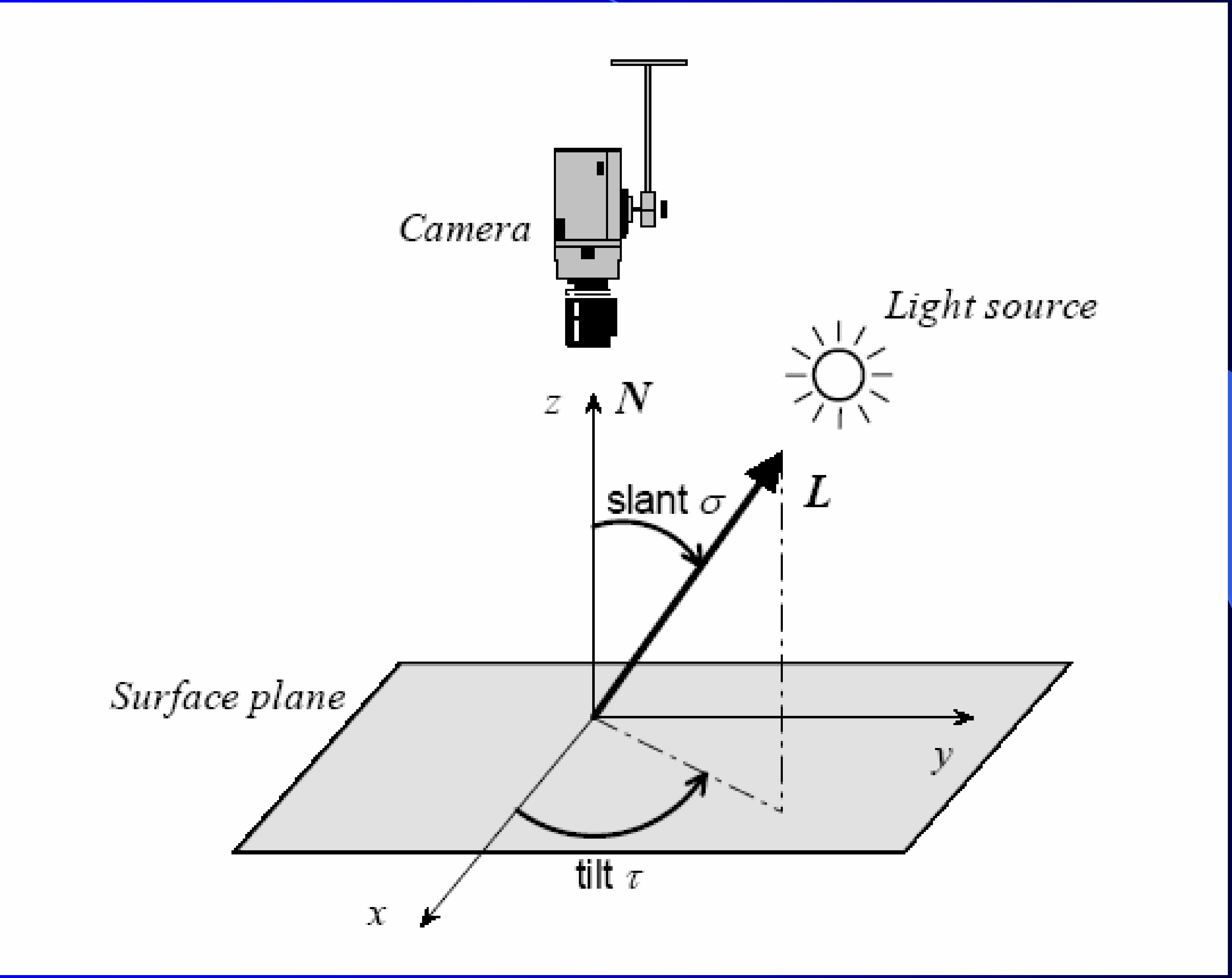
$$\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1\right), p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$

