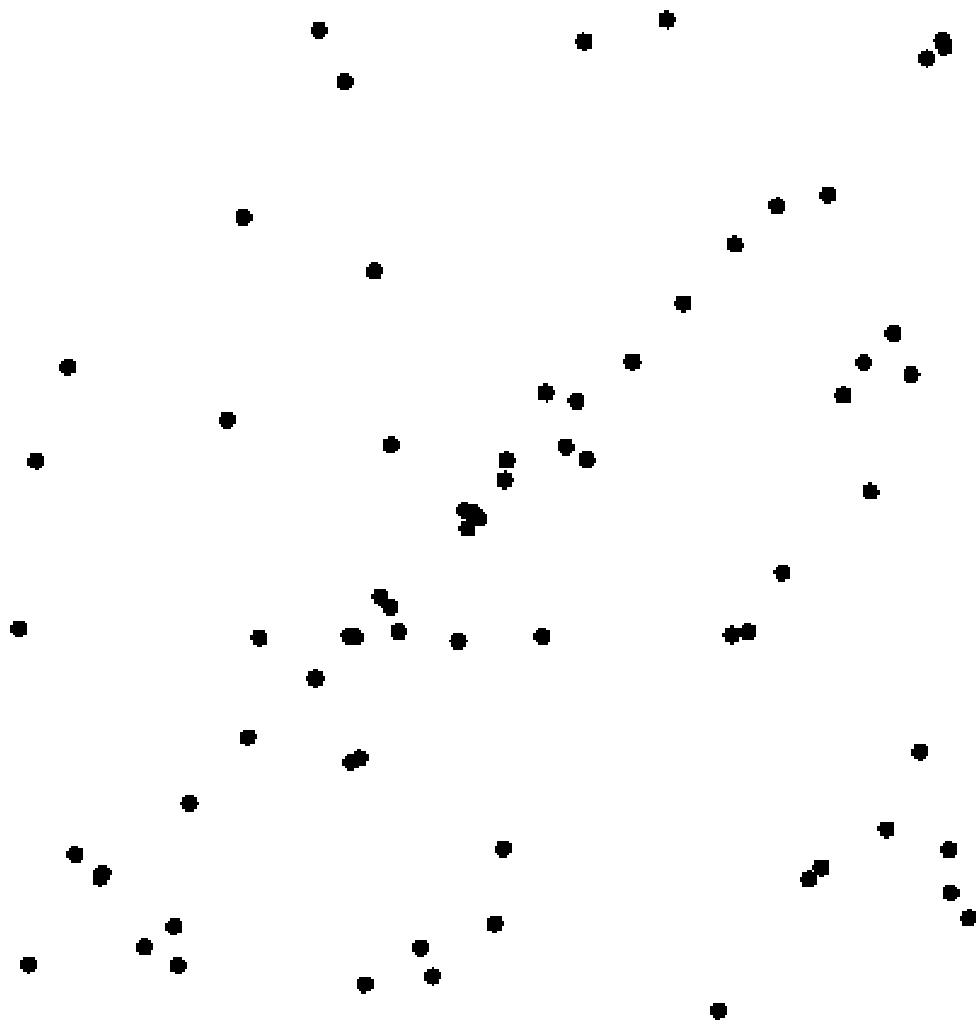
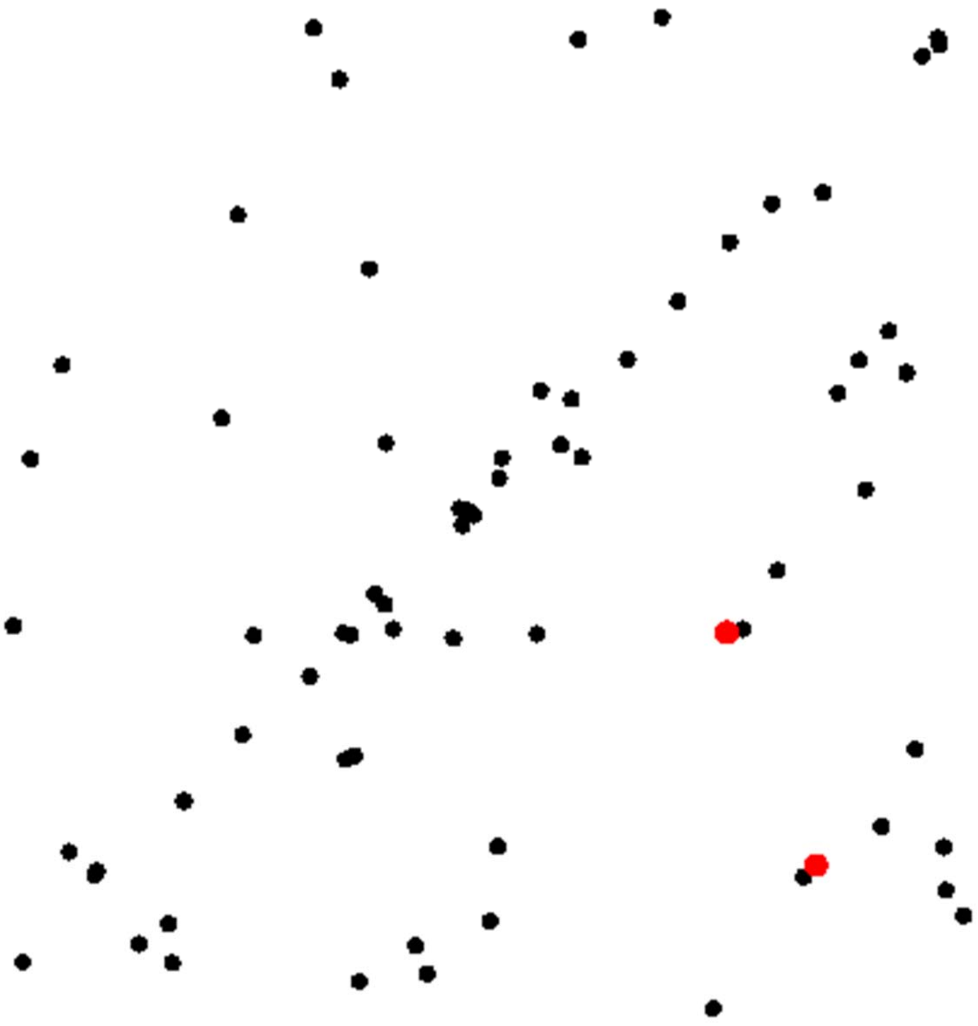
Example: line fitting

