Image Sequence (or Motion) Analysis

also referred as:

Dynamic Scene Analysis

Input: *A Sequence of Images (or frames)*

Output: ***Motion parameters*** **(3-D) of the objects/s in motion**

and/or

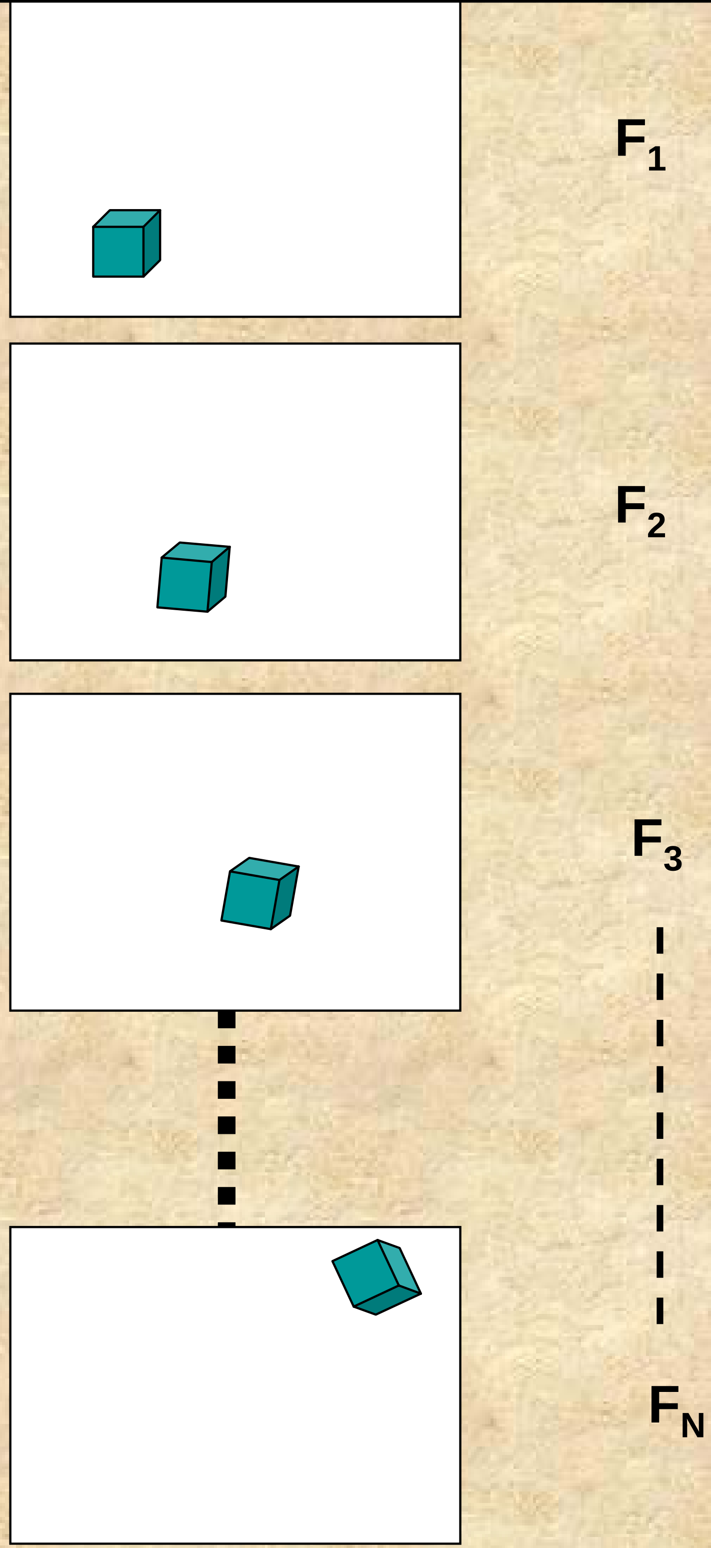
3-D Structure information **of the moving objects**

Assumption: **The time interval ΔT between two consecutive or successive pair of frames is small.**

Typical steps in Dynamic Scene Analysis

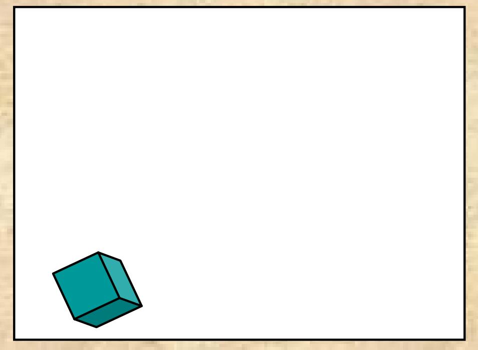
- **Image filtering and enhancement**
- **Tracking**
- **Feature detection**
- **Establish feature correspondence**
- **Motion parameter estimation (local and global)**
- **Structure estimation (optional)**
- **Predict occurrence in next frame**

TRACKING in a Dynamic Scene



Since ΔT is small,

the amount of motion / displacement of objects, between two successive pair of frames is also small.



At 25 frames per sec (fps): ΔT = 40 msec.

At 50 frames per sec (fps): ΔT = 20 msec.

Image Motion

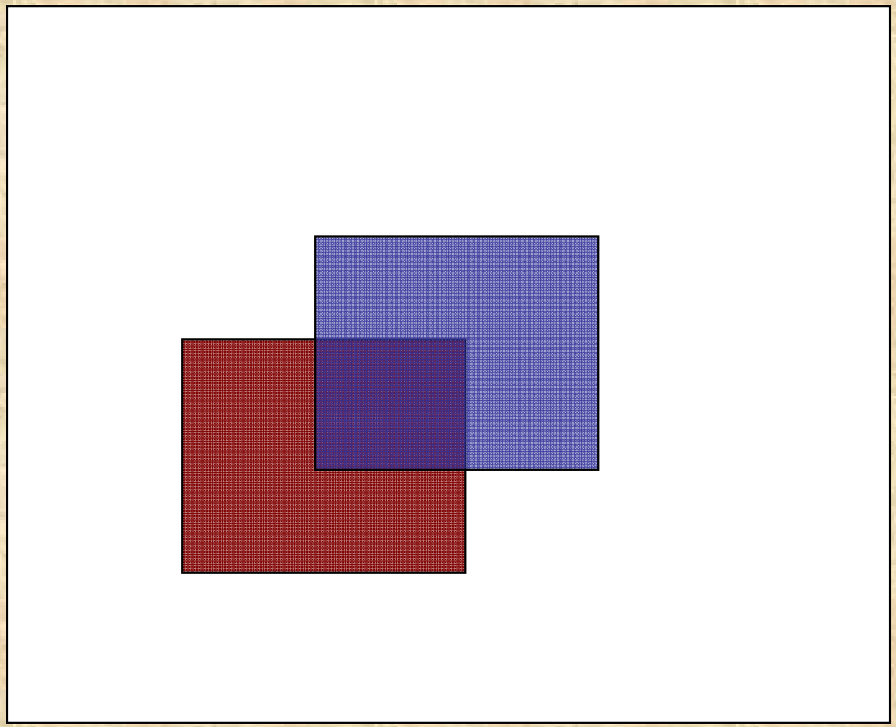


Image changes by difference equation:

$$f_d(x_1, x_2, t_i, t_j)$$
$$= f(x_1, x_2, t_i) - f(x_1, x_2, t_j)$$
$$= f(t_i) - f(t_i) = f_i - f_j$$

Accumulated difference image:

$$f_T(X, t_n) = f_d(X, t_{n-1}, t_n) - f_T(X, t_{n-1}); n \geq 3,$$
$$where,\ f_T(X, t_2) = f_d(X, t_2, t_1)$$