- **Slant and the Tilt angle (σ, τ)**
  - **σ is the angle made by the surface normal with z axis (3D world)**
  - **τ is the angle made by the projection of the normal on the image plane with the x axis (of image plane)**

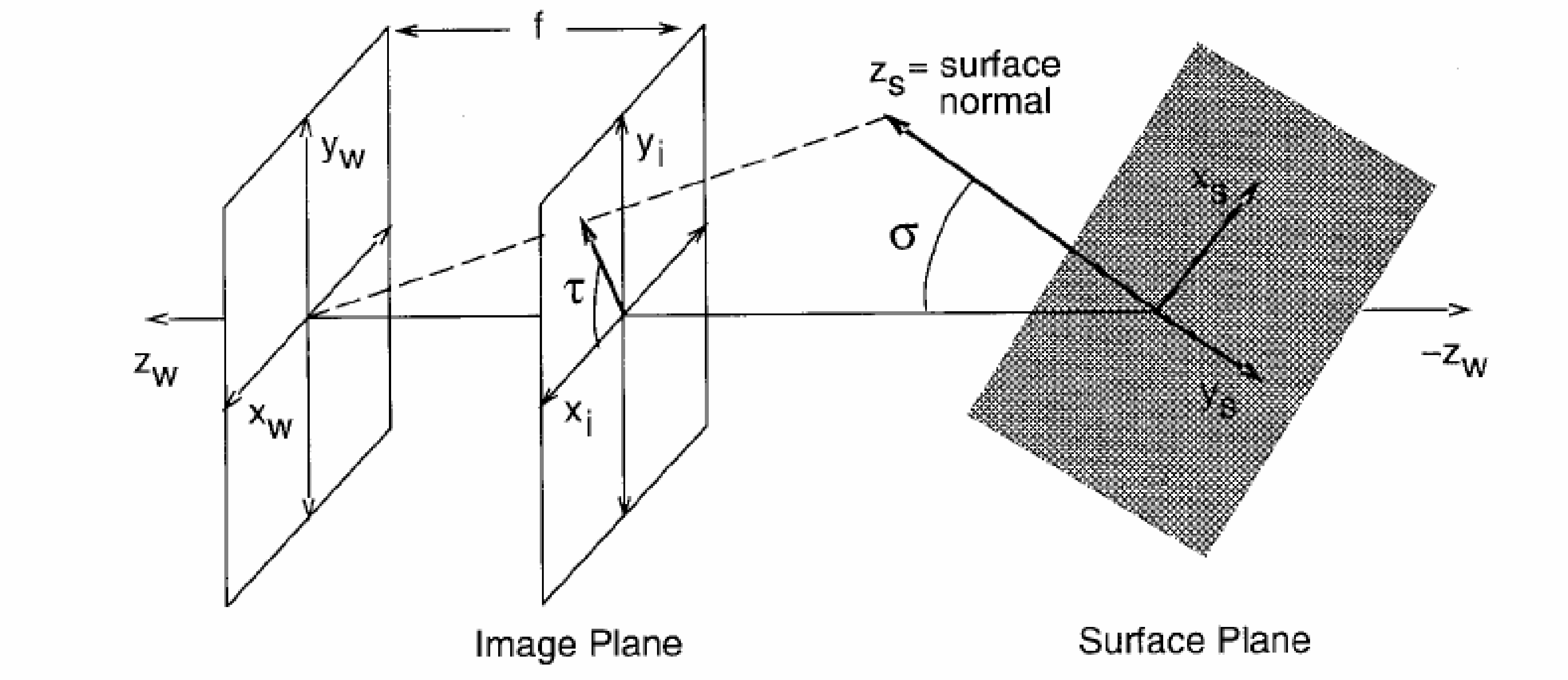
Slant and the tilt angle



Slant and the tilt angle