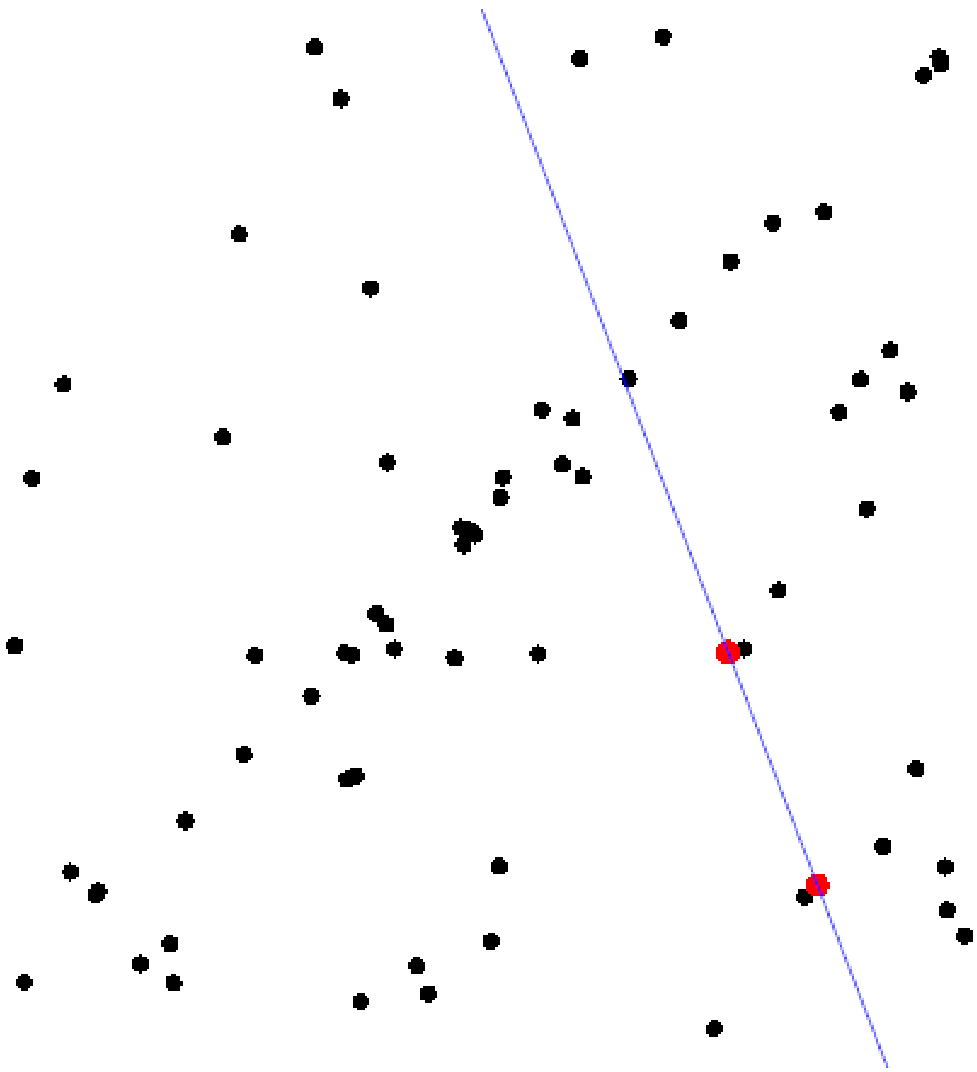


Example: line fitting

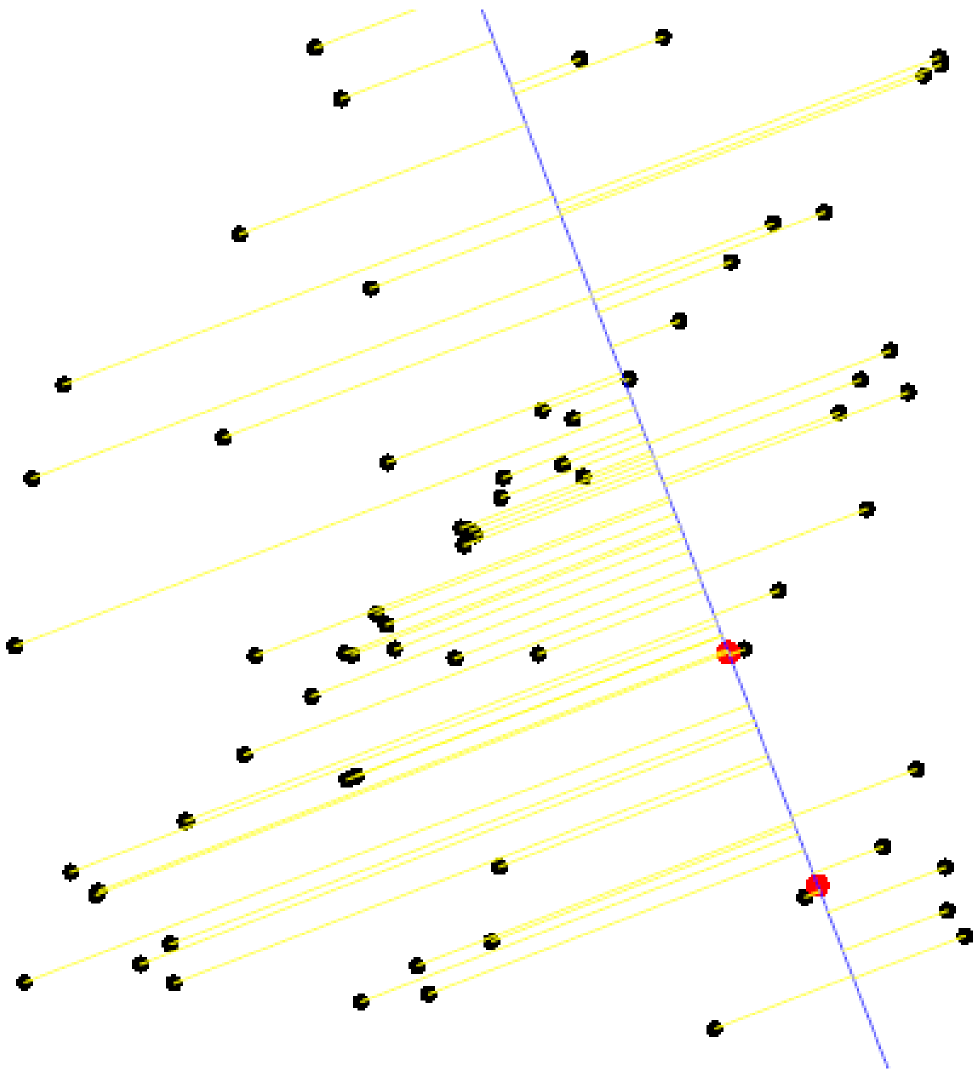


$n=2$

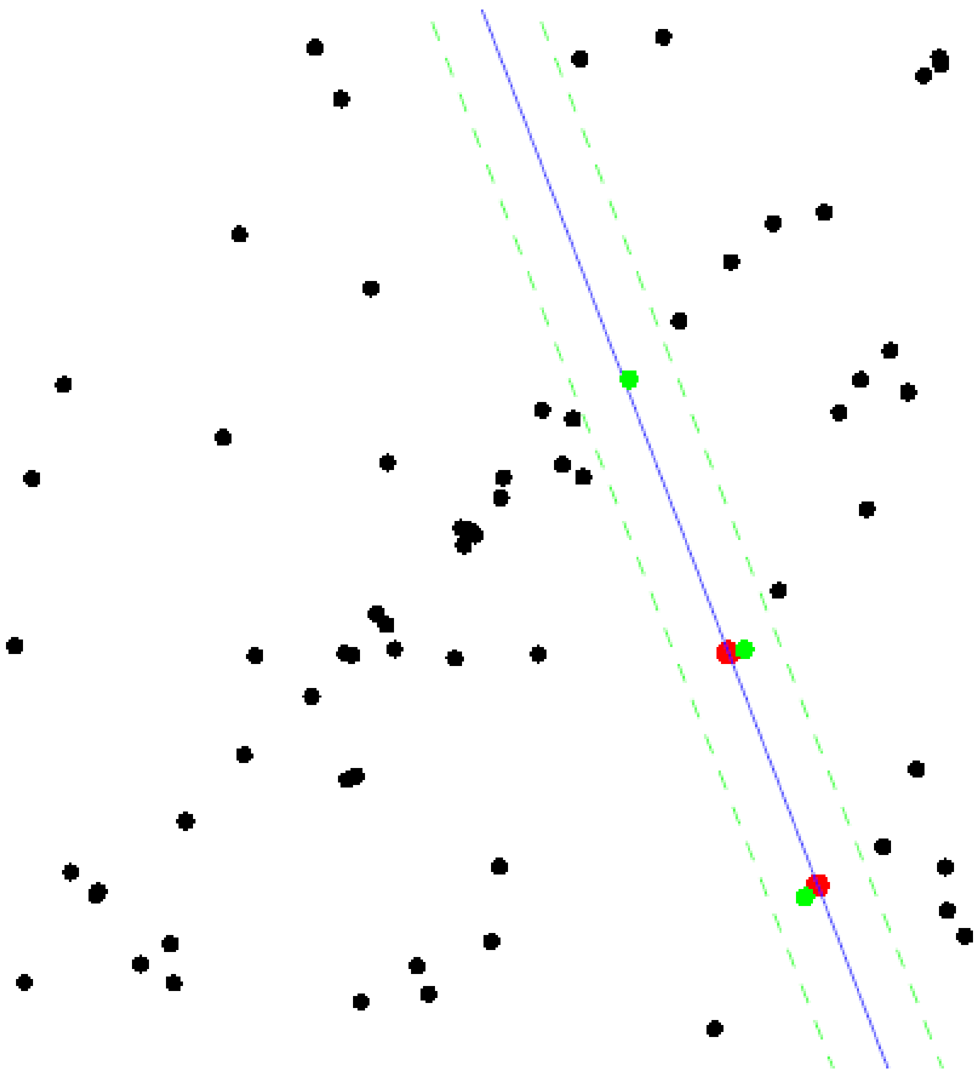
Model fitting



Measure distances

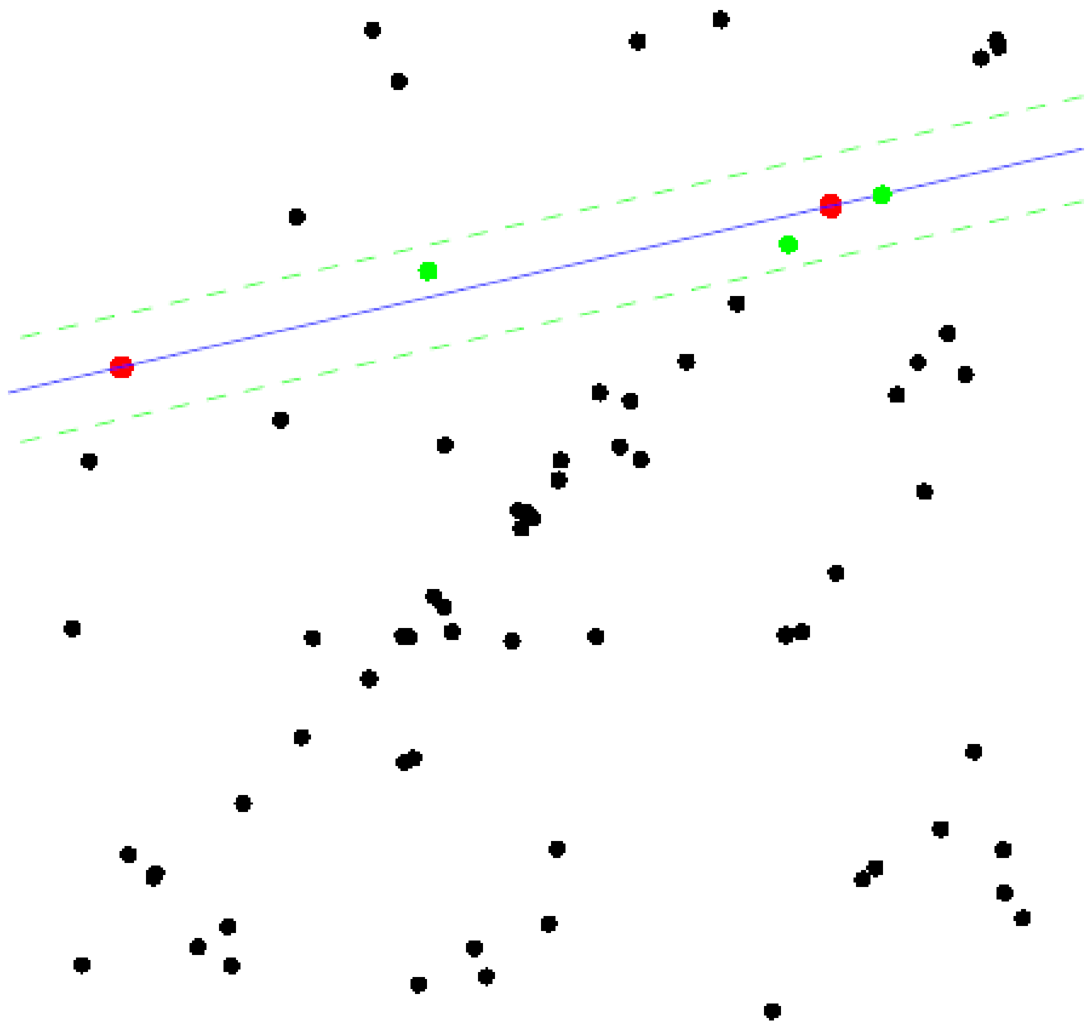


Count inliers



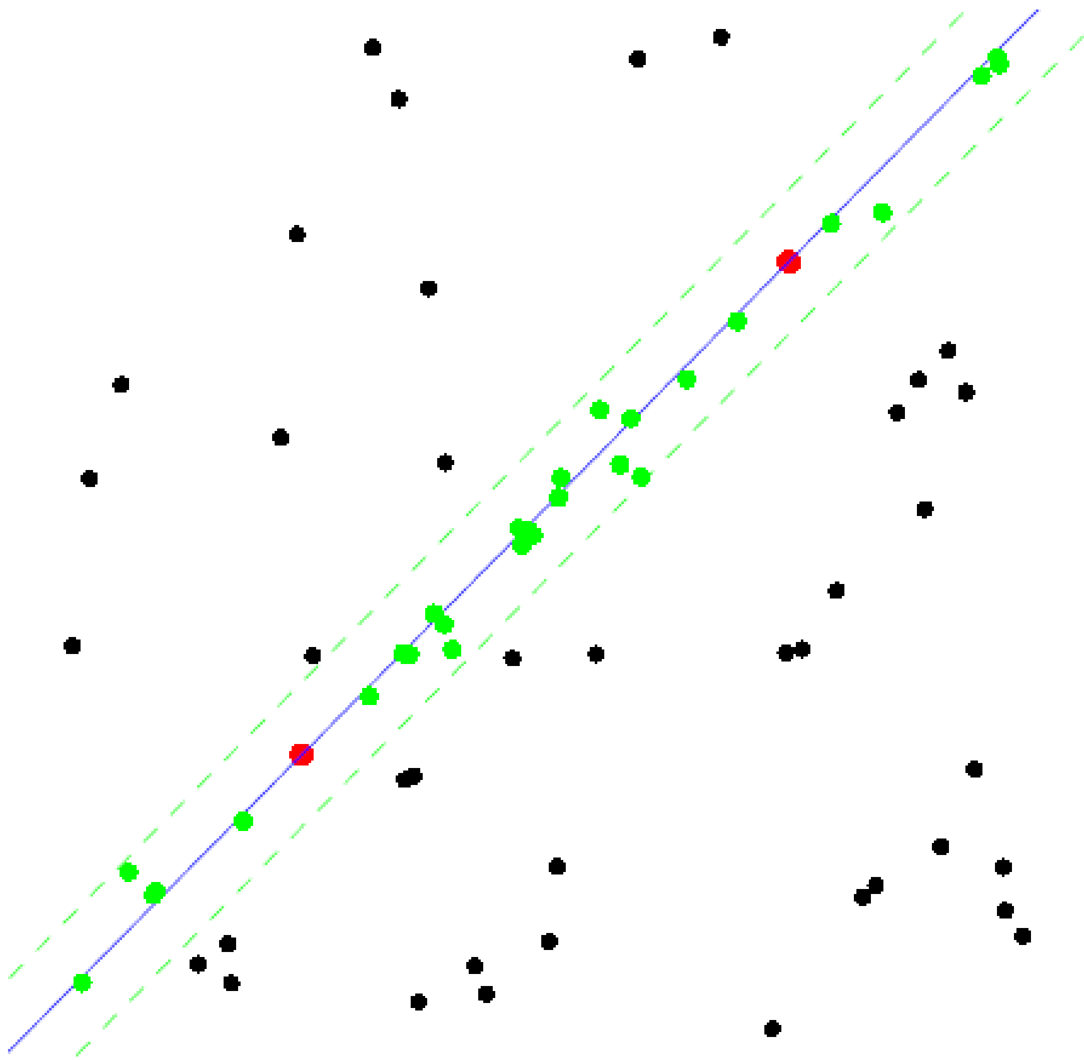
$c=3$

Another trial



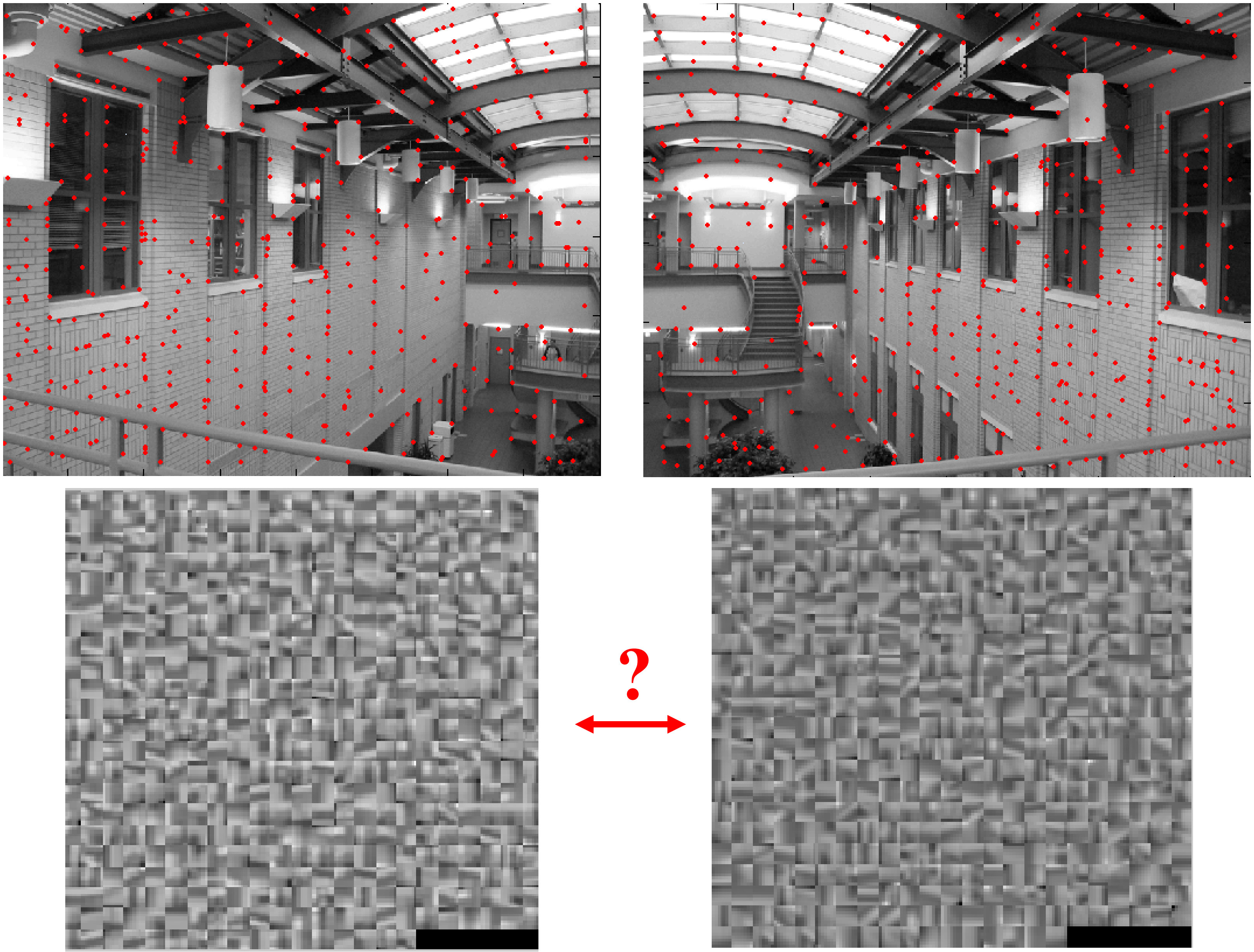
$c=3$

The best model



$c=15$

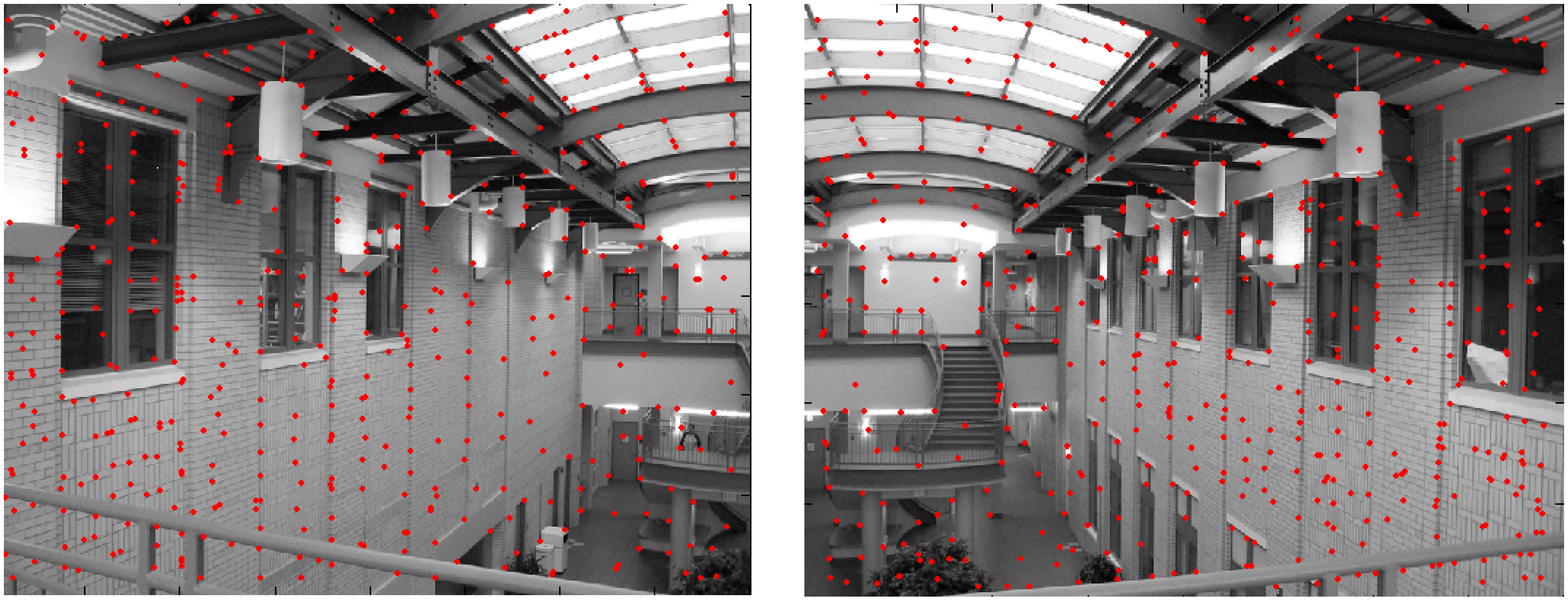
Feature matching

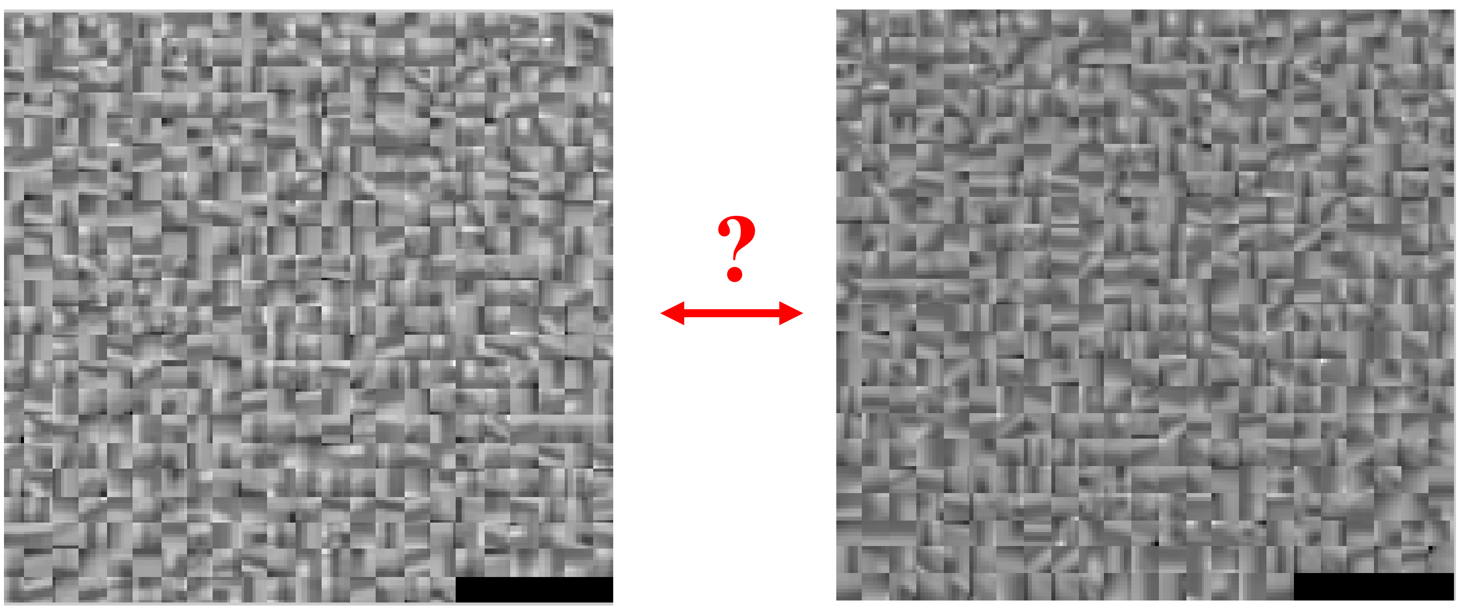


Feature matching

- Exhaustive search
  - for each feature in one image, look at *all* the other features in the other image(s)
- Hashing
  - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