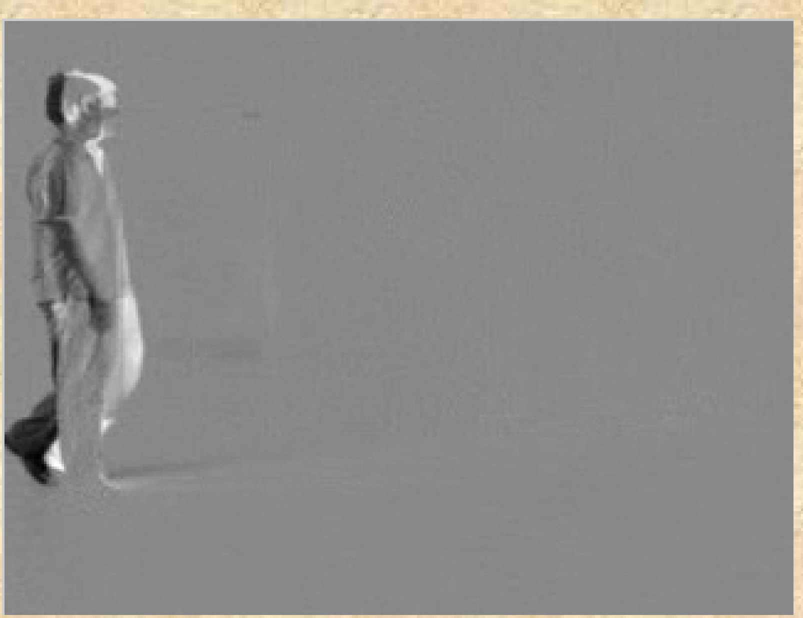
Moving Edge (or feature) detector:

$$F_{mov\_feat}(X, t_1, t_2) = \left|\frac{\partial f}{\partial X}\right| . \left| f_d(X, t_1, t_2) \right|$$

Recent methods include background and foreground modeling.

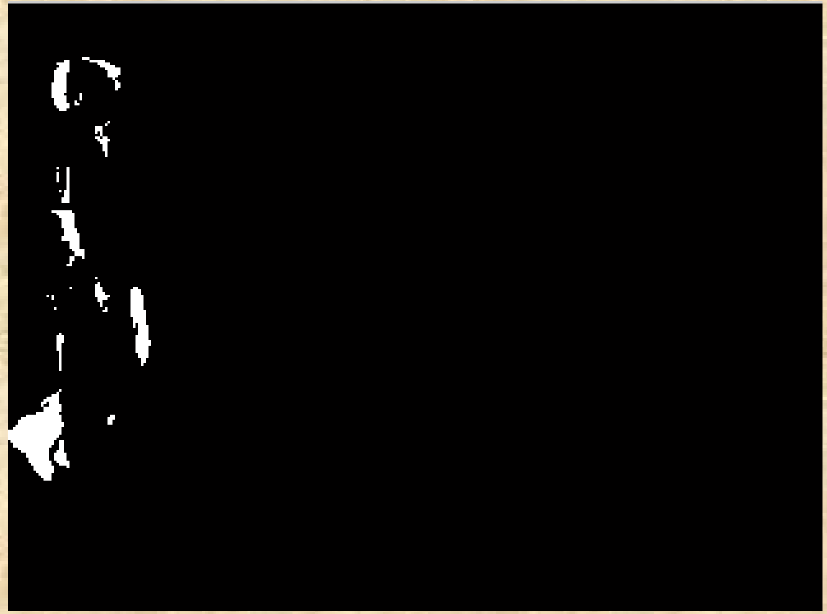
F_1

F_2

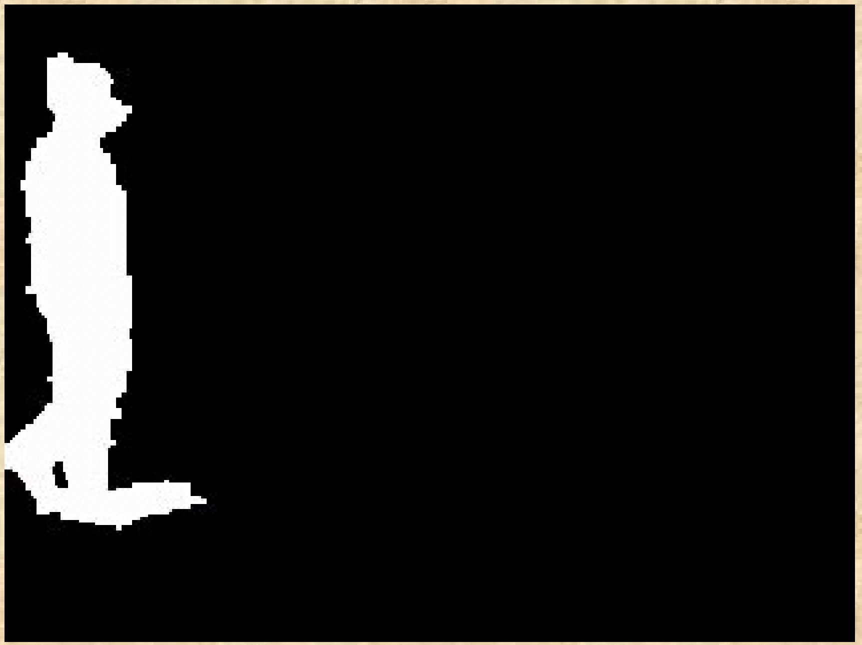


$F_1 - F_2$

$abs(F_1 - F_2)$



Input Video Frame



Segmented Video frame

Extracted moving object using Alpha matte

Two categories of Visual Tracking Algorithms:

Target Representation and Localization; Filtering and Data Association.

A. Some common ***Target Representation and Localization* algorithms**:

Blob tracking: Segmentation of object interior (for example **blob detection, block-based correlation or optical flow**)

Kernel-based tracking (**Mean-shift tracking**): An iterative localization procedure based on the maximization of a similarity measure (**Bhattacharyya coefficient**).

Contour tracking: Detection of object boundary (e.g. **active contours or Condensation algorithm**)

Visual **feature matching**: Registration

B. Some common ***Filtering and Data Association* algorithms:**

Kalman filter: An optimal recursive Bayesian filter for linear functions and Gaussian noise.

Particle filter: Useful for sampling the underlying state-space distribution of non-linear and non-Gaussian processes.

Also see: **Match moving; Motion capture; Swistrack**

BLOB Detection – Approaches used:

- **Corner detectors (Harris, Shi & Tomashi, Susan, Level Curve Curvature, Fast etc.)**
- **LOG, DOG, DOH (Det. Of Hessian), Ridge detectors, Scale-space, Pyramids**
- **Hessian affine, SIFT (Scale-invariant feature transform)**
- **SURF (Speeded Up Robust Features)**
- **GLOH (Gradient Location and Orientation Histogram)**
- **LESH (Local Energy based Shape Histogram).**

Complexities and issues in tracking:

Need to overcome difficulties that arise from noise, occlusion, clutter, moving cameras, multiple moving objects and changes in the foreground objects or in the background environment.

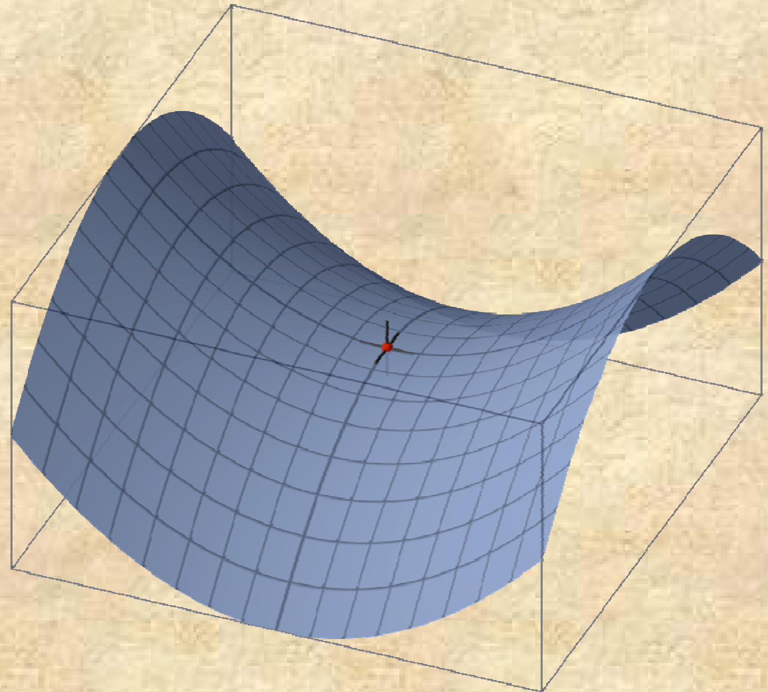
DOH - **the scale-normalized determinant of the Hessian, also referred to as the Monge–Ampère operator,**

$$\det H L(x, y; t) = t^2 (L_{xx} L_{yy} - L_{xy}^2)$$

where, *HL* denotes the Hessian matrix of *L* and then detecting scale-space maxima / minima of this operator one obtains another straightforward differential blob detector with automatic scale selection which also responds to saddles.