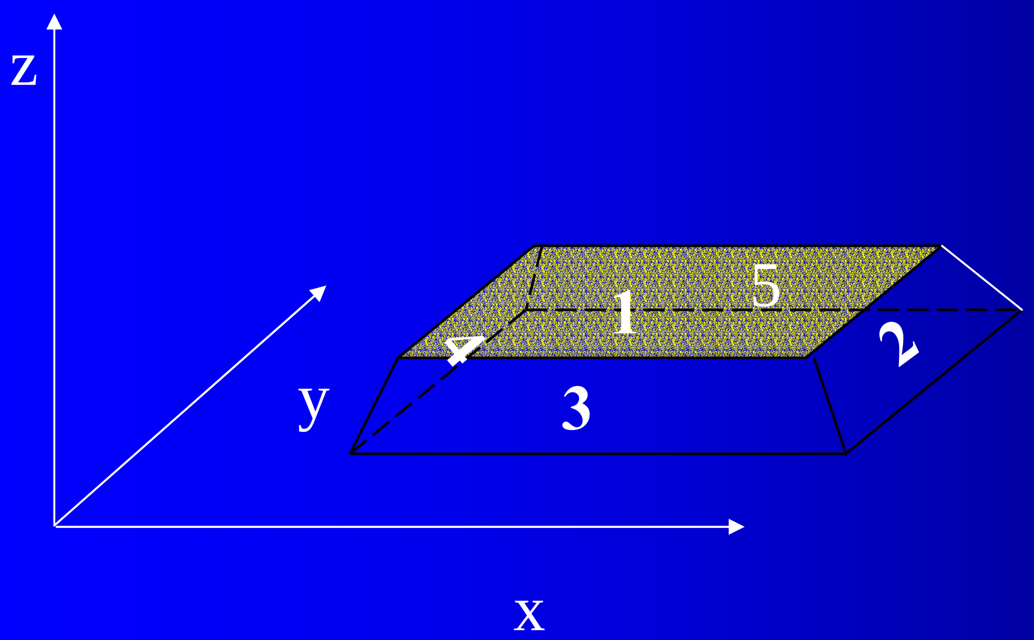
Slant and the tilt angle

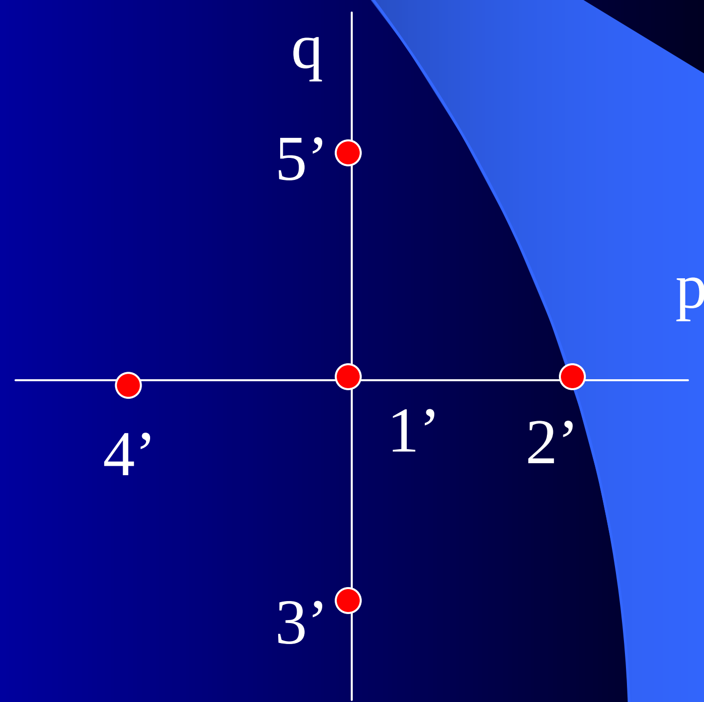


Gradient space representation

- **A plane parallel to x-y plane will have the gradient 0 in both x and y directions**
- **From p and q the equation of a plane can be recovered as**

$$z = px + qy + c$$





Reflectance map (Contd..)