  - *k*-trees and their variants

What about outliers?



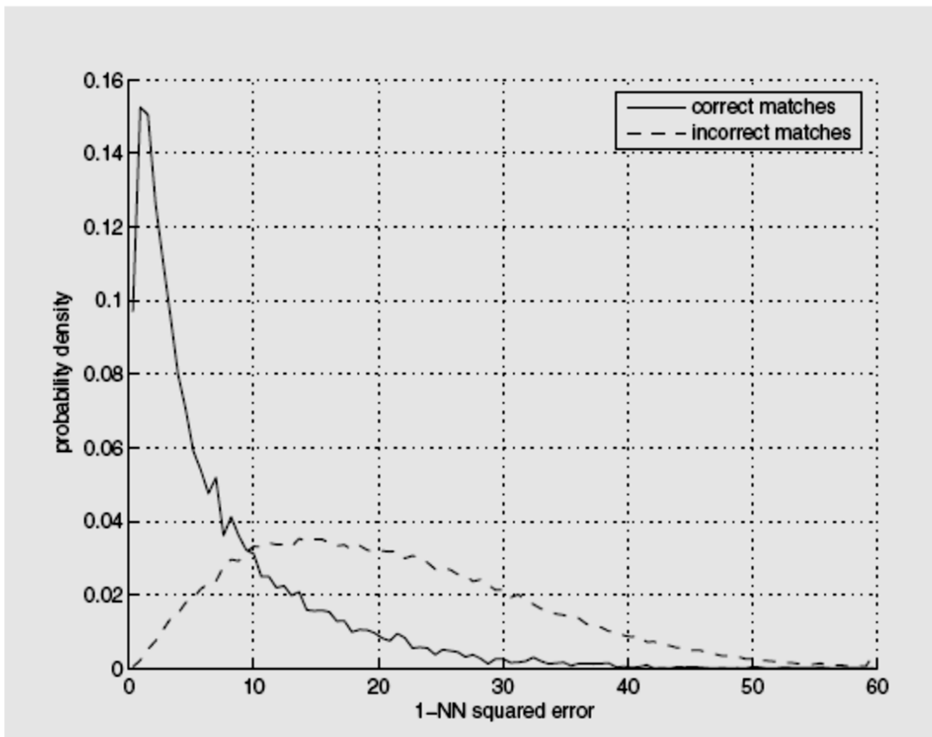


Feature-space outlier rejection

Let's not match all features, but only these that have "similar enough" matches?

How can we do it?

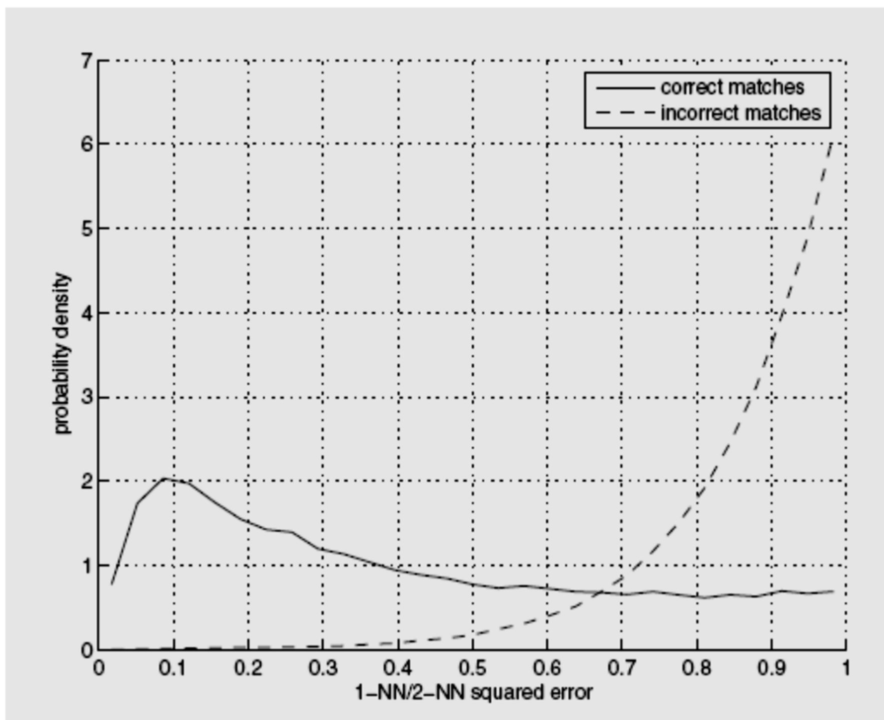
- SSD(patch1,patch2) < threshold
- How to set threshold?



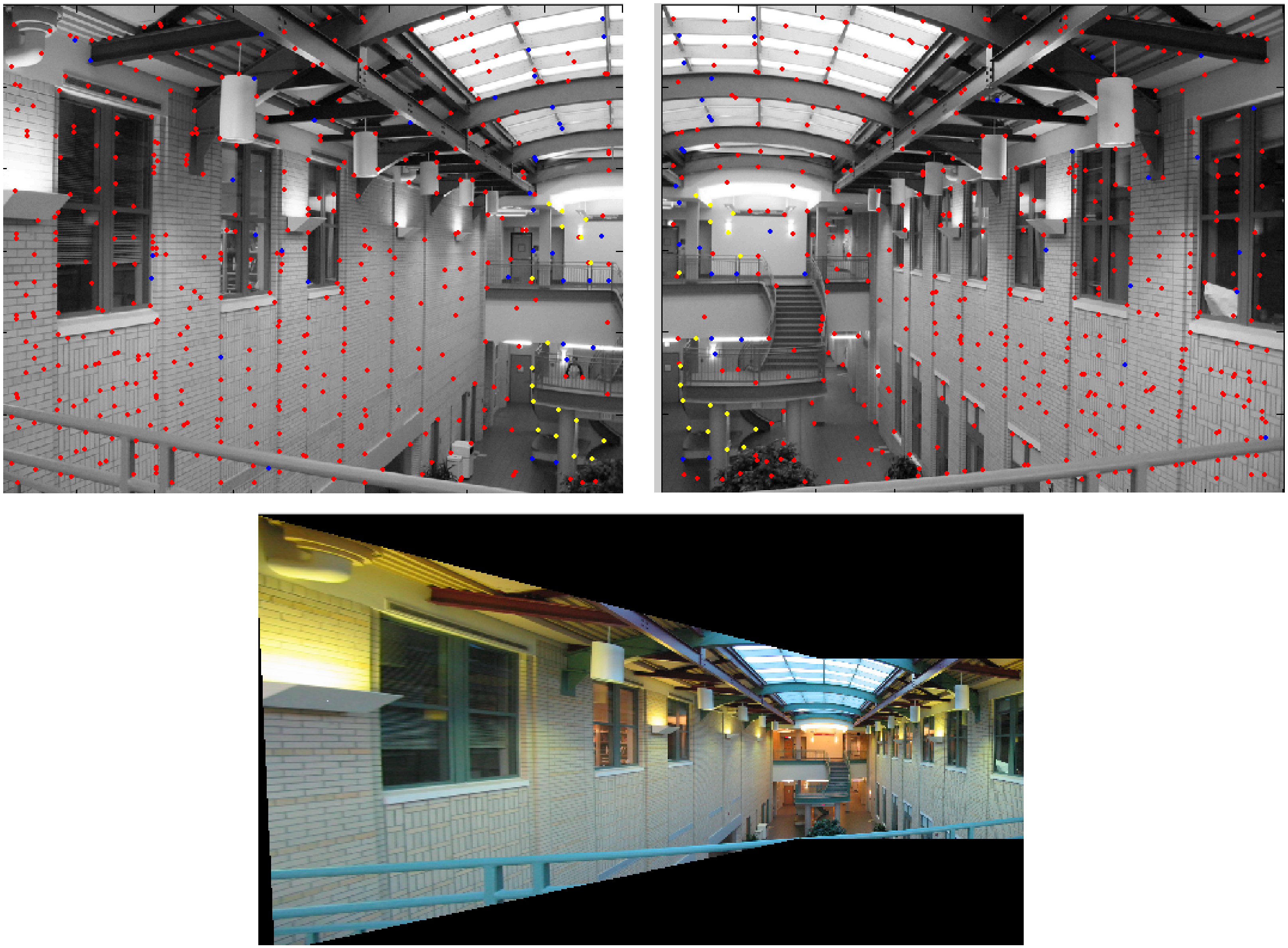
Feature-space outlier rejection

A better way [Lowe, 1999]:

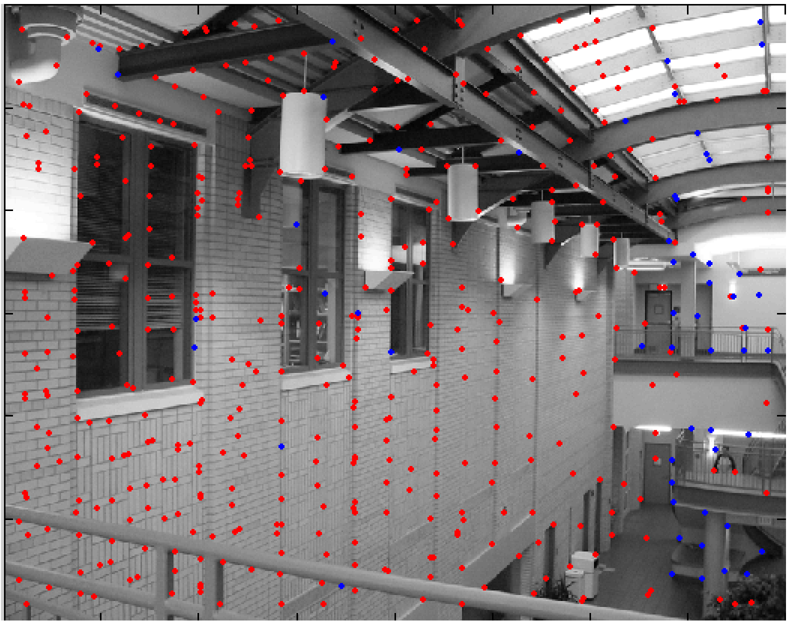
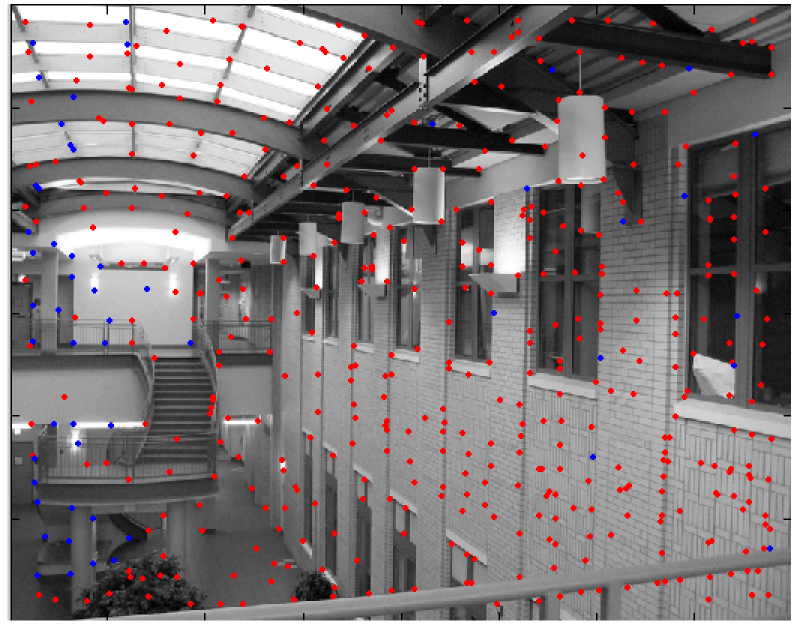
- 1-NN: SSD of the closest match
- 2-NN: SSD of the <u>second-closest</u> match
- Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
- That is, is our best match so much better than the rest?