$$(\hat{x}, \hat{y}; \hat{t}) = \operatorname{argmaxminlocal}_{(x,y;t)}(\det H L(x, y; t))$$

Hessian Affine: $H(\mathbf{x}) = \begin{bmatrix} L_{xx}(\mathbf{x}) & L_{xy}(\mathbf{x}) \\ L_{xy}(\mathbf{x}) & L_{yy}(\mathbf{x}) \end{bmatrix}$



SURF is based on a set of 2-D HAAR wavelets; implements DOH

SIFT:
Detect extremas at various scales:

Four major steps:

1. **Scale-space extrema detection**
2. **Keypoint localization**
3. **Orientation assignment**
4. **Keypoint descriptor**

Only three steps (1, 3 & 4) are shown below:

$$D(x,y,\sigma) = L(x,y,k_i\sigma) - L(x,y,k_j\sigma)$$

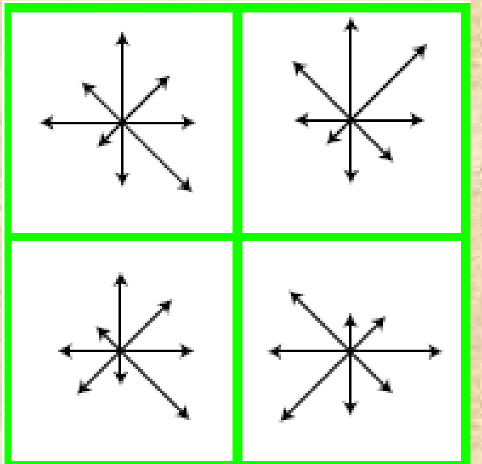
$$L(x,y,k\sigma) = G(x,y,k\sigma) * I(x,y)$$

$$\theta(x,y) = \tan^{-1}\left(\frac{L(x,y+1) - L(x,y-1)}{L(x+1,y) - L(x-1,y)}\right)$$

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

Histograms contain 8 bins each, and each descriptor contains an array of 4 histograms around the keypoint. This leads to a SIFT feature vector with (4 x 4 x 8 = 128 elements).

e.g.
2*2*8:



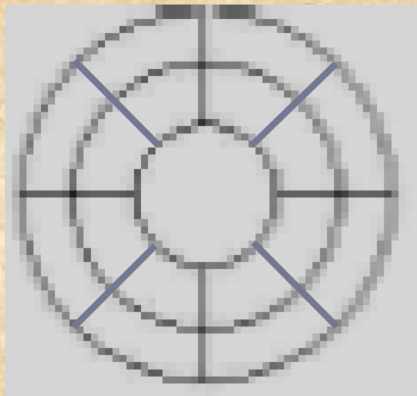
GLOH (Gradient Location and Orientation Histogram)

Gradient location-orientation histogram (GLOH) is an extension of the SIFT descriptor designed to increase its robustness and distinctiveness. The SIFT descriptor is computed for a log-polar location grid with 3 bins in radial direction (the radius set to 6, 11 and 15) and 8 in angular direction, which results 17 location bins.

Note that the central bin is not divided in angular directions. The gradient orientations are quantized in 16 bins. This gives a 272 bin histogram.

The size of this descriptor is reduced with PCA. The covariance matrix for PCA is estimated on 47 000 image patches collected from various images (see section III-C.1). The 128 largest eigenvectors are used for description.

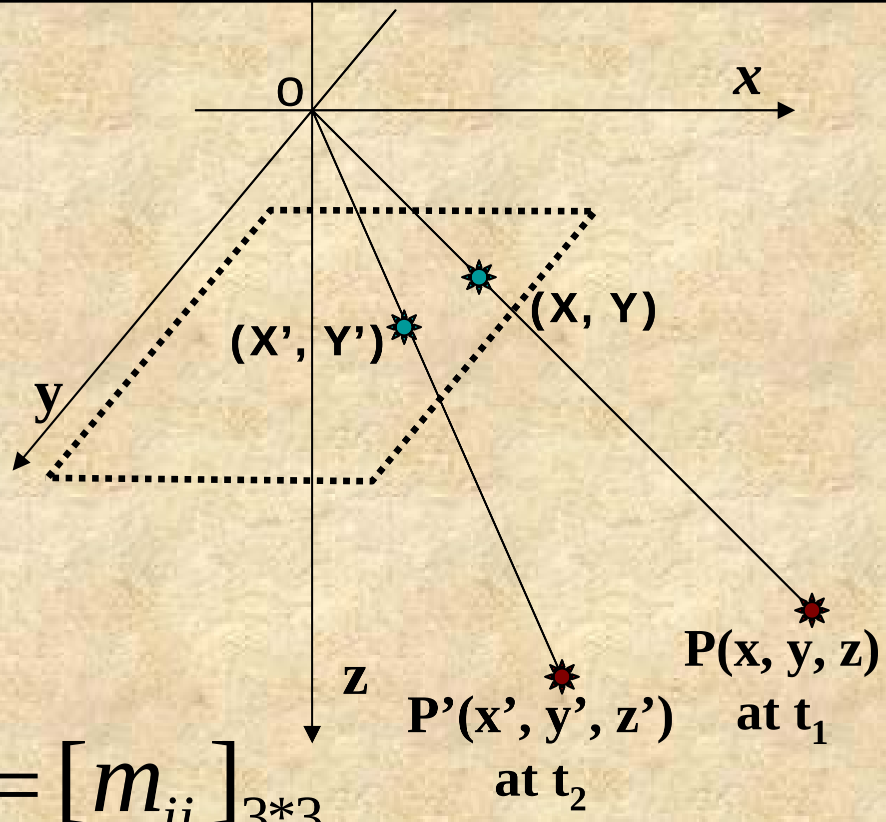
Gradient location and orientation histogram (*GLOH*) is a new descriptor which extends SIFT by changing the location grid and using PCA to reduce the size.



Motion Equations:

Models derived from mechanics: Euler's or Newton's equations.

P' = R.P + T, where:



$$m_{11} = n_1^2 + (1 - n_1^2)\cos\theta$$

$$m_{12} = n_1 n_2 (1 - \cos\theta) - n_3 \sin\theta$$

$$m_{13} = n_1 n_3 (1 - \cos\theta) + n_2 \sin\theta$$

$$m_{21} = n_1 n_2 (1 - \cos\theta) + n_3 \sin\theta$$

$$m_{22} = n_2^2 + (1 - n_2^2)\cos\theta$$

$$m_{23} = n_2 n_3 (1 - \cos\theta) - n_1 \sin\theta$$

$$m_{31} = n_1 n_3 (1 - \cos\theta) - n_2 \sin\theta$$

$$m_{32} = n_2 n_3 (1 - \cos\theta) + n_1 \sin\theta$$

$$m_{33} = n_3^2 + (1 - n_3^2)\cos\theta$$

$$R = [m_{ij}]_{3*3}$$

$$T = [\partial x \quad \partial y \quad \partial z]^T$$

where: $n_1^2 + n_2^2 + n_3^2 = 1$

Observation in the image plane:
U = X' – X; V = Y' – Y.

$$X = Fx / z; Y = Fy / z.$$

$$X' = Fx' / z'; Y = Fy' / z'.$$

Mathematically (for any two successive frames), the problem is:

Input: Given $(X, Y), (X', Y')$;

Output: Estimate $n_1, n_2, n_3, \theta, \delta x, \delta y, \delta z$

First look at the problem of estimating motion parameters using 3D knowledge only:

Given only <u>three (3) non-linear equations</u>, you have to obtain <u>seven (7) parameters</u>.

