Soft Object Modelling

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Models

Discrete Model:

- Polygon mesh is constructed.
- Particles (or mass) nodes at vertices of mesh.

Continuous Model:

- Use continuum mechanics to model deformable objects

Mass Spring System

- It is a Discrete Model.
- It consist of point masses connected by mass-less springs.
- Mesh Structure
- Types of Springs – structural, shear.
- Springs are modeled as linear elastic.

Mass Spring System

- Spring Force

$$f_{sij} = k_s \left(\left|x_j - x_i\right| - l_{ij}\right) \frac{\left(x_j - x_i\right)}{\left|x_j - x_i\right|}$$

where, f_{sij} is spring force acting on node i by spring between i and j, k_s is spring's stiffness, l_{ij} is rest length of string between node i & j, x_i, x_j are the node positions of i and j.

Mass Spring System

- Damping Force

$$f_{dij} = k_d\left(v_j - v_i\right)$$

$$f_{dij} = k_d\left[\frac{\left(v_j - v_i\right).\left(x_j - x_i\right)}{\left|x_j - x_i\right|}\right]\frac{\left(x_j - x_i\right)}{\left|x_j - x_i\right|}$$

where, f_{dij} is damping force acting on node i by spring between i and j, k_d is spring's damping coefficient, v_i, v_j are the node velocities of i and j.

Mass Spring System

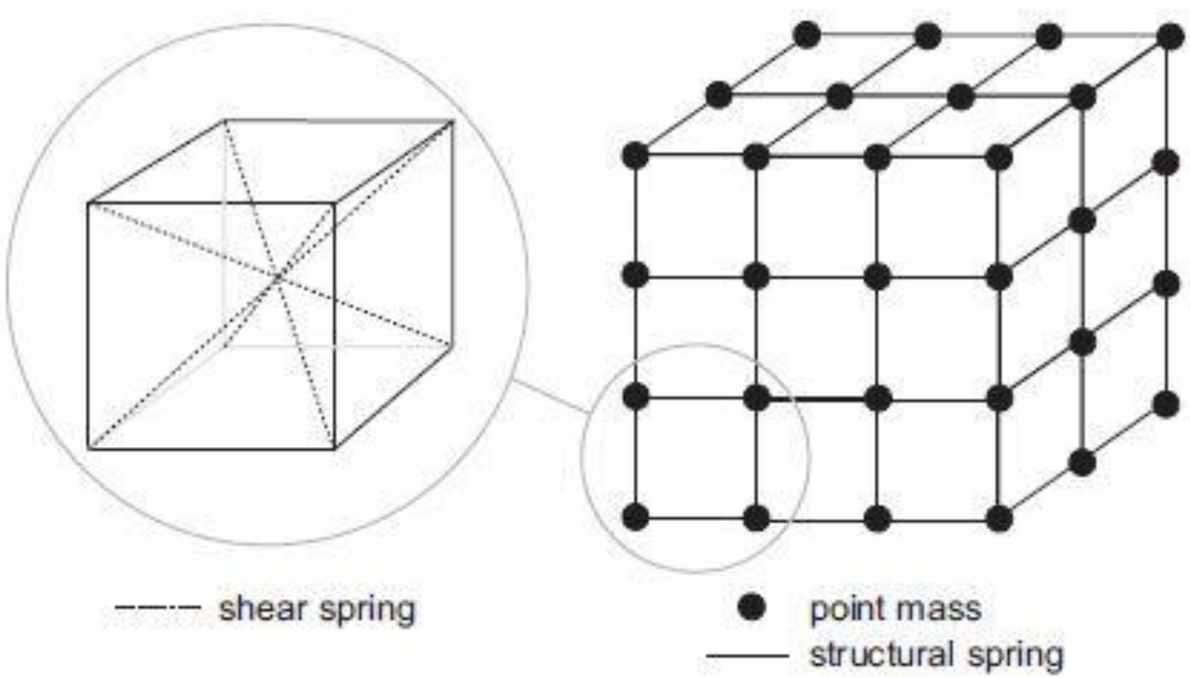
- Net Force

$$f_{neti} = \sum_j f_{sij} + \sum_j f_{dij} + f_{exti}$$

where, f_{exti} is the external force on node i, f_{neti} is the net force.