RANSAC



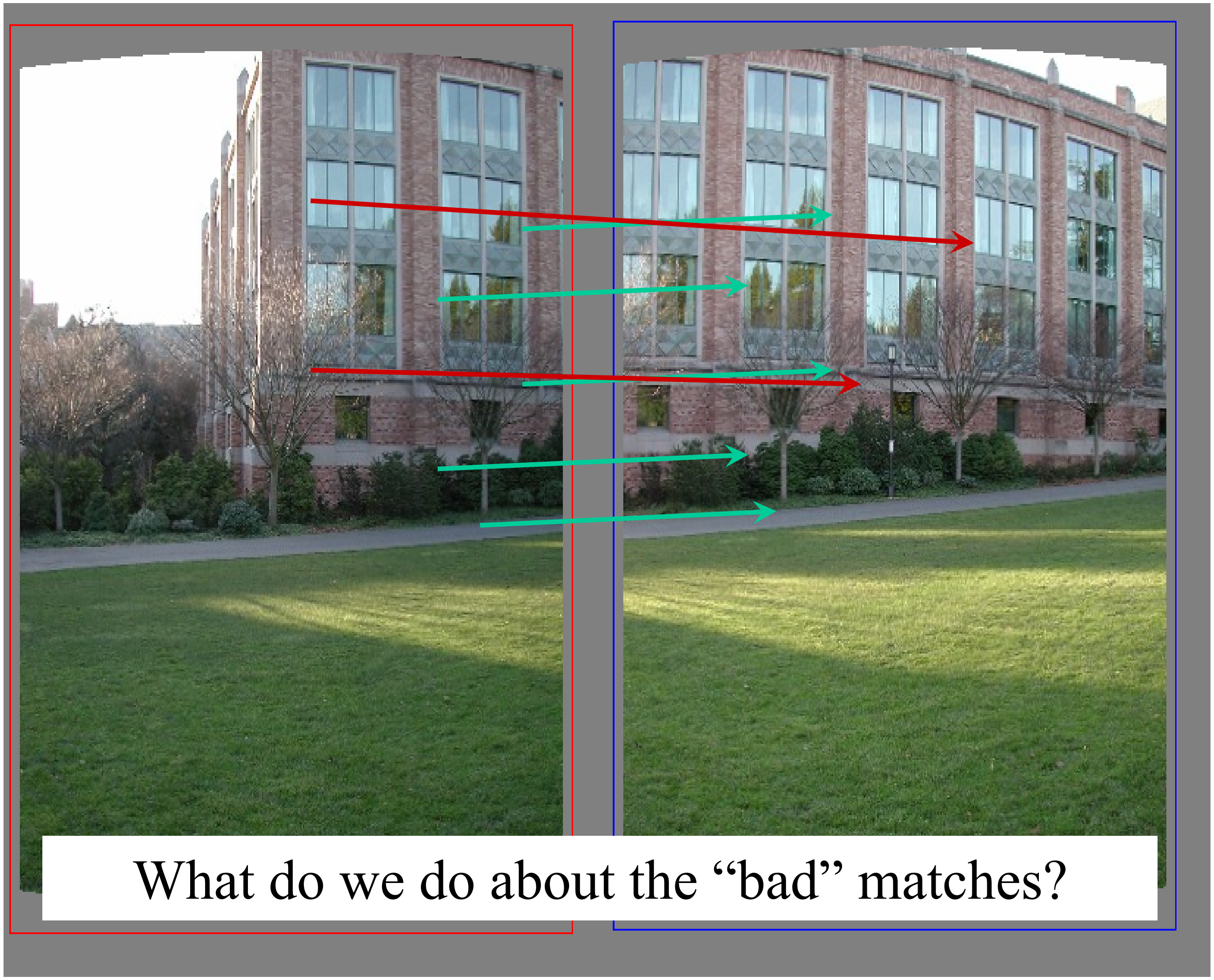
Feature-space outliner rejection

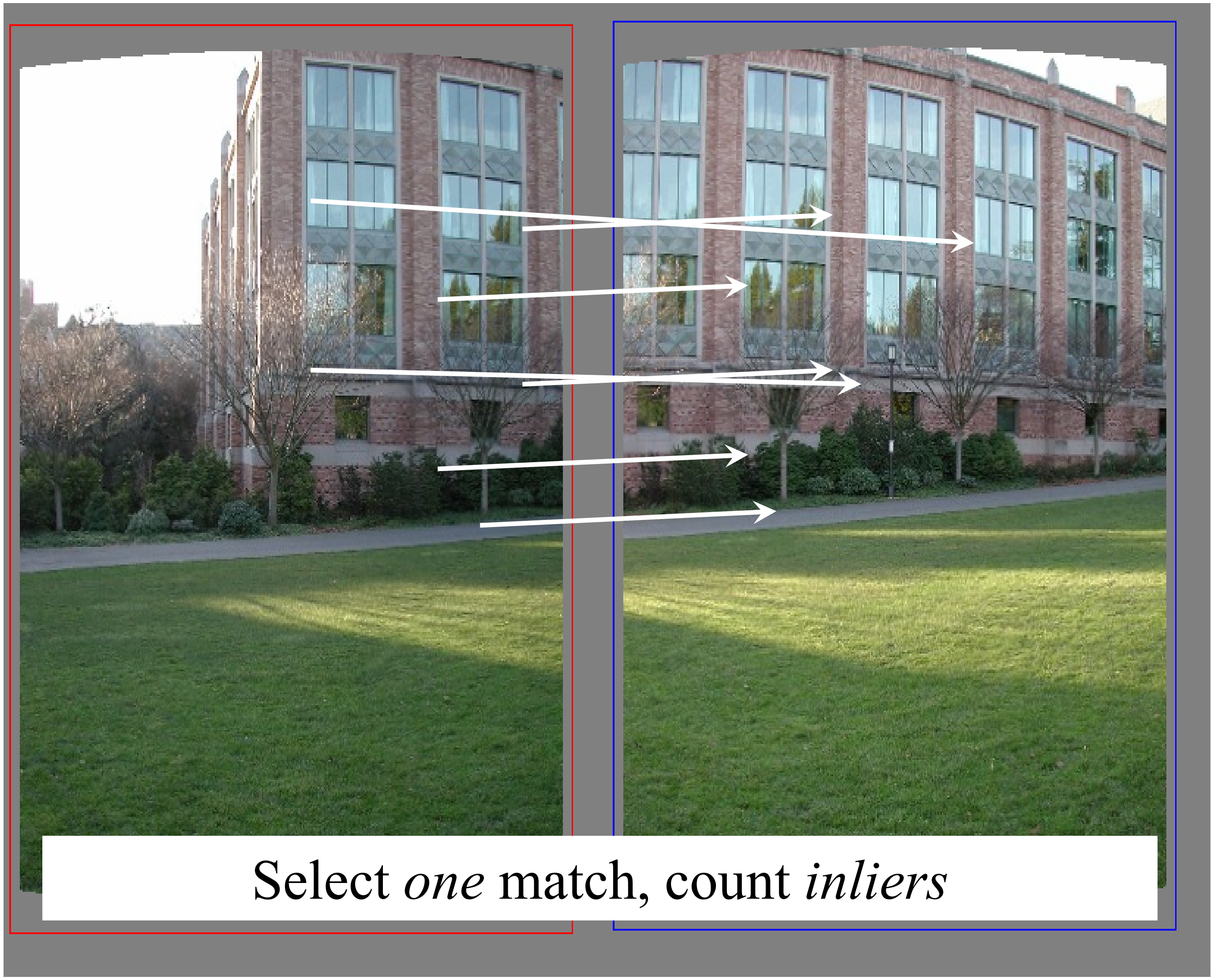
Can we now compute H from the blue points?

- No! Still too many outliers…
- What can we do?