- **The image irradiance can be related to the scene irradiance** $I(x, y) = R(\hat{n}(x, y))$
- **Since the surface normal can be represented using the gradient space representation**
  - **I(x, y) = R(p, q)**
- **R(p,q) is called the reflectance map of the image**
- **Our aim in the "shape from shading" problem is to recover the orientation (p, q) of the surface (or surface patch) given the image I(x, y)**

The shape from shading problem

- Each point in the image has only one attribute – the intensity and the surface orientation is defined by (p, q) . Is it possible to recover this from a single image?
- Yes… Provided …
  - We add some constraints on the object surface
    - Homogeneity assumption
  - Priory knowledge about the shape of the surface
- If homogeneity assumption is violated the shape perceived is quiet different from the one that actually is. e.g:- make up

The shape from shading problem (Contd…)

To formulate the shape from shading problem 2 issues need to be solved

- **Position of a point in the image with respect to its position in the 3D Scene**
  - **Projective Geometry is the answer**
- **What determines the brightness of each point on the surface**
  - **Reflecting properties of the surface (BRDF)**
  - **Illumination model used**

An example

- Lambertian (diffused) surface
- (p, q, 1) vector normal to the surface
- (p_s , q_s) vector in the direction of source s

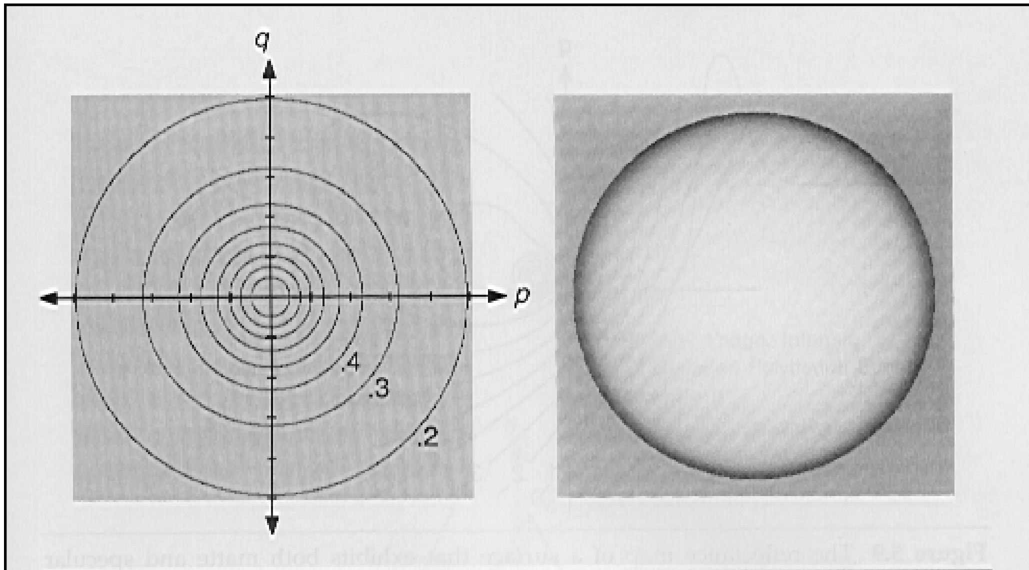
$$\cos i = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^2 + q_s^2}}$$

$$R(p,q) = \rho \cos i = \frac{\rho(1 + pp_s + qq_s)}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^2 + q_s^2}}$$