- Position: $x(t+\Delta t) = x(t) + \Delta t\, v(t+\Delta t)$
- Velocity: $v(t+\Delta t) = v(t) + \Delta t \frac{f_{neti}}{m_i}$

Mass Spring System



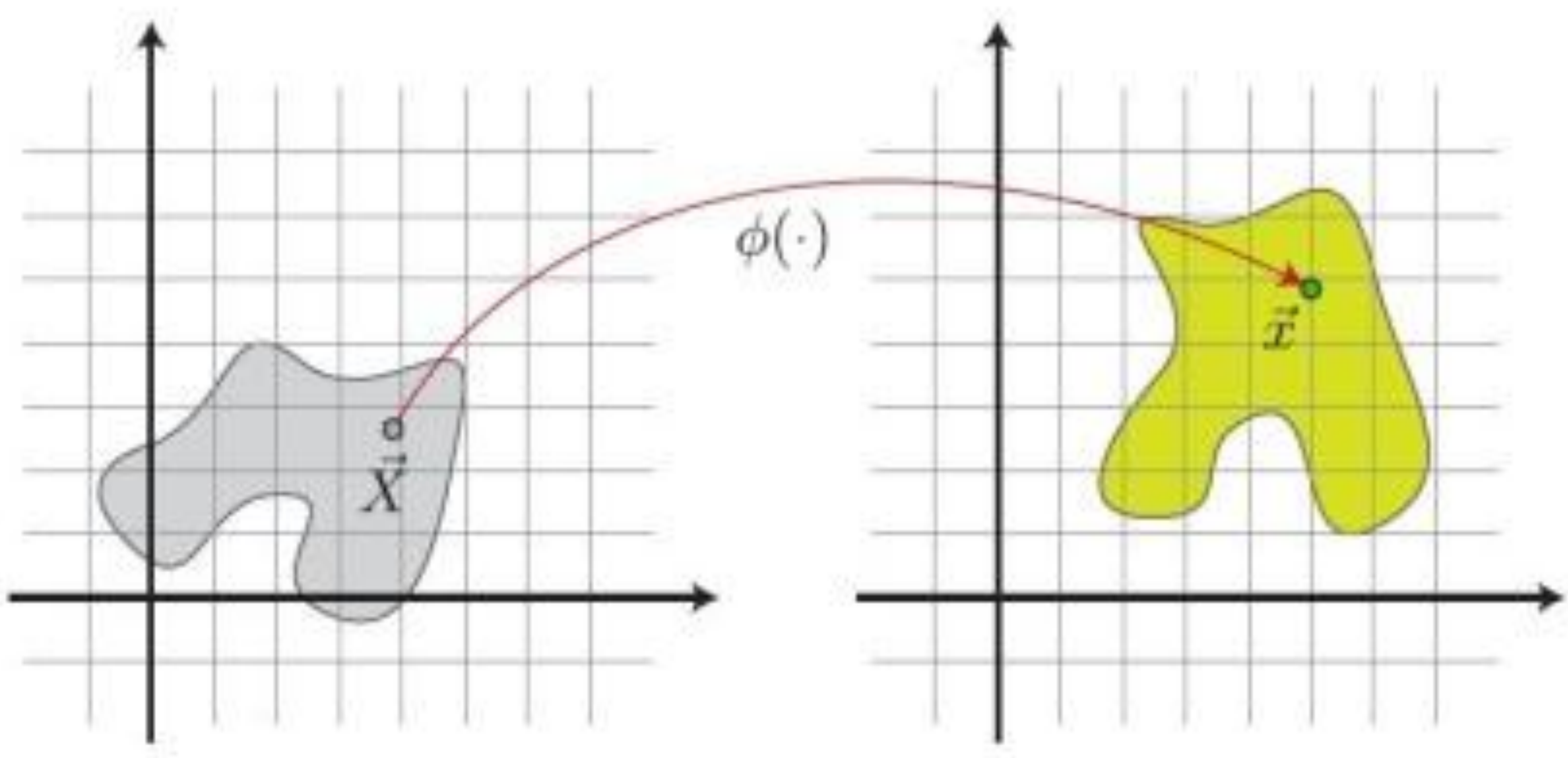
Finite Element Method

- It is a continuous method.
- Mesh can be regular or irregular.
- Nodes in mesh contains mass.
- First the deformation energy is determined.
- Forces on nodes are calculated as derivative of deformation energy.

Deformation Function

- Deformation Function gives the relationship between each material point and its deformed location.
- $\vec{x} = \phi(\vec{X})$
- $\phi(\vec{X}) = F\vec{X} + \vec{t}$
- $F = \frac{\partial\phi(\vec{X})}{\partial\vec{X}}$
- where, $\phi()$ is the deformation function (or deformation map),
- $\vec{X}$ is the original reference state,
- $\vec{x}$ is the deformed state,
- F is the deformation gradient.