Matching features

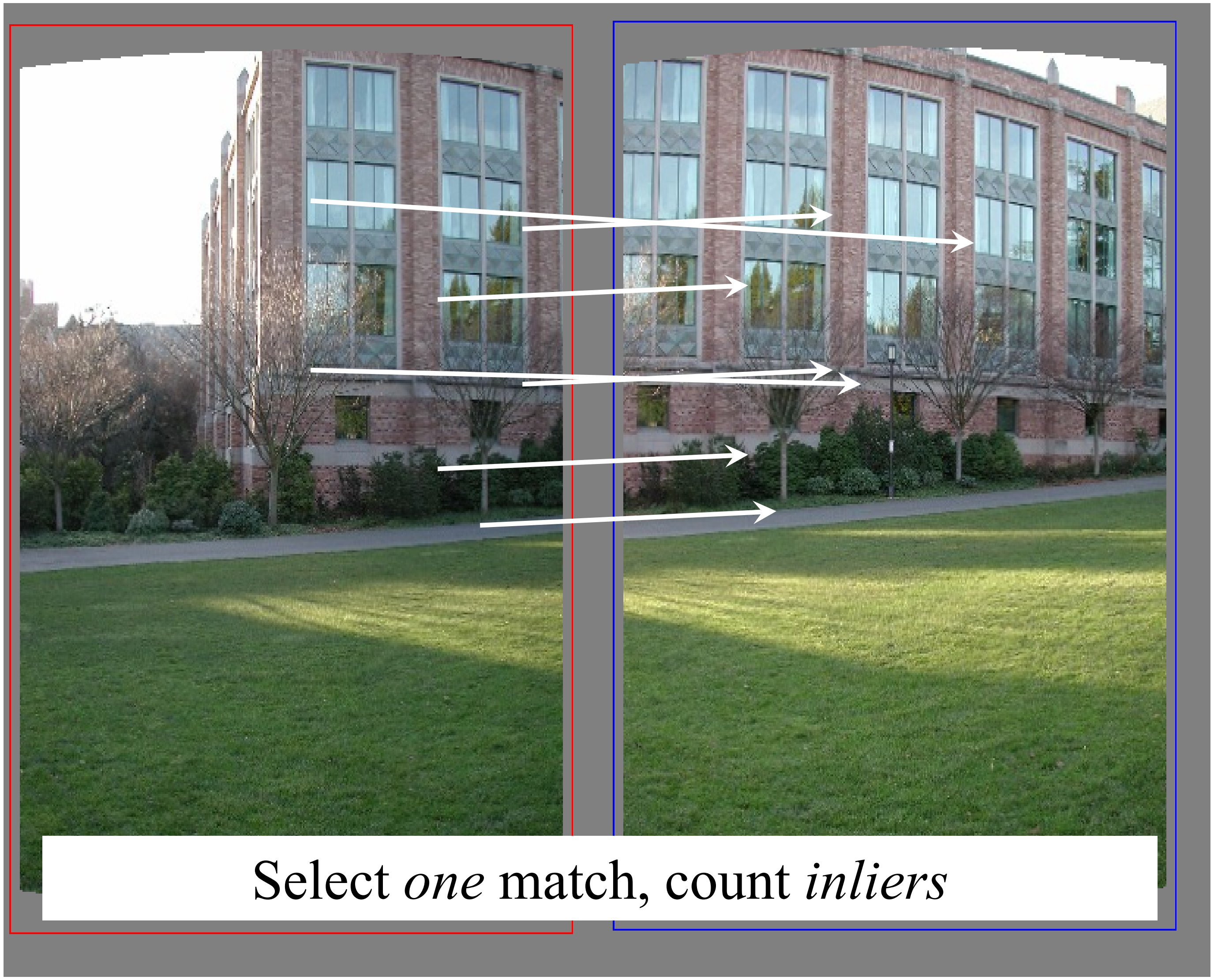


RAndom SAmple Consensus



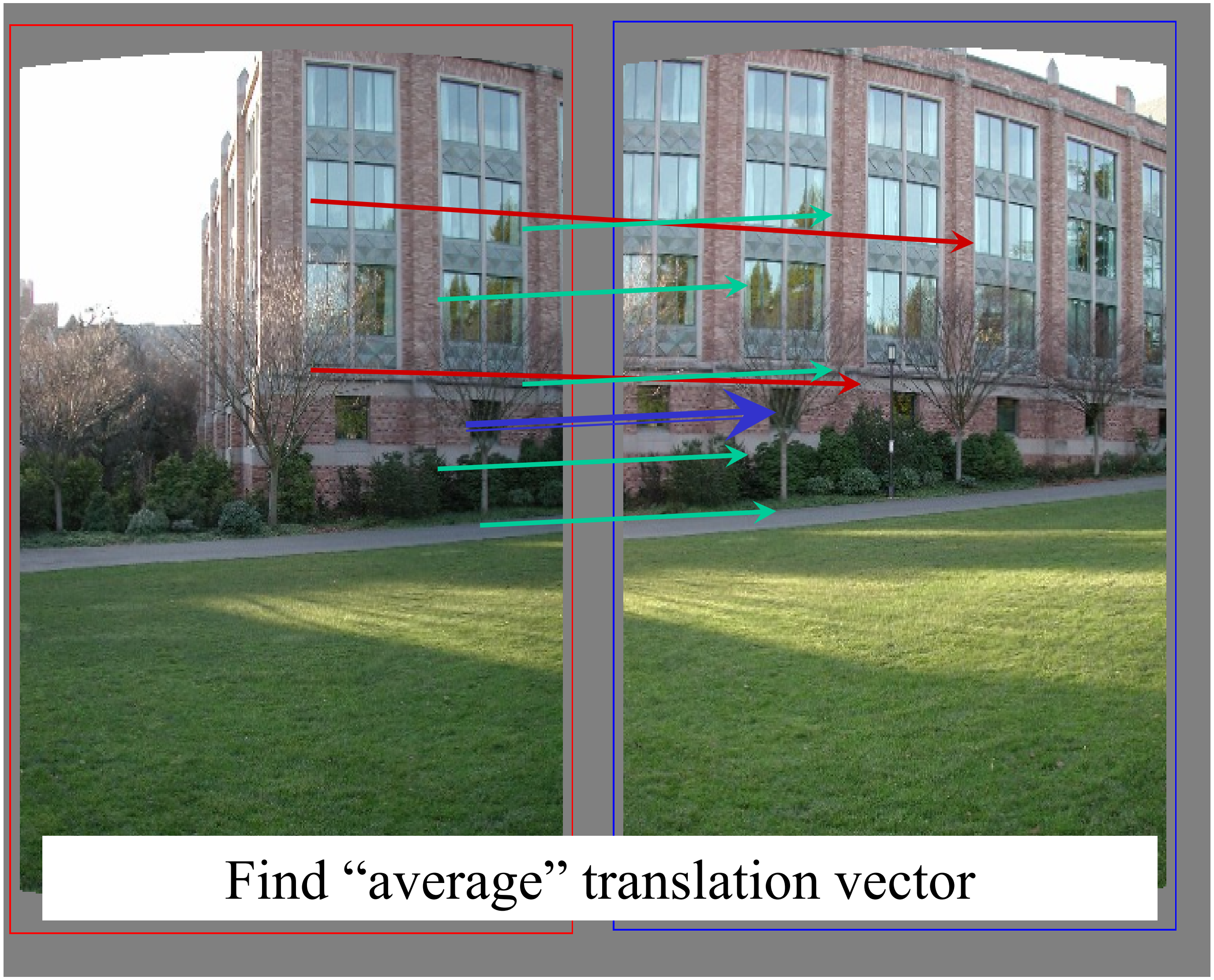
Select *one* match, count *inliers*

RAndom SAmple Consensus



Select *one* match, count *inliers*

Least squares fit



Find “average” translation vector

RANSAC for estimating homography

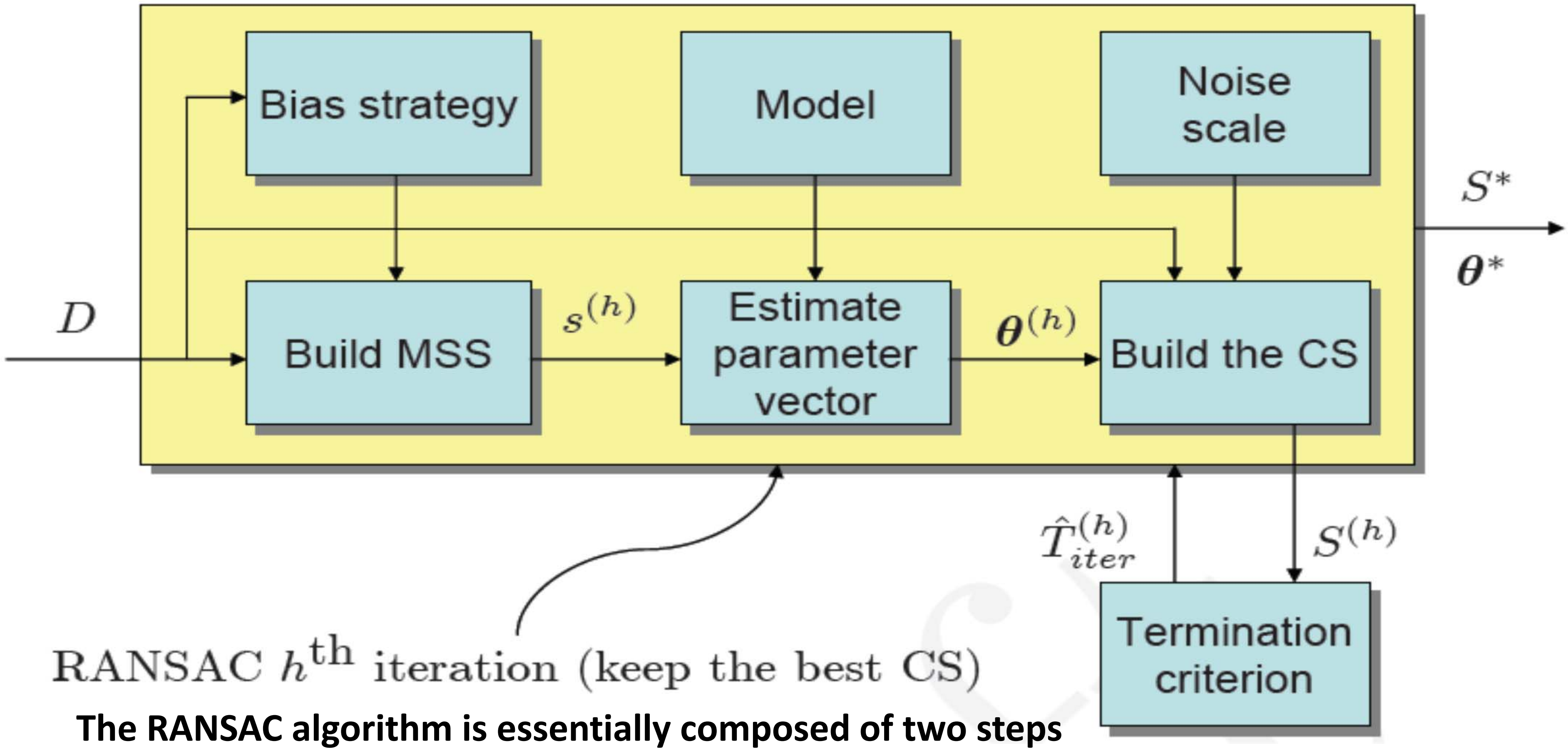
- RANSAC loop:

1. Select four feature pairs (at random)
2. Compute homography H (exact – DLT ?)
3. Compute *inliers* where $SSD(p_i', \boldsymbol{H}\, p_i) < \varepsilon$
4. Record the largest set of inliers so far
5. Re-compute least-squares H estimate on the largest set of the inliers

RANSAC in general

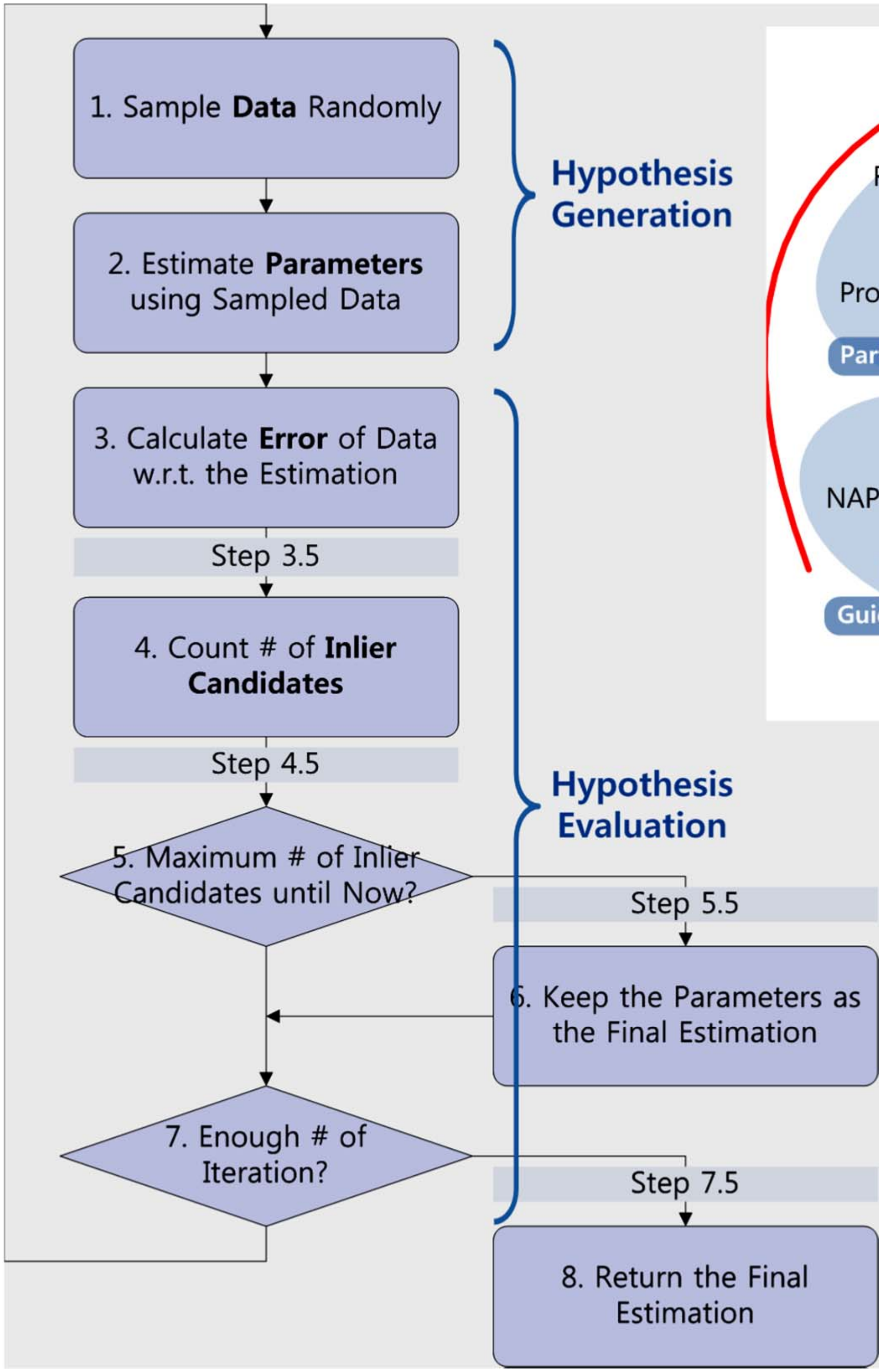
- RANSAC = Random Sample Consensus
- an algorithm for robust fitting of models in the presence of many data outliers
- Compare to robust statistics

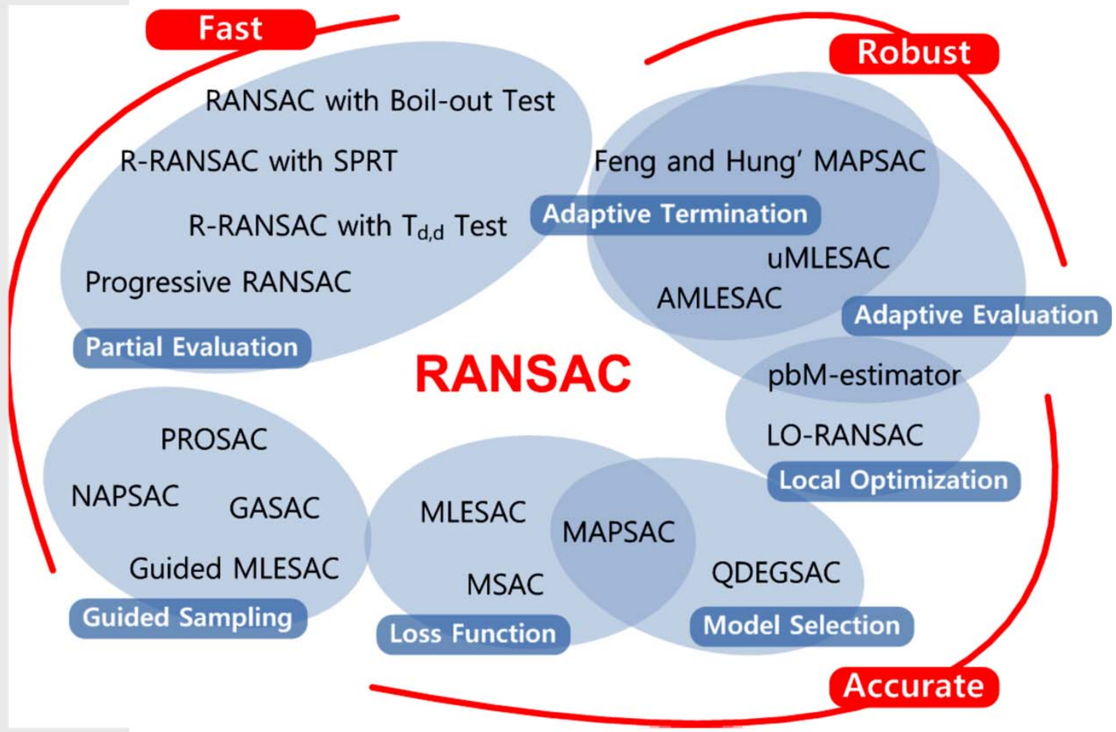
- Given N data points x_i, assume that majority of them are generated from a model with parameters Θ, try to recover Θ.

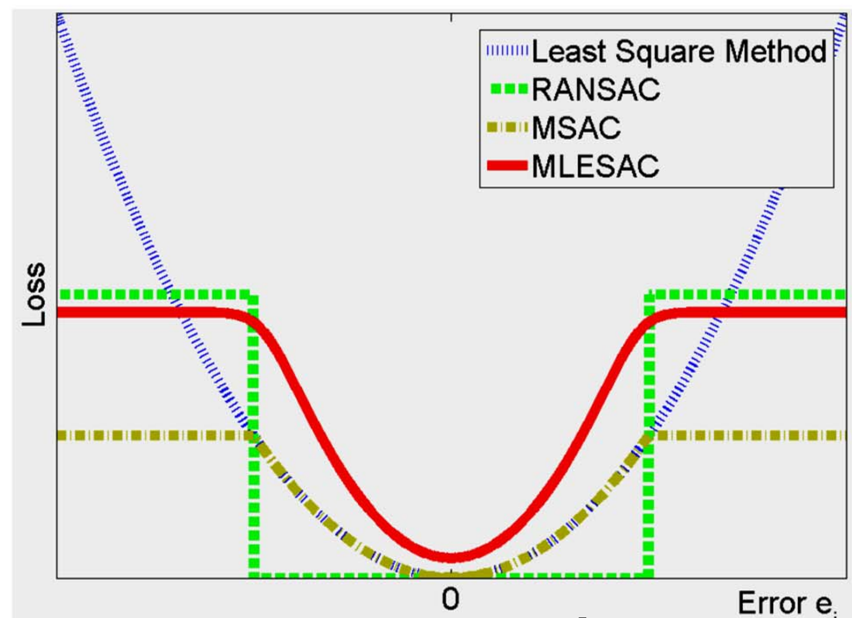


The RANSAC algorithm is essentially composed of two steps that are repeated in an iterative fashion (hypothesize{and{test framework):

- **Hypothesize.** **First minimal sample sets (MSSs) are randomly selected from the input dataset and the model parameters are computed using only the elements of the MSS. The cardinality of the MSS is the smallest sufficient to determine the model parameters (as opposed to other approaches, such as least squares, where the parameters are estimated using all the data available, possibly with appropriate weights).**
- **Test.** **In the second step RANSAC checks which elements of the entire dataset are consistent with the model instantiated with the parameters estimated in the rst step. The set of such elements is called consensus set (CS).**







Loss Functions

RANSAC converts a estimation problem in the continuous domain into a selection problem in the discrete domain. For example, there are 200 points to find a line and least square method uses 2 points. There are ${}_{200}\mathrm{C}_2 = 19,900$ available pairs. The problem is now to select the most suitable pair among huge number of pairs.