A contour (in p-q space) of constant intensity c (=R) is given by:

$$c = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^2 + q_s^2}}$$

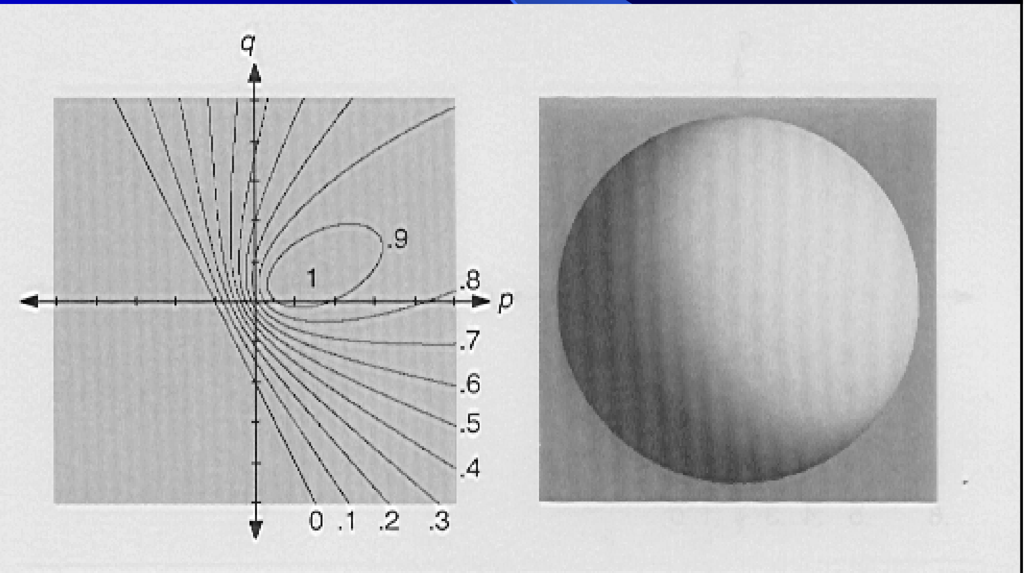
Check two cases: c = 0, 1.