Need a few more constraints and may be assumptions too.

Since ΔT is small, θ must also be small enough (in radians). Thus R simplifies (reduces) to:

$$R\Big|_{\theta \to 0} = \begin{bmatrix} 1 & -n_3\theta & n_2\theta \\ n_3\theta & 1 & -n_1\theta \\ -n_2\theta & n_1\theta & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\phi_3 & \phi_2 \\ \phi_3 & 1 & -\phi_1 \\ -\phi_2 & \phi_1 & 1 \end{bmatrix}$$

where, $\phi_1^2 + \phi_2^2 + \phi_3^2 = \theta^2$

Evaluate $|R|$.

Now (problem linearized), given three (3) linear equations, you have to obtain six (6) parameters. - Solution ?

Take two point correspondences: $P_1' = R.P_1 + T;\ \ P_2' = R.P_2 + T;$

Subtracting one from the other, gives:
(eliminates the translation component) $(P_1' - P_2') = R.(P_1 - P_2)$

$$\begin{bmatrix} \Delta x_{12}' \\ \Delta y_{12}' \\ \Delta z_{12}' \end{bmatrix} = \begin{bmatrix} 1 & -\phi_3 & \phi_2 \\ \phi_3 & 1 & -\phi_1 \\ -\phi_2 & \phi_1 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{12} \\ \Delta y_{12} \\ \Delta z_{12} \end{bmatrix},$$

Solve for: $\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix};$

$$\textit{where}, \Delta x_{12} = x_1 - x_2,$$

$$\Delta x_{12}' = x_1' - x_2';$$

and so on .... for y and z.

$$\nabla_{12} = \begin{bmatrix} 0 & \Delta z_{12} & -\Delta y_{12} \\ -\Delta z_{12} & 0 & \Delta x_{12} \\ \Delta y_{12} & -\Delta x_{12} & 0 \end{bmatrix}$$

Re-arrange to form:

$$[\nabla_{12}] \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \Delta_{12}^2 \quad \textit{and} \quad \Delta_{12}^2 = \begin{bmatrix} \Delta x_{12}' - \Delta x_{12} \\ \Delta y_{12}' - \Delta y_{12} \\ \Delta z_{12}' - \Delta z_{12} \end{bmatrix}$$

$$[\nabla_{12}]\begin{bmatrix}\phi_1\\\phi_2\\\phi_3\end{bmatrix}=\Delta_{12}^2$$

$$\nabla_{12}=\begin{bmatrix}0 & \Delta z_{12} & -\Delta y_{12}\\ -\Delta z_{12} & 0 & \Delta x_{12}\\ \Delta y_{12} & -\Delta x_{12} & 0\end{bmatrix} \text{ and } \Delta_{12}^2=\begin{bmatrix}\Delta x_{12}^{'}-\Delta x_{12}\\ \Delta y_{12}^{'}-\Delta y_{12}\\ \Delta z_{12}^{'}-\Delta z_{12}\end{bmatrix}$$

∇_{12} **is a skew-symmetric matrix.**

$$|\nabla_{12}|=0$$

Interprete, why is it so ?

So what to do ?

Contact a Mathematician?

Take two (one pair) more point correspondences:

$$[\nabla_{34}]\begin{bmatrix}\phi_1\\\phi_2\\\phi_3\end{bmatrix}=\Delta_{34}^2$$

$$\nabla_{34}=\begin{bmatrix}-\Delta z_{34} & 0 & \Delta x_{34}\\ \Delta y_{34} & -\Delta x_{34} & 0\\ 0 & \Delta z_{34} & -\Delta y_{34}\end{bmatrix} \text{ and } \Delta_{34}^2=\begin{bmatrix}\Delta y_{34}^{'}-\Delta y_{34}\\ \Delta z_{34}^{'}-\Delta z_{34}\\ \Delta x_{34}^{'}-\Delta x_{34}\end{bmatrix}$$

Using two pairs (4 points) of correspondences:

$$\left[\nabla_{12}\right]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix}=\Delta_{12}^2 \qquad \left[\nabla_{34}\right]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix}=\Delta_{34}^2$$

Adding:

$$\left[\nabla_{12}+\nabla_{34}\right]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix}=[\Delta_{12}^2+\Delta_{34}^2]\Rightarrow\left[\nabla_{1234}\right]\left[\Phi\right]=[\Delta_{1234}^2]$$

$$\nabla_{12}=\begin{bmatrix}0 & \Delta z_{12} & -\Delta y_{12}\\ -\Delta z_{12} & 0 & \Delta x_{12}\\ \Delta y_{12} & -\Delta x_{12} & 0\end{bmatrix} \qquad \nabla_{34}=\begin{bmatrix}-\Delta z_{34} & 0 & \Delta x_{34}\\ \Delta y_{34} & -\Delta x_{34} & 0\\ 0 & \Delta z_{34} & -\Delta y_{34}\end{bmatrix}$$

$$\text{and } \Delta_{12}^2=\begin{bmatrix}\Delta x_{12}^{'}-\Delta x_{12}\\ \Delta y_{12}^{'}-\Delta y_{12}\\ \Delta z_{12}^{'}-\Delta z_{12}\end{bmatrix} \qquad \text{and } \Delta_{34}^2=\begin{bmatrix}\Delta y_{34}^{'}-\Delta y_{34}\\ \Delta z_{34}^{'}-\Delta z_{34}\\ \Delta x_{34}^{'}-\Delta x_{34}\end{bmatrix}$$

$$\boldsymbol{\nabla_{1234}=\begin{bmatrix}-\Delta z_{34} & \Delta z_{12} & \Delta x_{34}-\Delta y_{12}\\ \Delta y_{34}-\Delta z_{12} & -\Delta x_{34} & \Delta x_{12}\\ \Delta y_{12} & -\Delta x_{12}-\Delta z_{34} & -\Delta y_{34}\end{bmatrix};\ \Delta_{1234}^2=\begin{bmatrix}\Delta y_{34}^{'}-\Delta y_{34}+\Delta x_{12}^{'}-\Delta x_{12}\\ \Delta z_{34}^{'}-\Delta z_{34}+\Delta y_{12}^{'}-\Delta y_{12}\\ \Delta x_{34}^{'}-\Delta x_{34}+\Delta z_{12}^{'}-\Delta z_{12}\end{bmatrix}}$$

$$\left[\nabla_{12}+\nabla_{34}\right]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix}=\left[\Delta_{12}^2+\Delta_{34}^2\right]\Rightarrow\left[\nabla_{1234}\right]\left[\Phi\right]=\left[\Delta_{1234}^2\right]$$

Condition for existence of a unique solution is based on a geometrical relationship of the coordinates of four points in space, at time t_1:

$$\nabla_{1234}=\begin{bmatrix}-\Delta z_{34} & \Delta z_{12} & \Delta x_{34}-\Delta y_{12}\\ \Delta y_{34}-\Delta z_{12} & -\Delta x_{34} & \Delta x_{12}\\ \Delta y_{12} & -\Delta x_{12}-\Delta z_{34} & -\Delta y_{34}\end{bmatrix}$$

$$\text{and}\quad \Delta_{1234}^2=\begin{bmatrix}\Delta y_{34}^{'}-\Delta y_{34}+\Delta x_{12}^{'}-\Delta x_{12}\\ \Delta z_{34}^{'}-\Delta z_{34}+\Delta y_{12}^{'}-\Delta y_{12}\\ \Delta x_{34}^{'}-\Delta x_{34}+\Delta z_{12}^{'}-\Delta z_{12}\end{bmatrix}$$

This solution is often used as an initial guess for the final estimate of the motion parameters.