2.2 Hypothesis Evaluation

RANSAC finally chooses the most probable hypothesis, which is supported by the most inlier candidates (Step 5 and 6). A datum is recognized as the inlier candidate, whose error from a hypothesis is within a predefined threshold (Step 4). In case of line fitting, error can be geometric distance from the datum to the estimated line. The threshold is the second tuning variable, which is highly related with magnitude of noise which contaminates inliers (shortly *the magnitude of noise*). However, the magnitude of noise is also unknown in almost all application.

RANSAC solves the selection problem as an optimization problem. It is formulated as

$$\hat{M} = \arg\min_{M} \left\{ \sum_{d \in \mathscr{D}} \mathrm{Loss}\Big(\mathrm{Err}(d;M)\Big) \right\}, \tag{2}$$

where $\mathscr{D}$ is data, Loss is a loss function, and Err is a error function such as geometric distance. The loss function of least square method is represented as $\mathrm{Loss}(e) = e^2$. In contrast, RANSAC uses

$$\mathrm{Loss}(e) = \begin{cases} 0 & |\, e \,| < c \\ \mathrm{const} & \mathrm{otherwise} \end{cases}, \tag{3}$$

where c is the threshold. Figure 3 shows difference of two loss functions. RANSAC has constant loss at large error while least square method has huge loss. Outliers disturb least squares because they usually have large error.

RANSAC algorithm

Run k times: ← How many times?

(1) draw n samples randomly ← How big? Smaller is better

(2) fit parameters Θ with these n samples

(3) for each of other N-n points, calculate its distance to the fitted model, count the number of inlier points, c

Output Θ with the largest c

How to define? Depends on the problem.

How to determine k

n: number of samples drawn each iteration

p: probability of real inliers

P: probability of at least 1 success after k trials

$$P = 1 - (1 - p^n)^k$$

n samples are all inliers

a failure

failure after k trials

$$k = \frac{\log(1-P)}{\log(1-p^n)}$$

for $P=0.99$

n	p	k
3	0.5	35
6	0.6	97
6	0.5	293

Applications

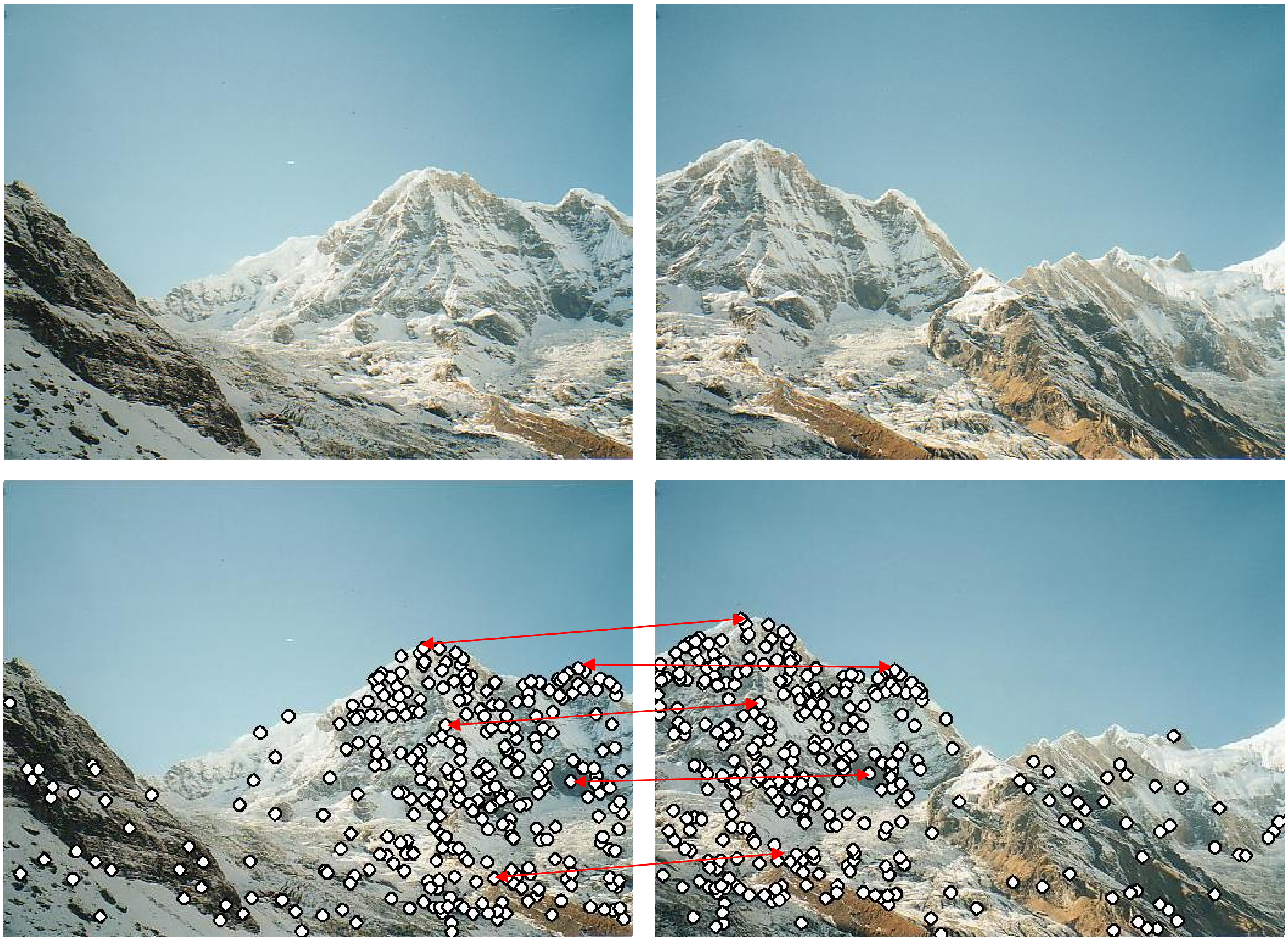
Feature Matching and RANSAC

© Krister Parmstrand

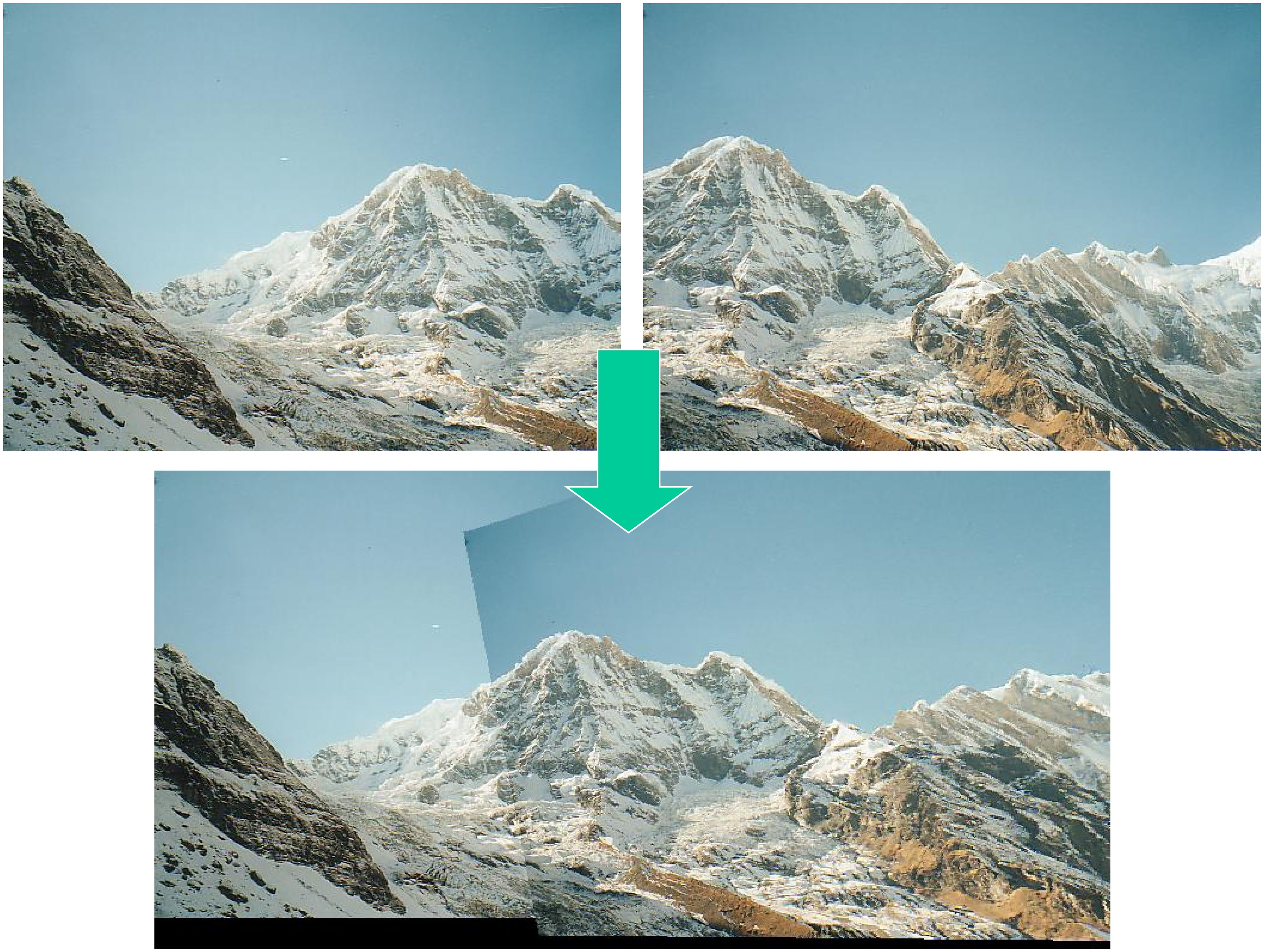
with a lot of slides stolen from Steve Seitz and Rick Szeliski