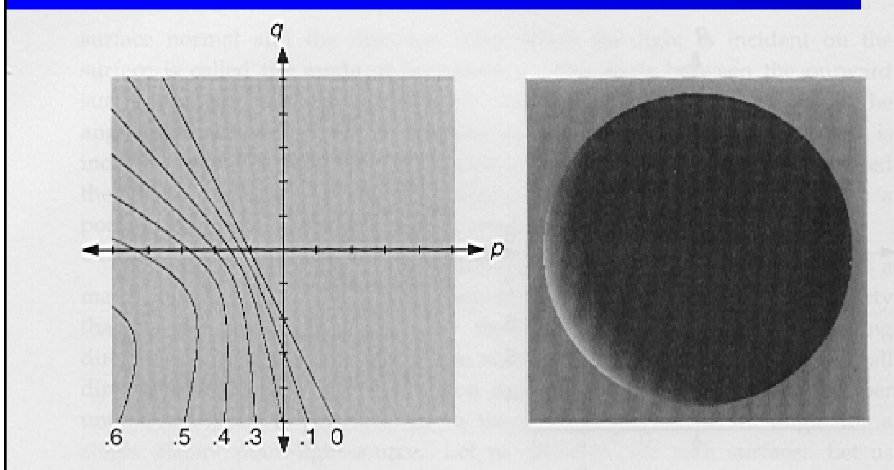


c=0	line
c=1	point
c=k	parabola
0 < c < k	hyperbola
k < c < 1	ellipse

$p_s = 0,\ \ q_s = 0.$

Circles are plotted for different values of c





$p_s = -2,\ \ q_s = -1.$

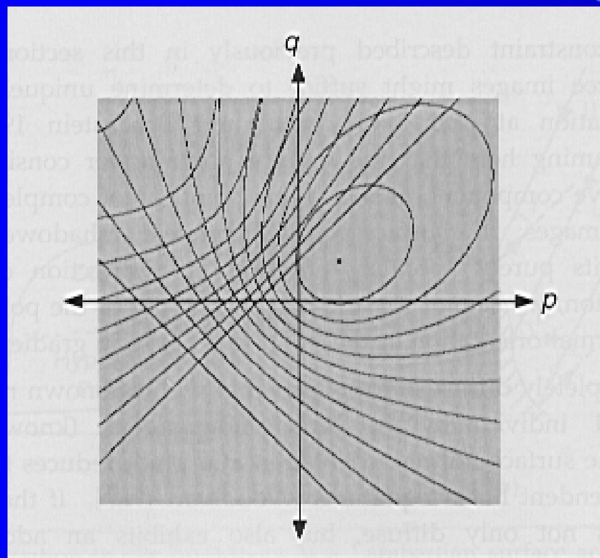
$p_s = 0.7,\ \ q_s = 0.$

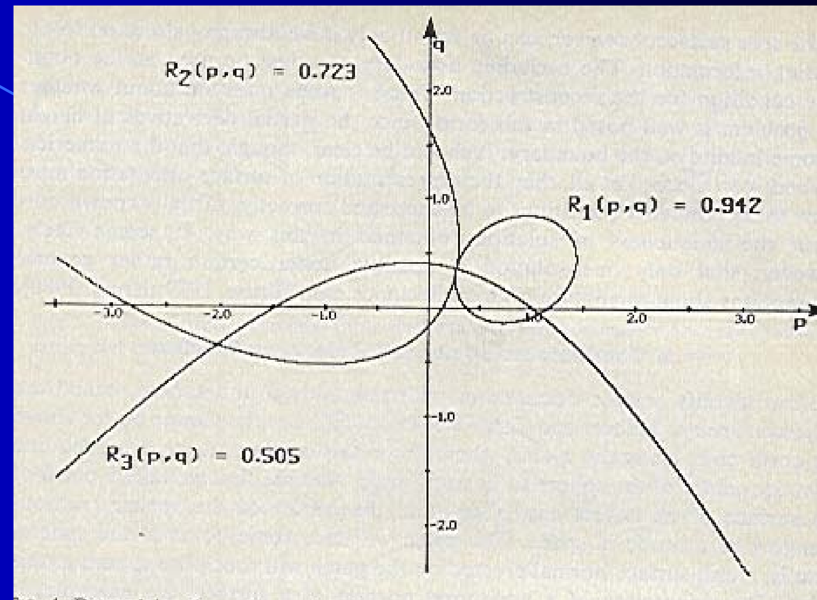
Contours are plotted for different values of c.

- **Hence for each intensity value and for each source direction we have a contour on which our orientation could lie.**
- **But a contour or a curve does not give a unique value of: p, q ……? what do we do?**

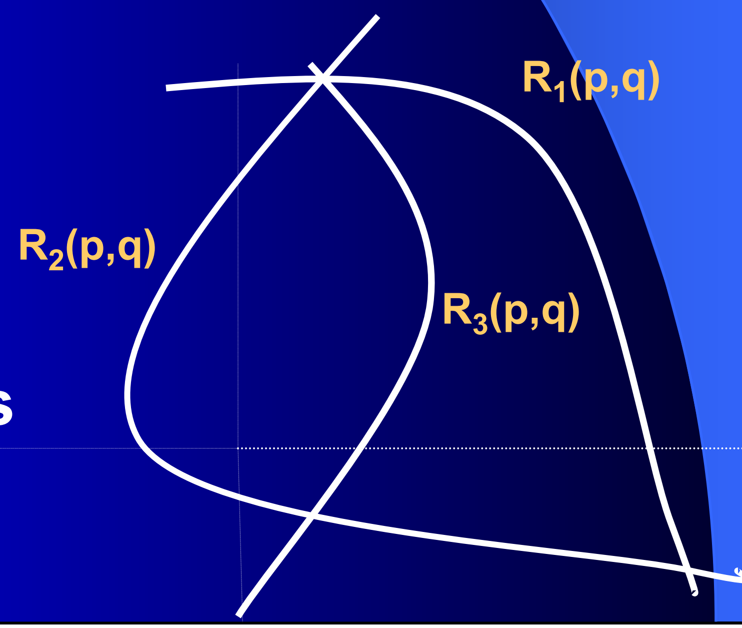
- **One solution is to have more than one image…. Photometric stereo**
- **Add constraints to the Object surface on the scene.**
  - **Parallel lines**
  - **Texture elements on the surface and its variation in the projected image**