Find geometric condition, when: $\left|\nabla_{1234}\right|=0$

OPTICAL FLOW

A point in 3D space:

$$X_0 = \begin{bmatrix} kx_o & ky_o & kz_o & k \end{bmatrix}; k \neq 0, \text{an arbitary constant.}$$

Image point: $X_i = \begin{bmatrix} wx_i & wy_i & w \end{bmatrix}$ **where,** $X_i = PX_0$

Assuming normalized focal length, f = 1:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} x_o / z_o \\ y_o / z_o \end{bmatrix} \quad ....(1)$$

Assuming *linear* motion model (no acceleration), between successive frames:

$$\begin{bmatrix} x_o(t) \\ y_o(t) \\ z_o(t) \end{bmatrix} = \begin{bmatrix} x_o + ut \\ y_o + vt \\ z_o + wt \end{bmatrix} \quad ....(2)$$

Combining equations (1) and (2):

$$\begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} (x_o + ut) / (z_o + wt) \\ (y_o + vt) / (z_o + wt) \end{bmatrix} \quad ....(3)$$

Assume in equation (3), that w (=dz/dt) < 0.

$$\begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} (x_o + ut) \big/ (z_o + wt) \\ (y_o + vt) \big/ (z_o + wt) \end{bmatrix} \quad ....(3)$$

In that case, the object (or points on the object) will appear to come closer to you. Now visualize, where does these points may come out from?

$$\underset{t \to -\infty}{Lt} \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} =$$

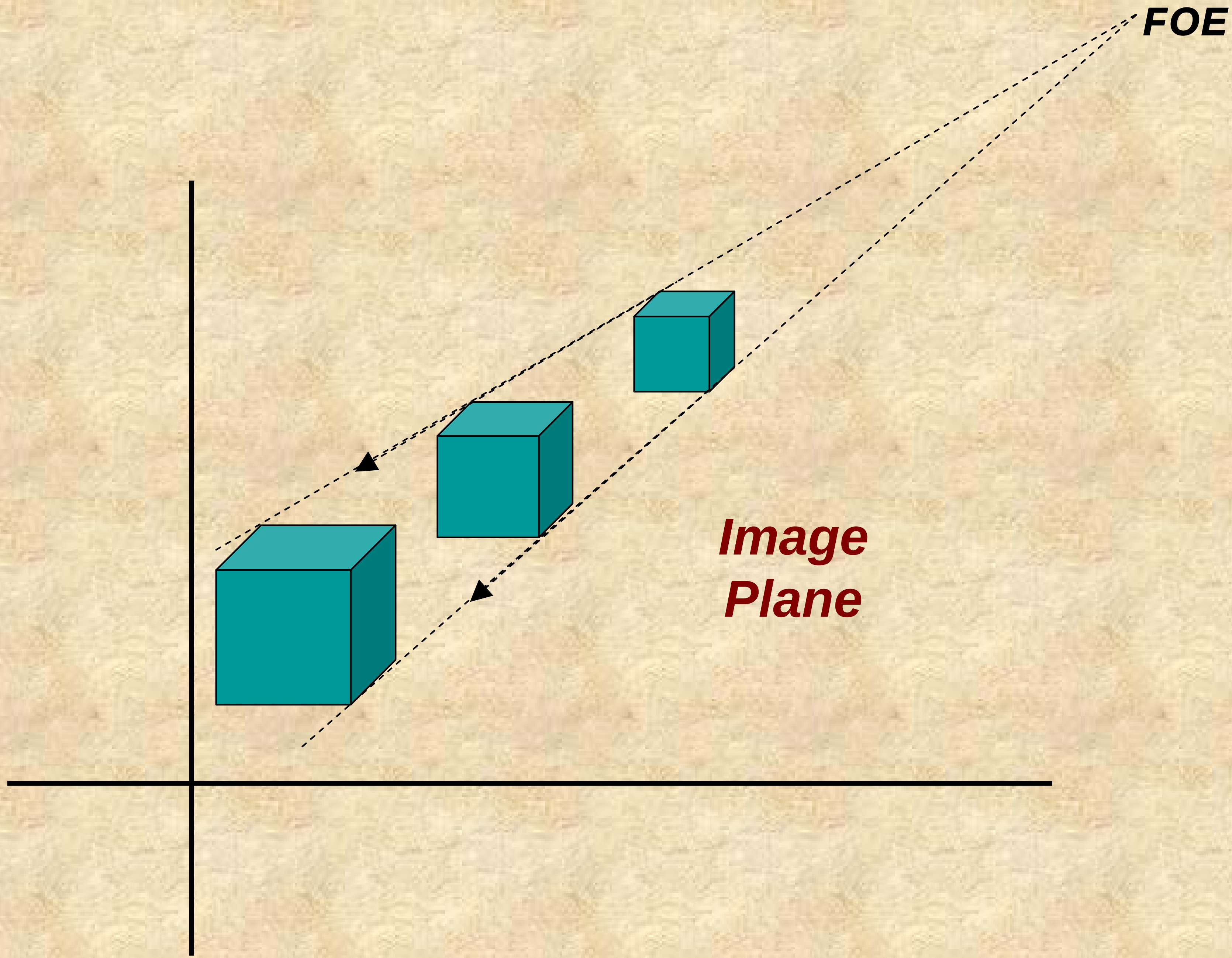
This point "*e*" is a point in the image plane, known as the:

<u>FOE (Focus of Expansion).</u>

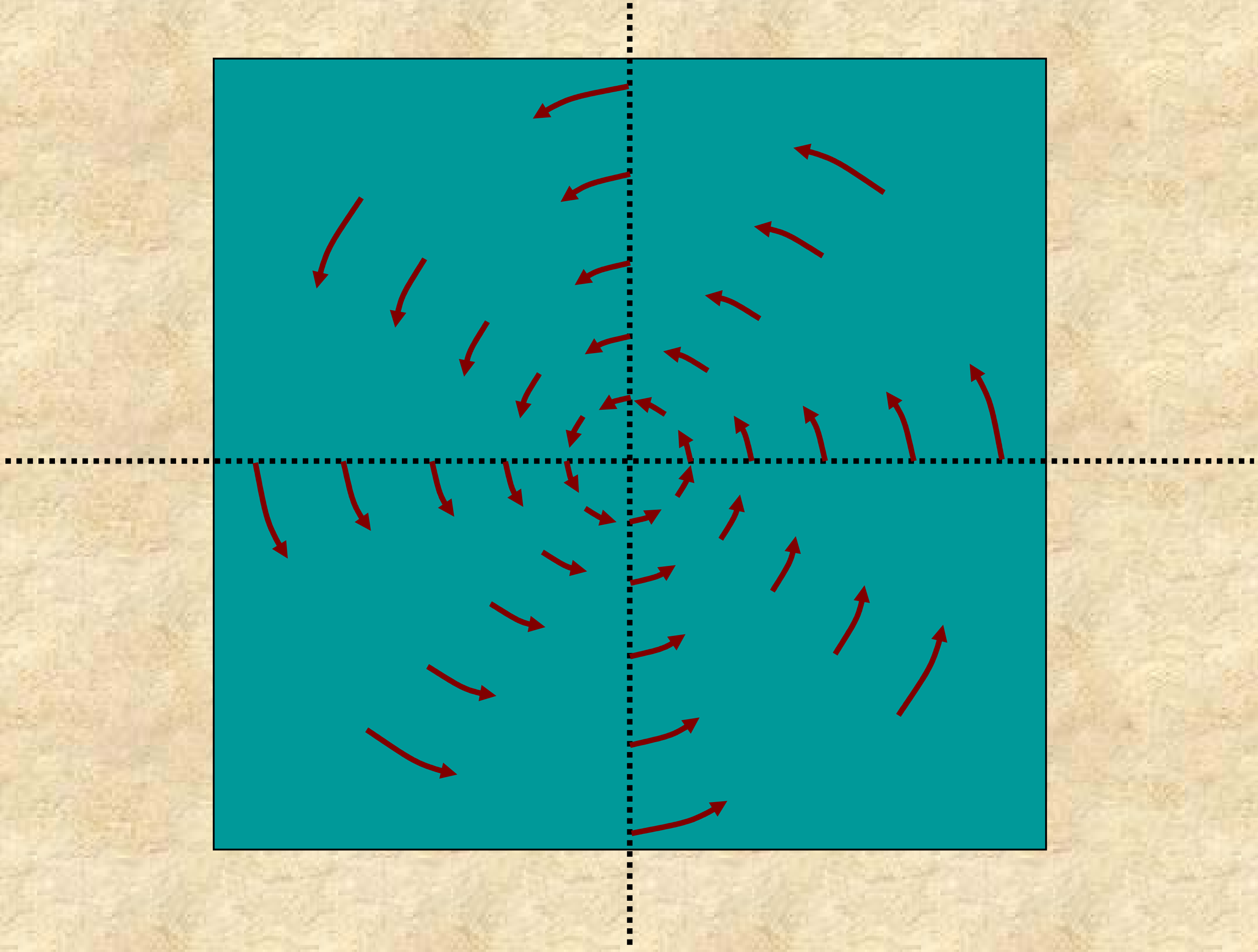
The motion of the object points appears to emanate from a fixed point in the image plane. This point is known as <u>FOE</u>.