15-463: Computational Photography
Alexei Efros, CMU, Fall 2005

Automatic image stitching



Automatic image stitching



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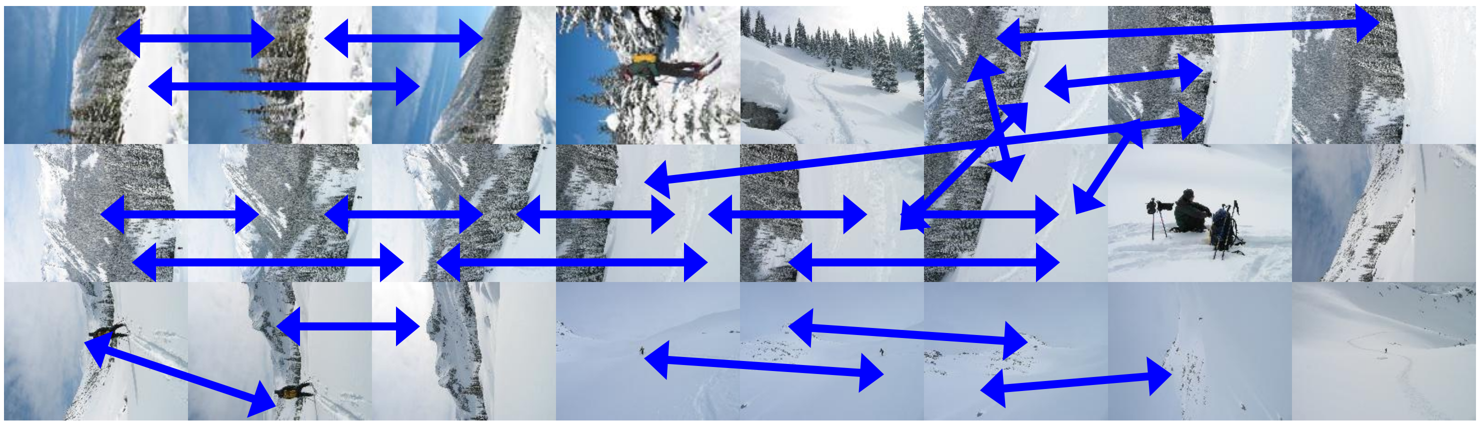
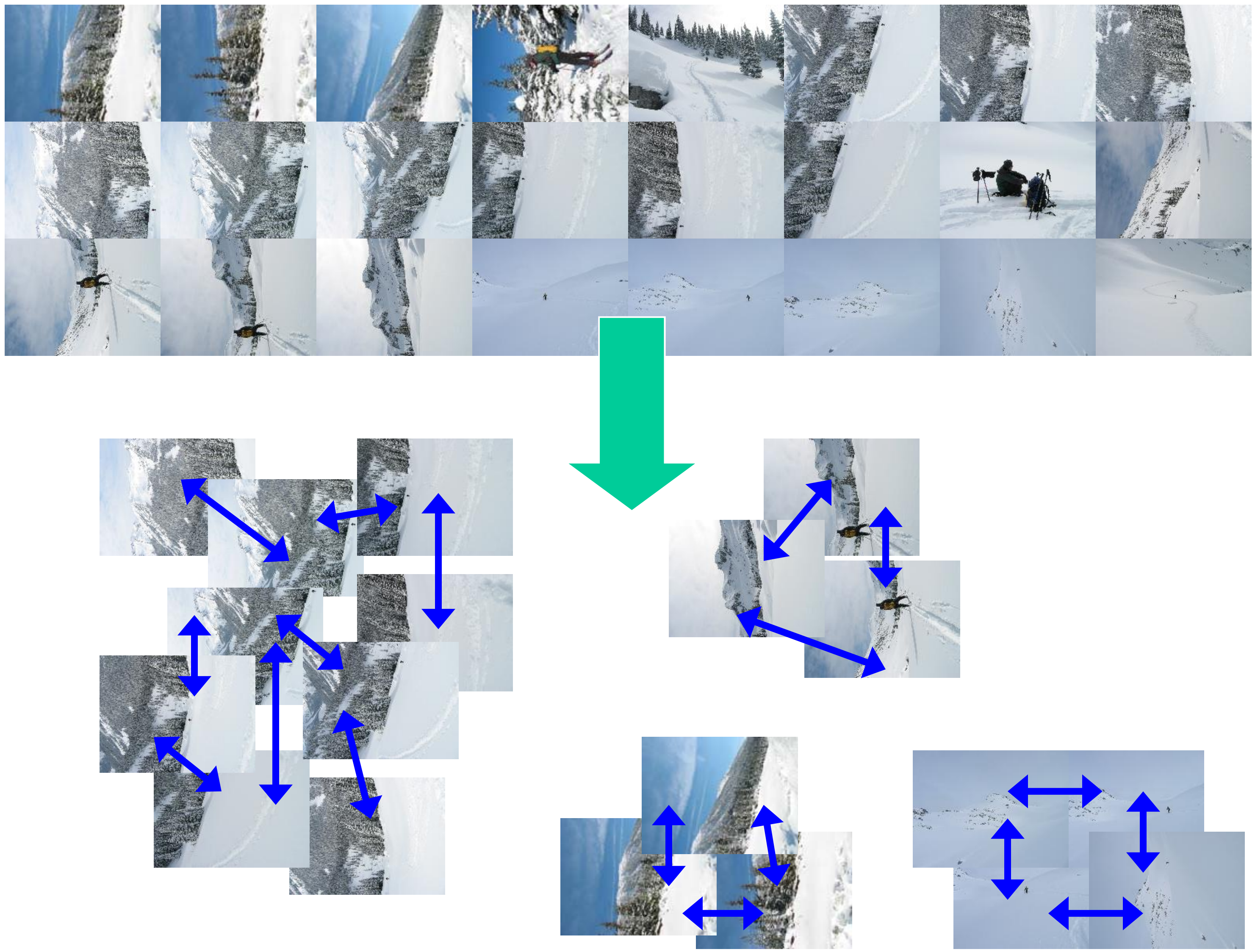
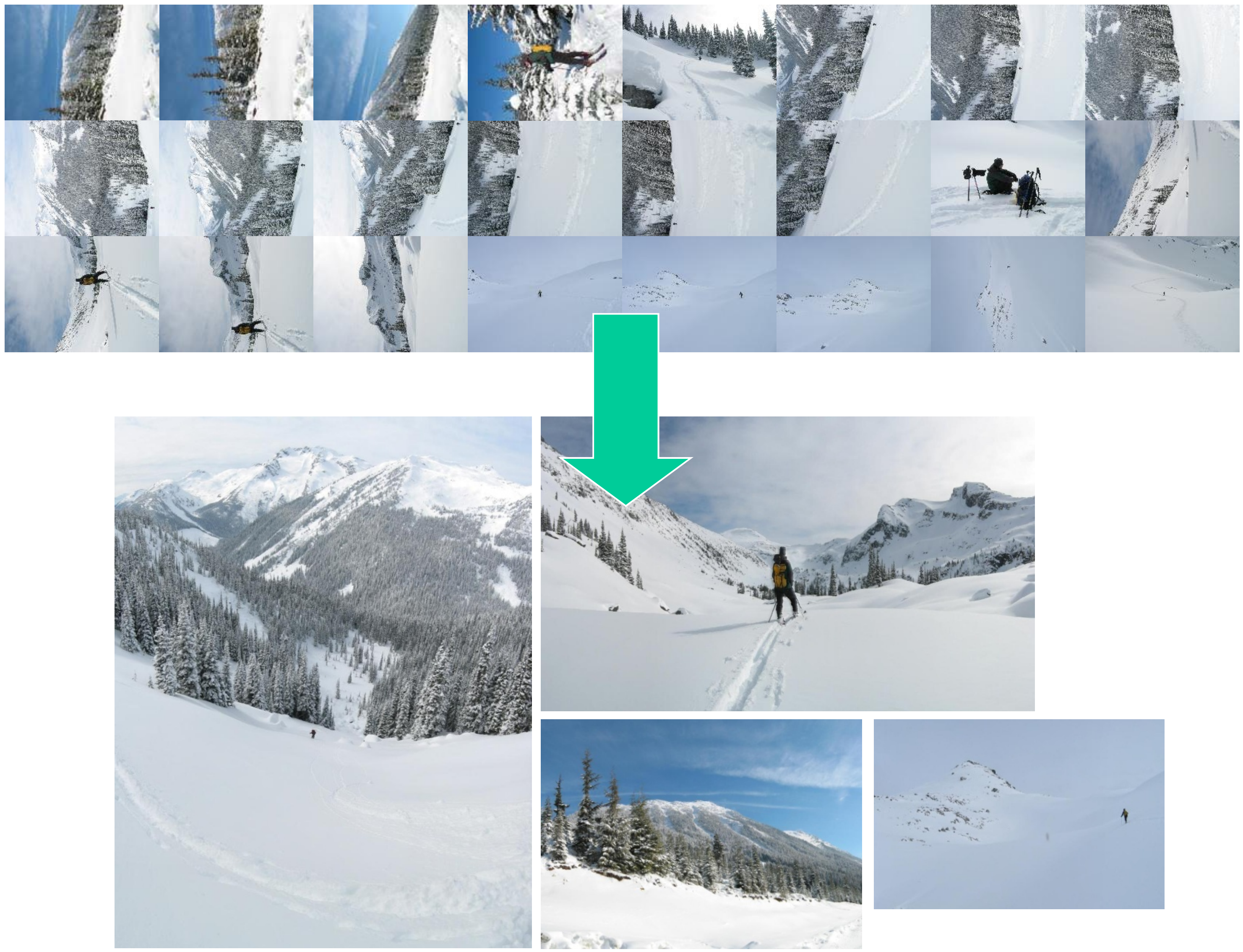