Photometric stereo





- Use more than one image.
- Find the contours or curves for each one.
- The intersection of 2 curves gives 2 such possible points
- Intersection of 3 or more curves will give one unique value for (p, q)



Photometric stereo (Mathematical formulation)

- So we have 3 sets of
  - light source directions (s_{11},s_{12},s_{13}) (s_{21},s_{22},s_{23}) (s_{31},s_{32},s_{33})
  - resulting images $E_1(x,y)$, $E_2(x,y)$, $E_3(x,y)$
  - resulting reflectance maps $R_1(p,q)$, $R_2(p,q)$, $R_3(p,q)$

$$\overline{E}(x,y) = \rho \cos(\overline{s}_k . \overline{n})$$

$$\rho = \left| S^{-1}\overline{E} \right|$$

$$\begin{bmatrix} E_1(x,y) \\ E_2(x,y) \\ E_3(x,y) \end{bmatrix} = \rho \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{bmatrix} n_1(x,y) \\ n_2(x,y) \\ n_3(x,y) \end{bmatrix}$$

$$\overline{n} = \left(\frac{1}{\rho} \right) S^{-1}\overline{E}$$

Adding Geometric Constraints to the scene

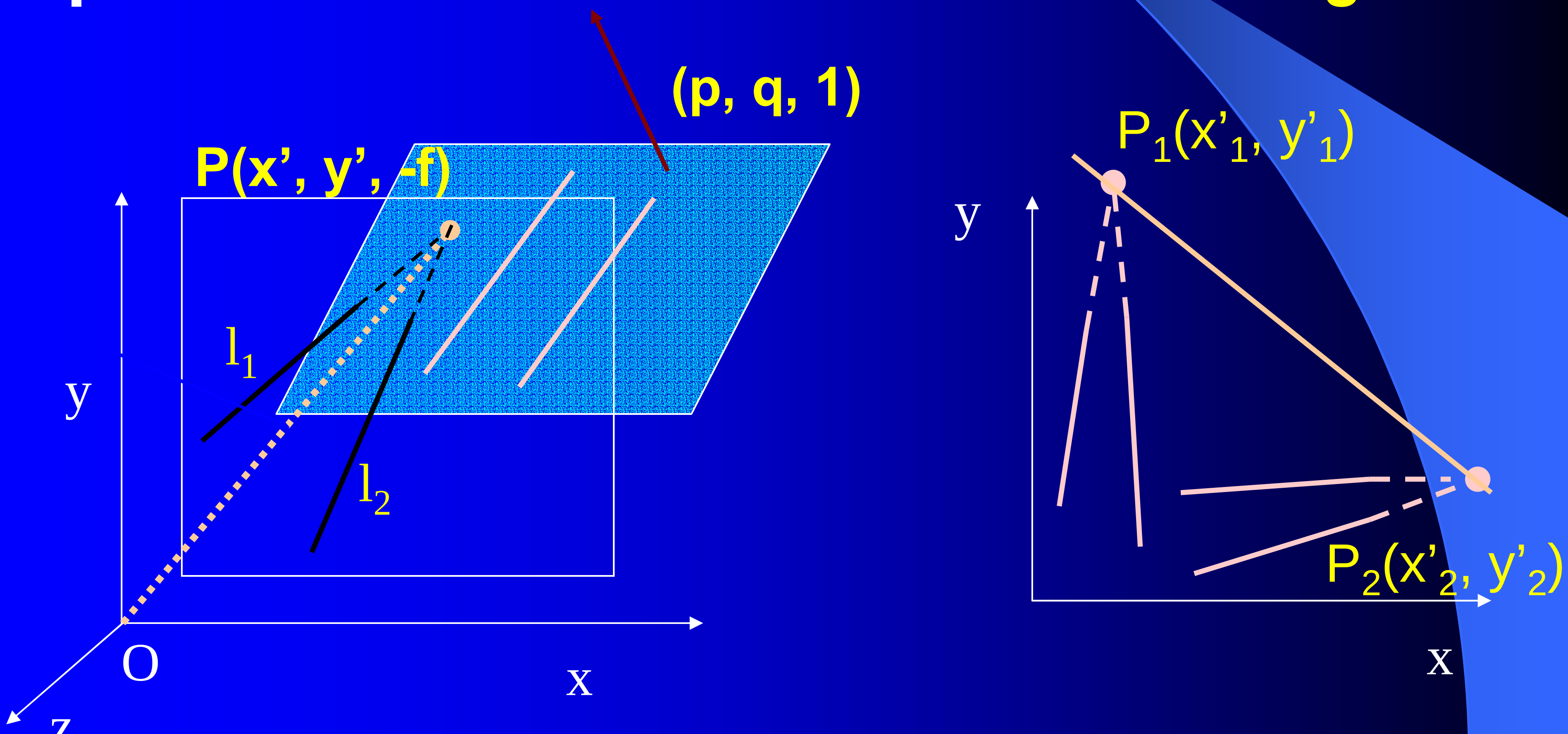
- **X = (x, y, z)** **3D world coordinate**
- **X' = (x', y')** **Image point**
- **X' = f(X)** **Perspective projection**
- **If we know m constraints relating n points in the scene then we have the following set of simultaneous equations.**