Approaches to calculate FOE are based on the exploitation of the fact that for constant velocity object motion all image plane flow vectors intersect at the FOE.

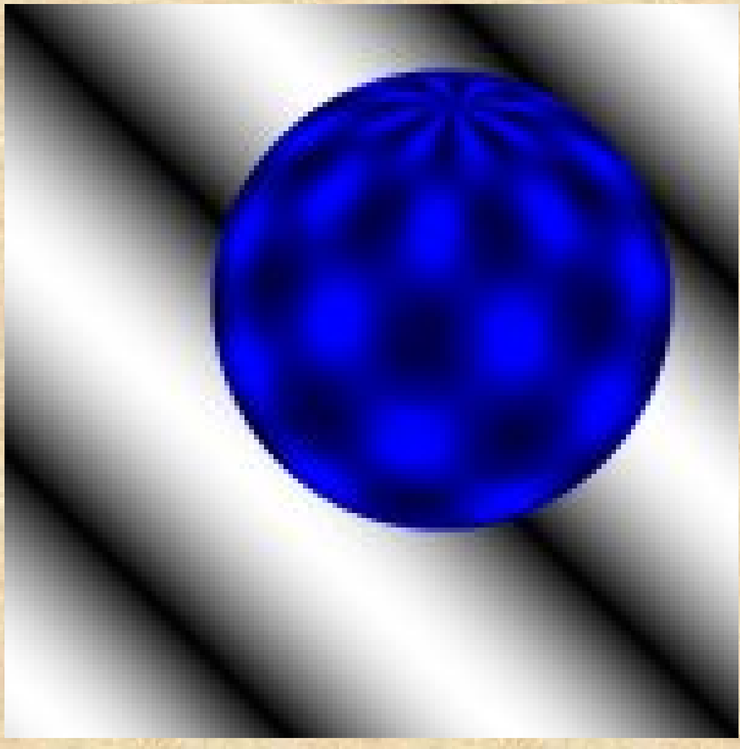
Plot the vectors and extrapolate them to obtain FOE.

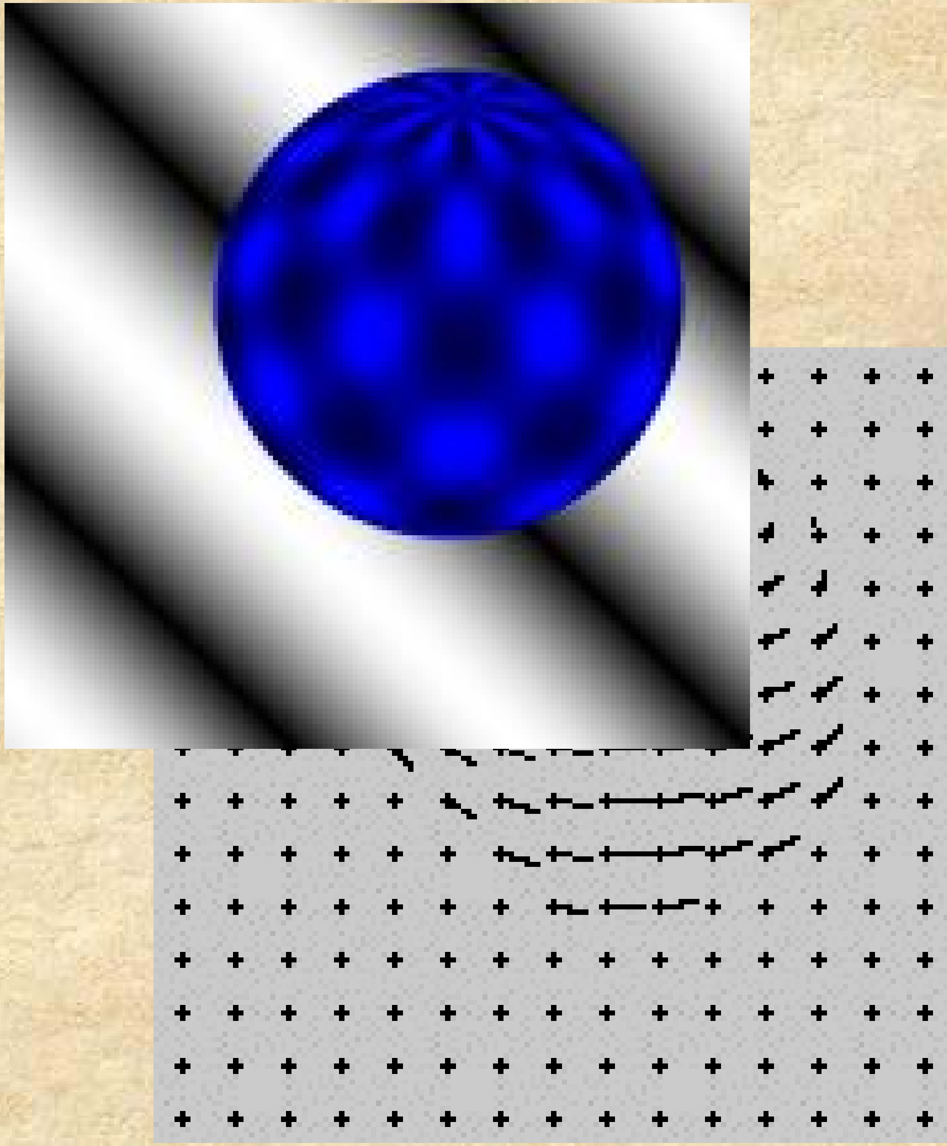

FOE
Image
Plane

FOE may not exist for all types of motion – say pure ROTATION, as shown below.



Multiple FOE's may exist for multiple object motion and occlusion.





Depth from Motion

Time varying distance, D(t), in the image plane , is the distance of an image point from the FOE:

$$D(t) = \left\| X_i - e \right\| = \sqrt{[x_i(t) - {}^{u}\!/_{w}]^2 + [y_i(t) - {}^{v}\!/_{w}]^2}$$

$$\underset{t \to -\infty}{Lt}\ D(t) = 0$$

Rate of change of distance D(t) is: $V(t) = \dfrac{d[D(t)]}{dt}$

Derive this to obtain (home assignment): $V(t) = \dfrac{d[D(t)]}{dt} = -\dfrac{w.D(t)}{z_o(t)}$

This helps to define, TIME-TO-ADJACENCY equation:

$$T_A = \frac{D(t)}{V(t)} = -\frac{z(t)}{w}$$

$$T_A = \frac{D(t)}{V(t)} = -\frac{z(t)}{w}$$

Assuming z is +ve and w is –ve. D(t) is different for different pixels.

This equation holds for each corresponding object and image point pair.