Deformation Function



Reference Configuration

Deformed Configuration

Deformation Gradient Calculation

- We have, $\vec{x} = \phi(\vec{X}) = F\vec{X} + \vec{t}$
- One of the method to represent deformable object is tetrahedral mesh. In each tetrahedron, the vertices satisfy,
- $\vec{x}_i = \phi(\vec{X}_i) = F\vec{X}_i + \vec{t}$
- So we have,

$$\begin{aligned} \vec{x}_1 &= \phi(\vec{X}_1) = F\vec{X}_1 + \vec{t} \\ \vec{x}_2 &= \phi(\vec{X}_2) = F\vec{X}_2 + \vec{t} \\ \vec{x}_3 &= \phi(\vec{X}_3) = F\vec{X}_3 + \vec{t} \\ \vec{x}_4 &= \phi(\vec{X}_4) = F\vec{X}_4 + \vec{t} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \vec{x}_1 - \vec{x}_4 &= F(\vec{X}_1 - \vec{X}_4) \\ \vec{x}_2 - \vec{x}_4 &= F(\vec{X}_2 - \vec{X}_4) \\ \vec{x}_3 - \vec{x}_4 &= F(\vec{X}_3 - \vec{X}_4) \end{aligned}$$

Deformation Gradient Calculation

- Previous expression can be written as,

$$D_d = FD_r$$

- where, D_d is termed as deformed shape matrix,

D_r is termed as reference shape matrix

- So, we have deformation gradient as,

$$F = D_d {D_r}^{-1}$$

Strain Measure

- Strain measure shows how far current configuration is from rest configuration. It is derived from deformation gradient.
- Green Strain Measure

$$E = \frac{1}{2}\left(FF^T - I\right)$$

- Small Strain Measure

$$\varepsilon = \frac{1}{2}\left(F + F^T\right) - I$$

- where, *I* represents rest configuration
- *F* is the deformation gradient.

Linear model

- Strain Energy Density

$$\psi(F) = \mu\varepsilon^2 + \frac{\lambda}{2}\, tr^2(\varepsilon)$$

- First Piola Stress

$$P(F) = \frac{\partial\psi(F)}{\partial F}$$

$$P(F) = 2\mu\varepsilon + \lambda tr(\varepsilon)I$$

$$P(F) = \mu(F + F^T - 2I) + \lambda tr(F - I)I$$

- where, μ and λ are Lame' constants
- ε is the small strain measure
- *F* is the deformation gradient.

St. Venant-Kirchhoff model

- Strain Energy Density

$$\psi(F) = \mu E^2 + \frac{\lambda}{2}\, tr^2(E)$$

- First Piola Stress

$$P(F) = F[2\mu E + \lambda tr(E) I]$$

- where, μ and λ are Lame' constants
- E is the green strain measure
- *F* is the deformation gradient.

Strain Energy

- Strain Energy Density, $\psi(F)$, denotes the strain energy per unit deformed volume.
- Total Strain Energy is calculated by integrating energy density function, over the entire domain as,

$$E[\phi] = \int \psi(F)\, d\vec{X}$$

- Strain Energy on discrete nodes

$$E[x] = E\left[\hat{\phi}\right] = \int \psi\left(\hat{F}\right) d\vec{X}$$

- Force acting on nodes can be calculated as

$$f = \frac{\partial E(x)}{\partial x}$$

Force

- Total Energy of body is sum of strain energy (E) and kinetic energy (K) as
- $E_{total} = E(x) + K(v) = E(x) + \sum_{i=1}^{N} \frac{1}{2}\, m_i |v_i|^2$
- Total energy of body is conserved over time, thus
- $\frac{\partial E_{total}}{\partial t} = 0$
- $\sum_{i=1}^{N} \left[\frac{\partial E(x)}{\partial \vec{x}_i} . \vec{v}_i + m_i \vec{a}_i . \vec{v}_i \right] = 0$, where N is total nodes.
- $\frac{\partial E(x)}{\partial \vec{x}_i} + m_i \vec{a}_i = 0$
- $\vec{f}_i = m_i \vec{a}_i = -\frac{\partial E(x)}{\partial \vec{x}_i}$, where $\vec{f}_i$ is force on node i.

Steps in Finite Element Method

- Deformation Gradient: $F = \frac{\partial \phi(\vec{X})}{\partial \vec{X}}$
- Green Strain Measure: $E = \frac{1}{2}\,(FF^T - I)$
- Strain Energy Density: $\psi(F) = \mu E^2 + \frac{\lambda}{2}\,tr^2(E)$
- Strain Energy: $E[\phi] = \int \psi(F)\,d\vec{X}$
- Discrete Strain Energy: $E[x] = E\left[\hat{\phi}\right] = \int \psi\left(\hat{F}\right)d\vec{X}$
- Internal Force: $f = \frac{\partial E(x)}{\partial x}$

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