$$h_1(X_1, X_2 \dots X_n) = 0 \qquad X_1' = f(X_1)$$
$$h_2(X_1, X_2 \dots X_n) = 0 \qquad X_2' = f(X_2)$$
$$\vdots \qquad \& \qquad \vdots$$
$$h_m(X_1, X_2 \dots X_n) = 0 \qquad X_n' = f(X_n)$$

Solve to get equation relating (x,y,z) to equation of the curve.

An example

- Projections of parallel lines in the scene meet at a **vanishing point** in the image.
- Any pair of parallel lines meet on the vanishing point. Line OP is termed as the **vanishing line.**



Generating the constraints

- P_1 and P_2 are both vanishing points, and OP_1 and OP_2 are perpendicular to the surface normal

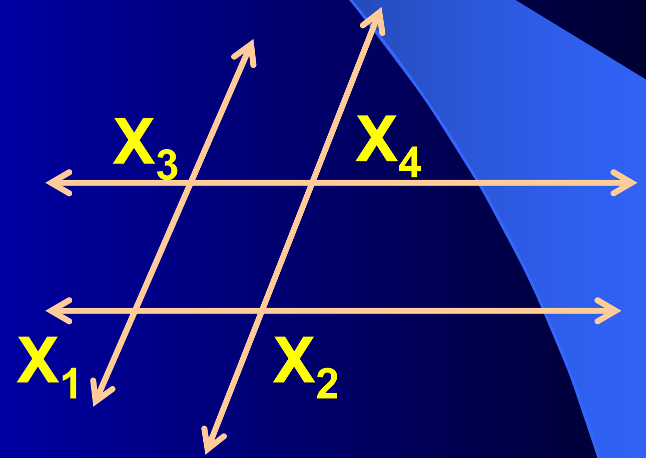
$$x_1' p + y_1' q - f = 0,$$
$$x_2' p + y_2' q - f = 0$$

Generating the constraints

- Consider these set of parallel lines

$$\overline{X}_2 - \overline{X}_1 = \overline{X}_4 - \overline{X}_3 \quad \Rightarrow 3$$

$$\left.\begin{aligned} X_1' &= f(\overline{X}_1) \\ X_1' &= f(\overline{X}_2) \\ X_1' &= f(\overline{X}_3) \\ X_1' &= f(\overline{X}_4) \end{aligned}\right\} \Rightarrow 8$$



11 constraints to find 12 unknowns

End of lectures on

Shape from Shading

Slides courtesy: *Shivani G. Rao*