Consider two object points, $z_1(t)$ and $z_2(t)$. Then:

$$\frac{D_1(t)}{V_1(t)} = -\frac{z_1(t)}{w}; \frac{D_2(t)}{V_2(t)} = -\frac{z_2(t)}{w}; \Rightarrow z_2(t) = z_1(t)\left[\frac{D_2(t)}{D_1(t)}\right]\left[\frac{V_1(t)}{V_2(t)}\right]$$

$D_i(t)$ and $V_i(t)$ values for any object point can be obtained from the image plane, once e (FOE) is obtained.

Hence it is possible to determine the *<u>relative 3-D depths :</u>* of any two object points, solely from the image plane motion data.

$$\frac{Z_2(t)}{Z_1(t)}$$

This is the key idea of Structure from motion (SFM) problem, and you are able to extract the shape (structure) information of the object in motion up to a certain scale factor, from a single perspective view only.

Another important idea of optical flow is based on the Horn's (Horn-Schunk, 1980) equations. A global energy function is sought to be minimized, whose functional form is given as:

$$f = \int ((\nabla I \cdot \vec{V} + I_t)^2 + \alpha(|\nabla V_x|^2 + |\nabla V_y|^2 + |\nabla V_z|^2)) \mathrm{d}x \mathrm{d}y \mathrm{d}z$$

$$V_x^{k+1} = \overline{V_x^k} - \frac{I_x(I_x \overline{V_x^k} + I_y \overline{V_y^k} + I_z \overline{V_z^k} + I_t)}{\alpha^2 + I_x^2 + I_y^2 + I_z^2}$$

KLT tracker:

$$\frac{\partial I}{\partial x} V_x + \frac{\partial I}{\partial y} V_y + \frac{\partial I}{\partial z} V_z + \frac{\partial I}{\partial t} = 0$$ or, $$\nabla I \cdot \vec{V} = -I_t$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \sum I_{x_i}^2 & \sum I_{x_i} I_{y_i} & \sum I_{x_i} I_{z_i} \\ \sum I_{x_i} I_{y_i} & \sum I_{y_i}^2 & \sum I_{y_i} I_{z_i} \\ \sum I_{x_i} I_{z_i} & \sum I_{y_i} I_{z_i} & \sum I_{z_i}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum I_{x_i} I_{t_i} \\ -\sum I_{y_i} I_{t_i} \\ -\sum I_{z_i} I_{t_i} \end{bmatrix}$$

- ***Lucas B D and Kanade T, 1981, An iterative image registration technique with an application to stereo vision. Proceedings of Imaging understanding workshop, pp 121—130.***

- ***Horn, B.K.P. and Schunck, B.G., "Determining optical flow." Artificial Intelligence, vol 17, pp 185-203, 1981.***

Concepts from above are left for self-study ……..

Motion Analysis using rigid body assumption

Rigid Body Assumption:

$$\left\| x_i - x_j \right\|^2 = c_{ij}, \forall t, \forall (i, j), \text{where } c_{ij} \text{ are constants.}$$

Motion Equation:

$$X(t_2) = M.X(t_1), \text{where, M} = \begin{bmatrix} \mathrm{m}_{11} & \mathrm{m}_{12} & \mathrm{m}_{13} & l_1 \\ \mathrm{m}_{21} & \mathrm{m}_{22} & \mathrm{m}_{23} & l_2 \\ \mathrm{m}_{31} & \mathrm{m}_{32} & \mathrm{m}_{33} & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, m_{ij} = f(n_1, n_2, n_3, \theta).$$

$$X(t_i) = \begin{bmatrix} x(t_i) & y(t_i) & z(t_i) & 1 \end{bmatrix}^T$$

Form matrix A(t_i), using four points as:

$$A(t_i) = \begin{bmatrix} X_1(t_i) & X_2(t_i) & X_3(t_i) & X_4(t_i) \end{bmatrix}$$

Thus obtain the matrix M, using: $$M = A(t_j).A(t_i)^{-1}$$

Points must be selected in such a fashion that A(t_i) is a non-singular matrix.

Can you guess, when A(t_i) will be singular ?

After m_{ij}'s are obtained:

$$\cos(\theta) = \frac{m_{11} + m_{22} + m_{33} - 1}{2}; \; \sin(\theta) = \frac{m_{32} - m_{23}}{2n_1}.$$

$$n_1 = \sqrt{\frac{m_{11} - \cos(\theta)}{1 - \cos(\theta)}}; n_2 = \frac{m_{21} + m_{12}}{2n_1(1 - \cos(\theta))}; n_3 = \frac{m_{31} + m_{13}}{2n_1(1 - \cos(\theta))}.$$

This is fine in an ideal case. In noisy situations (or even with numerical errors):

$$A(t_i) = M.A(t_j) + N_{ij}$$

Need to formulate an optimization function to minimize a cost function, to satisfy equations in the least square sense:

$$X_k(t_i) = M.X_k(t_j), \; k = 1,2,\ldots,K$$

For example, minimize:

$$\sum_{k=1}^{K} [X_k(t_i) - M.X_k(t_j)]^2$$

along with Rigidity constraint.

Use linearized solution as your initial estimate.

Heard about *SA or GA*?

Work out the following (2nd method):

$$X_k(t_i) = R.X_k(t_j),\ k = 1,2,\ldots,K$$

where,

$$R = \begin{bmatrix} R_p & T \\ 0 & 1 \end{bmatrix} \qquad R_p = R_\alpha R_\beta R_\lambda$$

Again, we have 12 unknown elements in R, which are functions of six unknown parameters.

First obtain the 12 unknown elements and then get the six parameters.

Motion Analysis

using

Image plane coordinates
of the features
of the moving object.

$$P_k(t_i) = R.P_k(t_j),\ \ k = 1,2,\dots,K$$

$$R = \begin{bmatrix} R_p & T \\ 0 & 1 \end{bmatrix}$$

$$P_k = \begin{bmatrix} x_k & y_k & z_k & 1 \end{bmatrix}$$

Thus:

$$x_{k2} = r_{11}x_{k1} + r_{12}y_{k1} + r_{13}z_{k1} + t_x;$$

$$y_{k2} = r_{21}x_{k1} + r_{22}y_{k1} + r_{23}z_{k1} + t_y;$$

$$z_{k2} = r_{31}x_{k1} + r_{32}y_{k1} + r_{33}z_{k1} + t_z;$$

Projective Equations:

$$X_k(t) = x_k / z_k;$$

$$Y_k(t) = y_k / z_k.$$

$$x_{k2} = (r_{11}X_{k1} + r_{12}Y_{k1} + r_{13})z_{k1} + t_x;$$

$$y_{k2} = (r_{21}X_{k1} + r_{22}Y_{k1} + r_{23})z_{k1} + t_y;$$

$$z_{k2} = (r_{31}X_{k1} + r_{32}Y_{k1} + r_{33})z_{k1} + t_z;$$

$$X_{k2} = \frac{x_{k2}}{z_{k2}} = \frac{(r_{11}X_{k1} + r_{12}Y_{k1} + r_{13})z_{k1} + t_x}{(r_{31}X_{k1} + r_{32}Y_{k1} + r_{33})z_{k1} + t_z};$$

$$Y_{k2} = \frac{y_{k2}}{z_{k2}} = \frac{(r_{21}X_{k1} + r_{22}Y_{k1} + r_{23})z_{k1} + t_y}{(r_{31}X_{k1} + r_{32}Y_{k1} + r_{33})z_{k1} + t_z};$$

$$X_{k2} = \frac{x_{k2}}{z_{k2}} = \frac{(r_{11}X_{k1} + r_{12}Y_{k1} + r_{13})z_{k1} + t_x}{(r_{31}X_{k1} + r_{32}Y_{k1} + r_{33})z_{k1} + t_z};$$

$$Y_{k2} = \frac{y_{k2}}{z_{k2}} = \frac{(r_{21}X_{k1} + r_{22}Y_{k1} + r_{23})z_{k1} + t_y}{(r_{31}X_{k1} + r_{32}Y_{k1} + r_{33})z_{k1} + t_z};$$

Solve for Z_{k1}, from the above two equations to obtain:

$$\begin{bmatrix} X_{k2} & Y_{k2} & 1 \end{bmatrix} \begin{bmatrix} E \end{bmatrix} \begin{bmatrix} X_{k1} \\ Y_{k1} \\ 1 \end{bmatrix} = 0;$$

After manipulation obtain:

$$\overrightarrow{\boldsymbol{c}^T}\, \vec{\boldsymbol{e}} = -\mathbf{1}$$

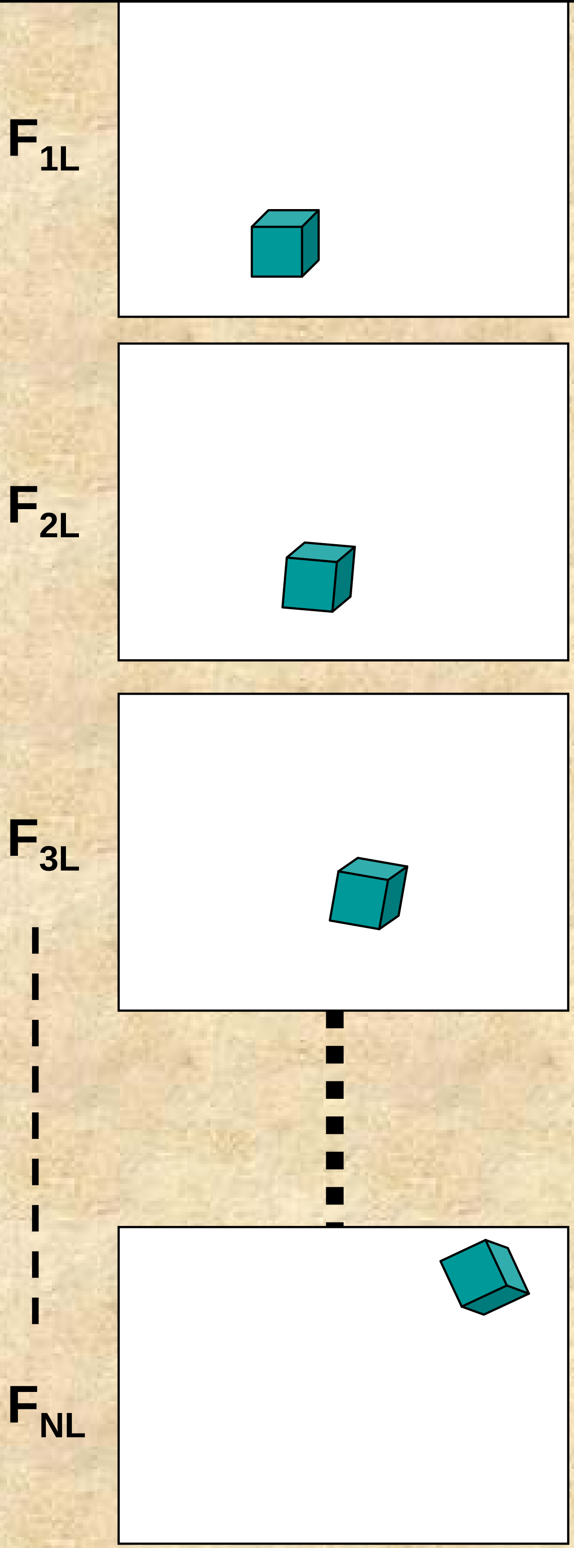
where:

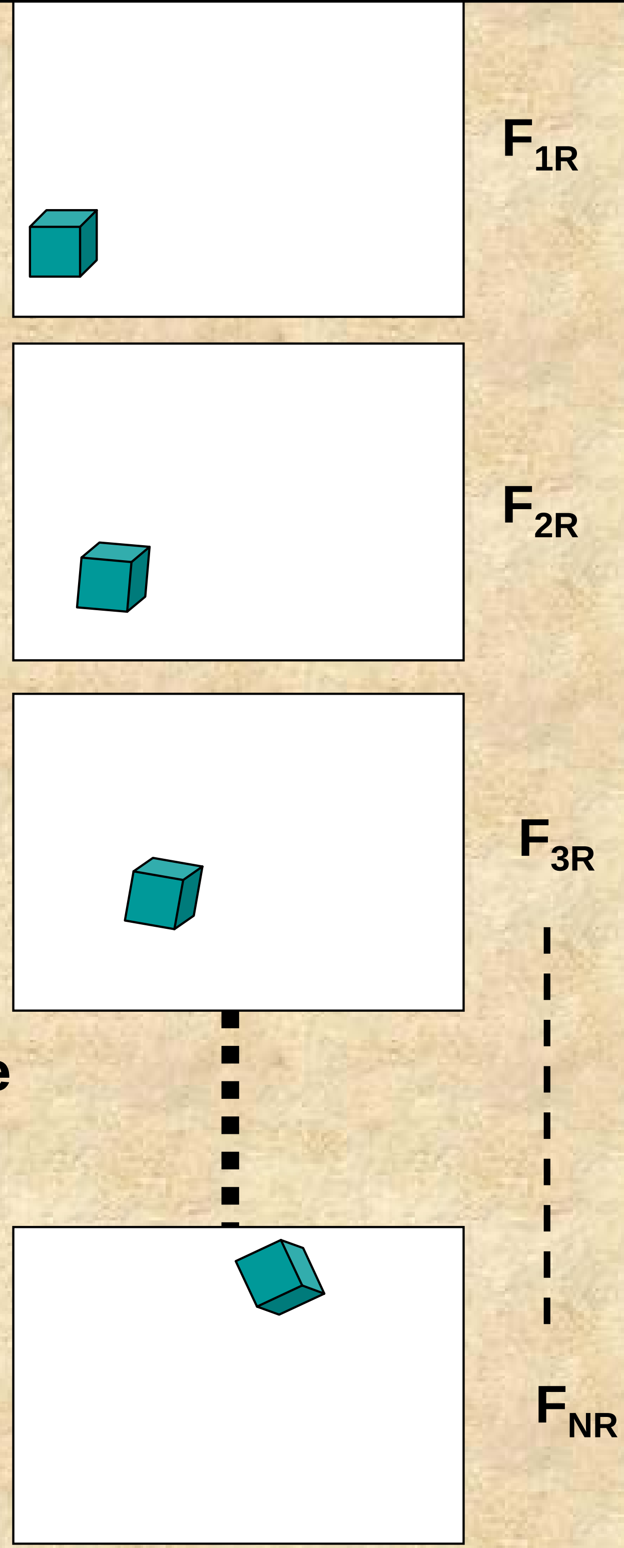
$$\vec{c}^{\,T} = \begin{bmatrix} X_{k2}X_{k1} & X_{k2}Y_{k1} & X_{k2} & Y_{k2}X_{k1} & Y_{k2}Y_{k1} & Y_{k2} & X_{k1}Y_{k1} & X_{k2}Y_{k2} \end{bmatrix}$$

e **is a vector (matrix) of 8 essential parameters, called the** ***<u>essential matrix</u>*****. Solve to get** ***e*** **first.**

Hence we require 8 point correspondences in this case to obtain the (i) essential parameters first and then (ii) the motion parameters.

DYNAMIC STEREO





Environment similar to the Human Vision System (minus brain-power)

Motion equations for dynamic stereo:

Let A_l and A_r be the perspective transformation matrices for the pair of stereo cameras. R be the composite motion matrix with 12 unknown elements.

$$\left.\begin{aligned} I_i^l(t_2) &= A_l R \overrightarrow{X}_i(t_1); \\ I_i^r(t_2) &= A_r R \overrightarrow{X}_i(t_1); \end{aligned}\right| \quad i = 1,2,\ldots,N$$

Each point provides: ***eight (8)*** **equations**

Thus N points provide: ***8N equations*** **(linear).**

Number of unknowns: ***12 + 3N***

To provide a solution: N $\geq$ 3

Solve for obtaining the optimal solution

That's all for now –

Let's MOVE ON