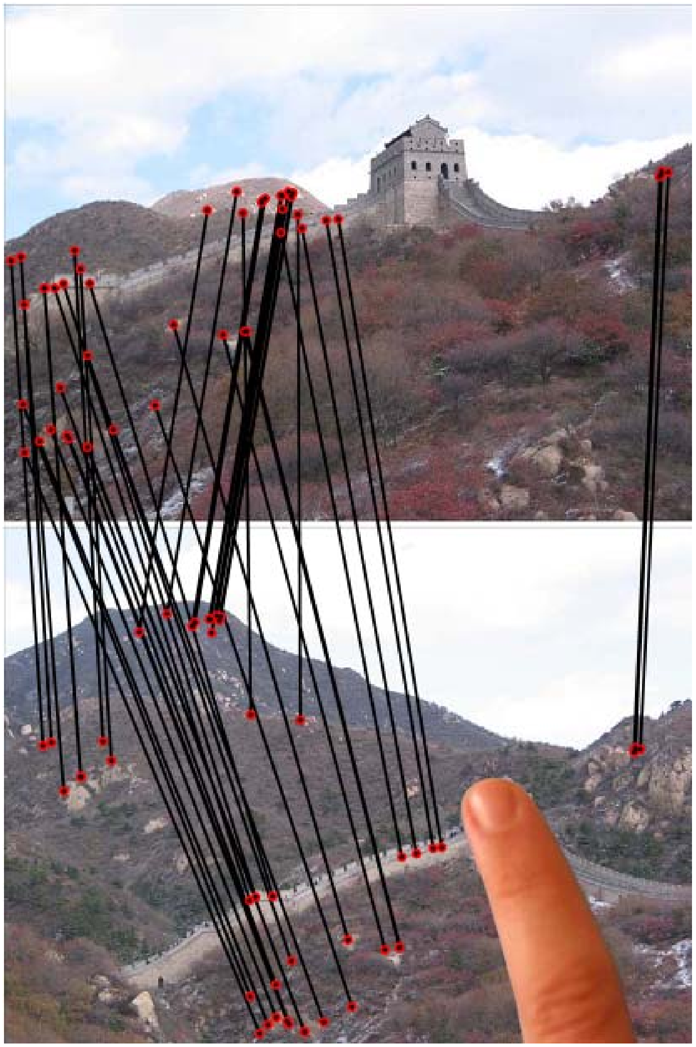
Automatic image stitching



Automatic image stitching

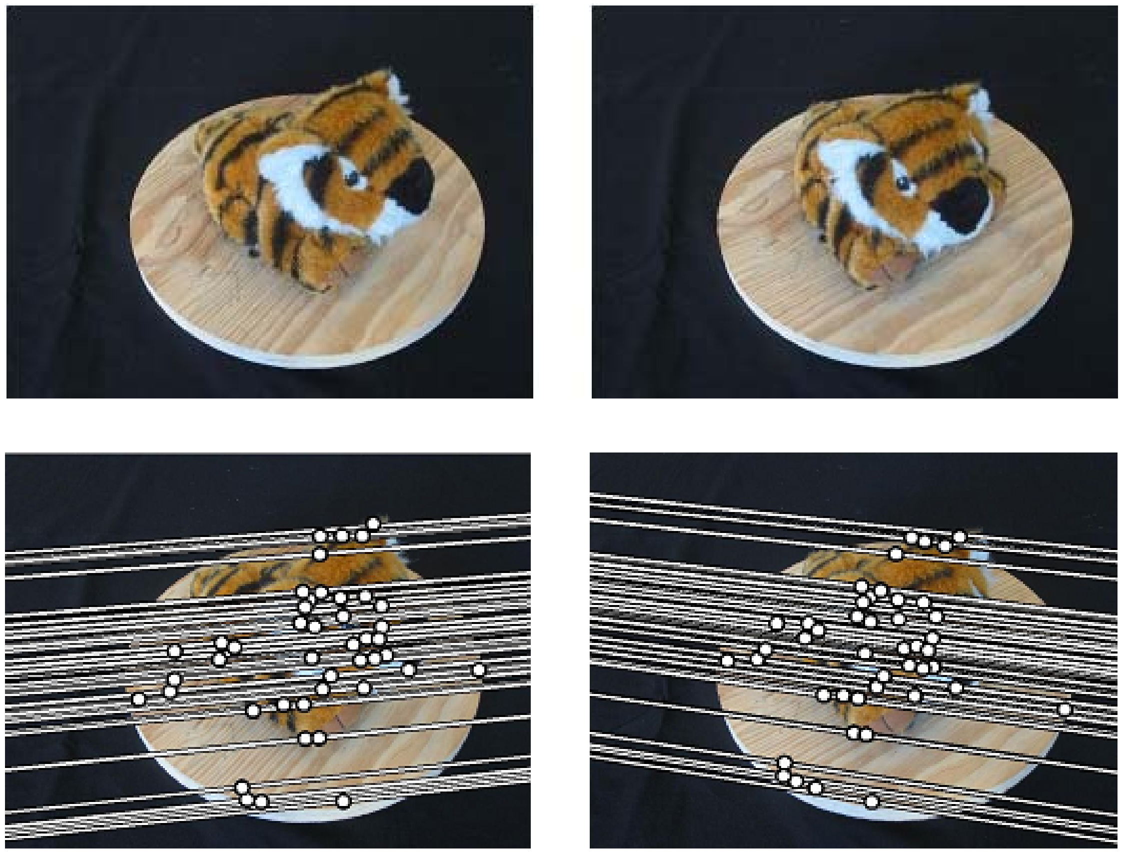


Correspondence Results

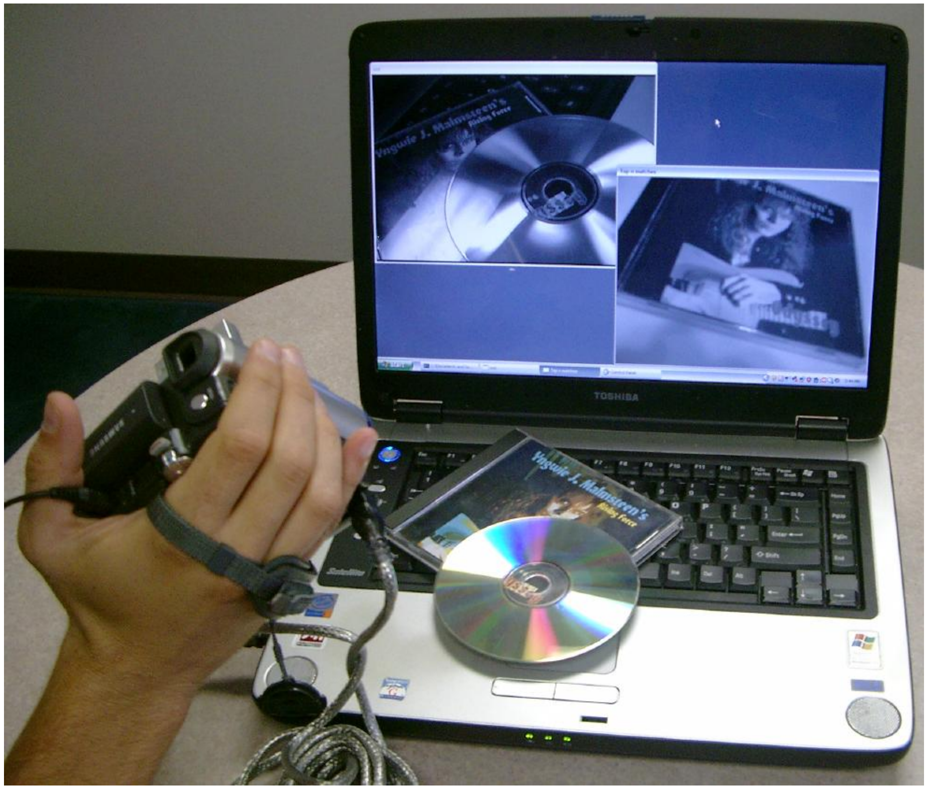
Chum & Matas 2005

Object Recognition Results



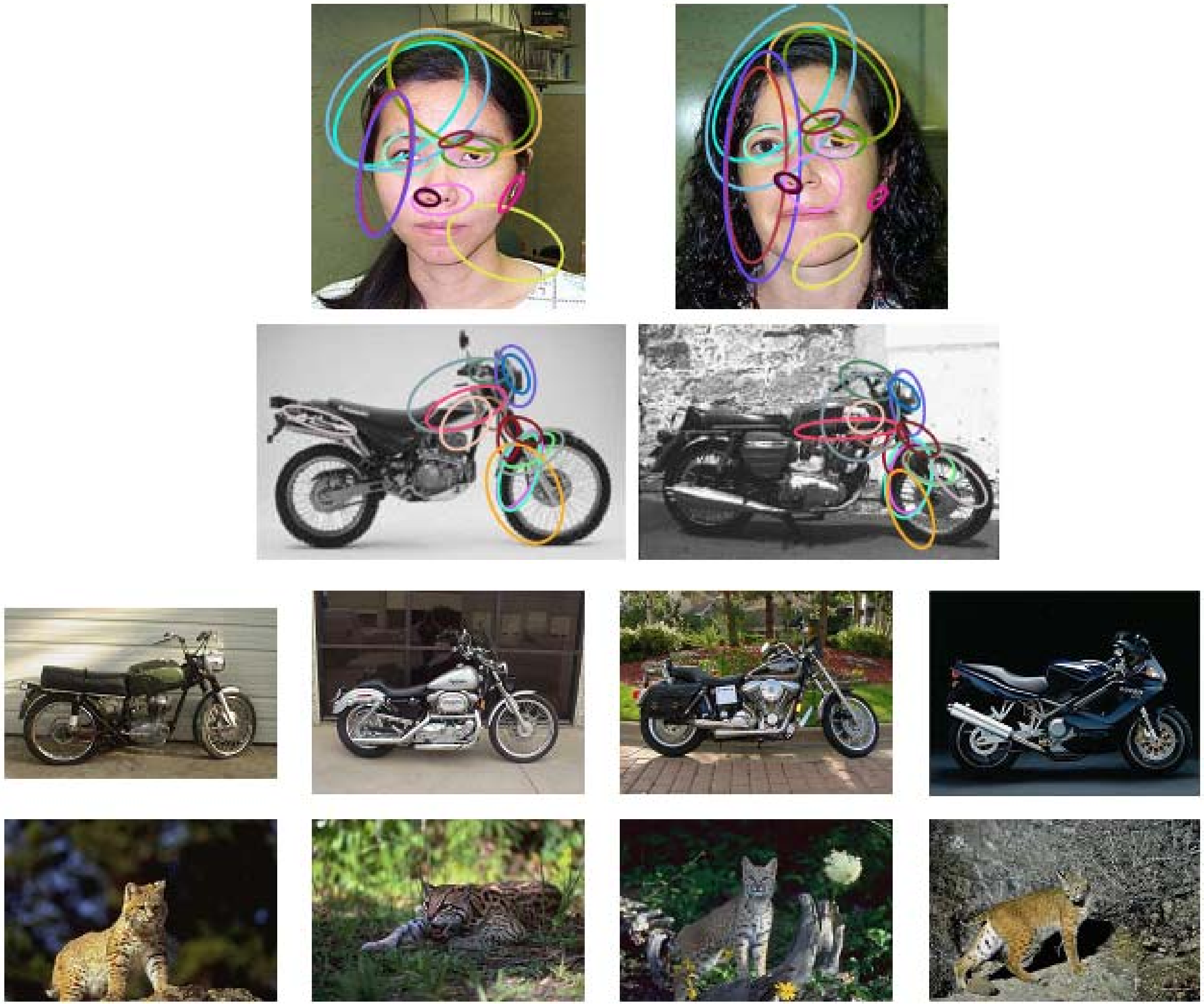
Brown & Lowe 2005

Object Recognition Results



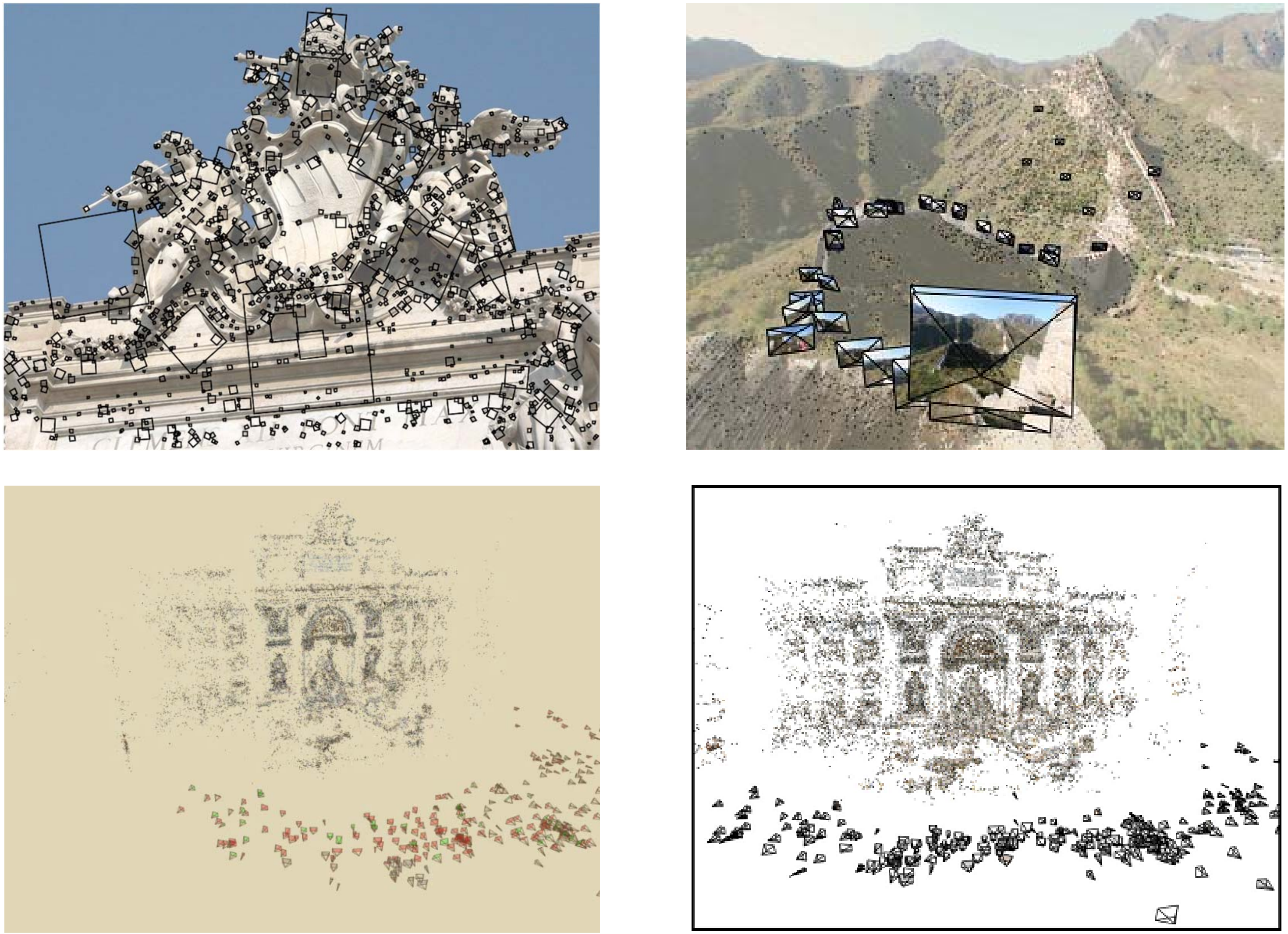
Nister & Stewenius 2006

Object Classification Results



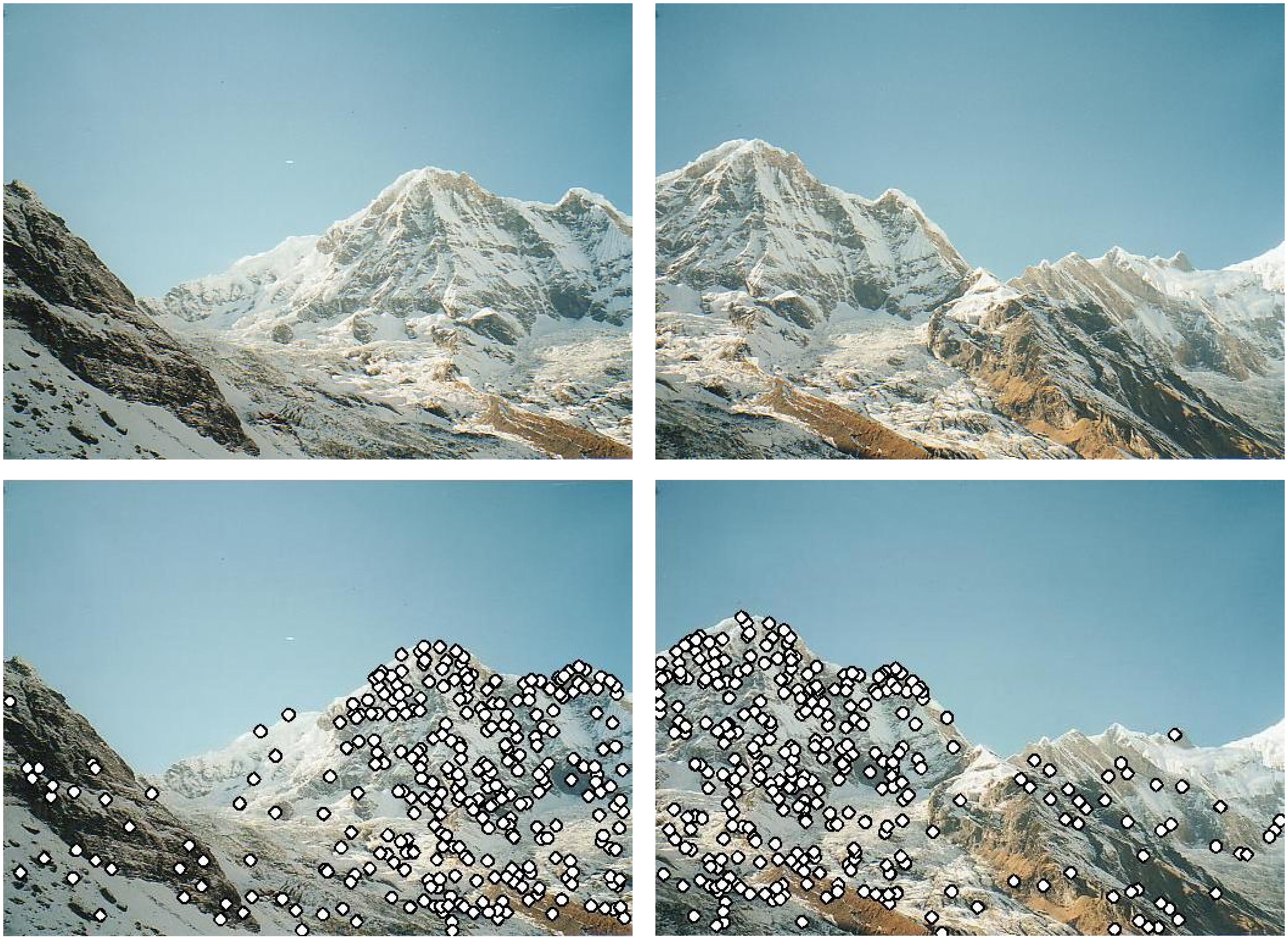
Grauman & Darrell 2006, Dorko & Schmid 2004

Geometry Estimation Results



Snavely, Seitz, & Szeliski 2006

RANSAC for Homography



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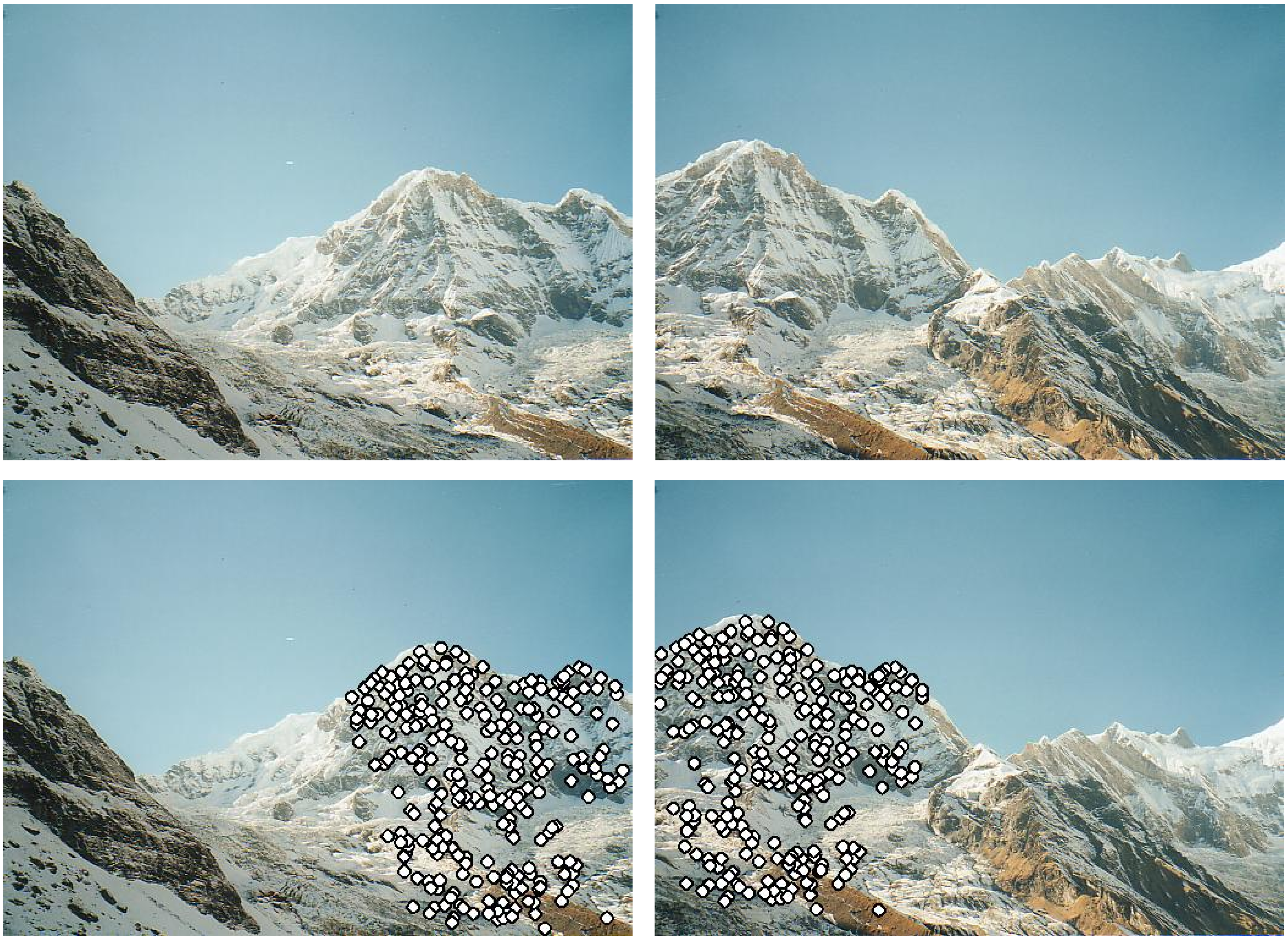
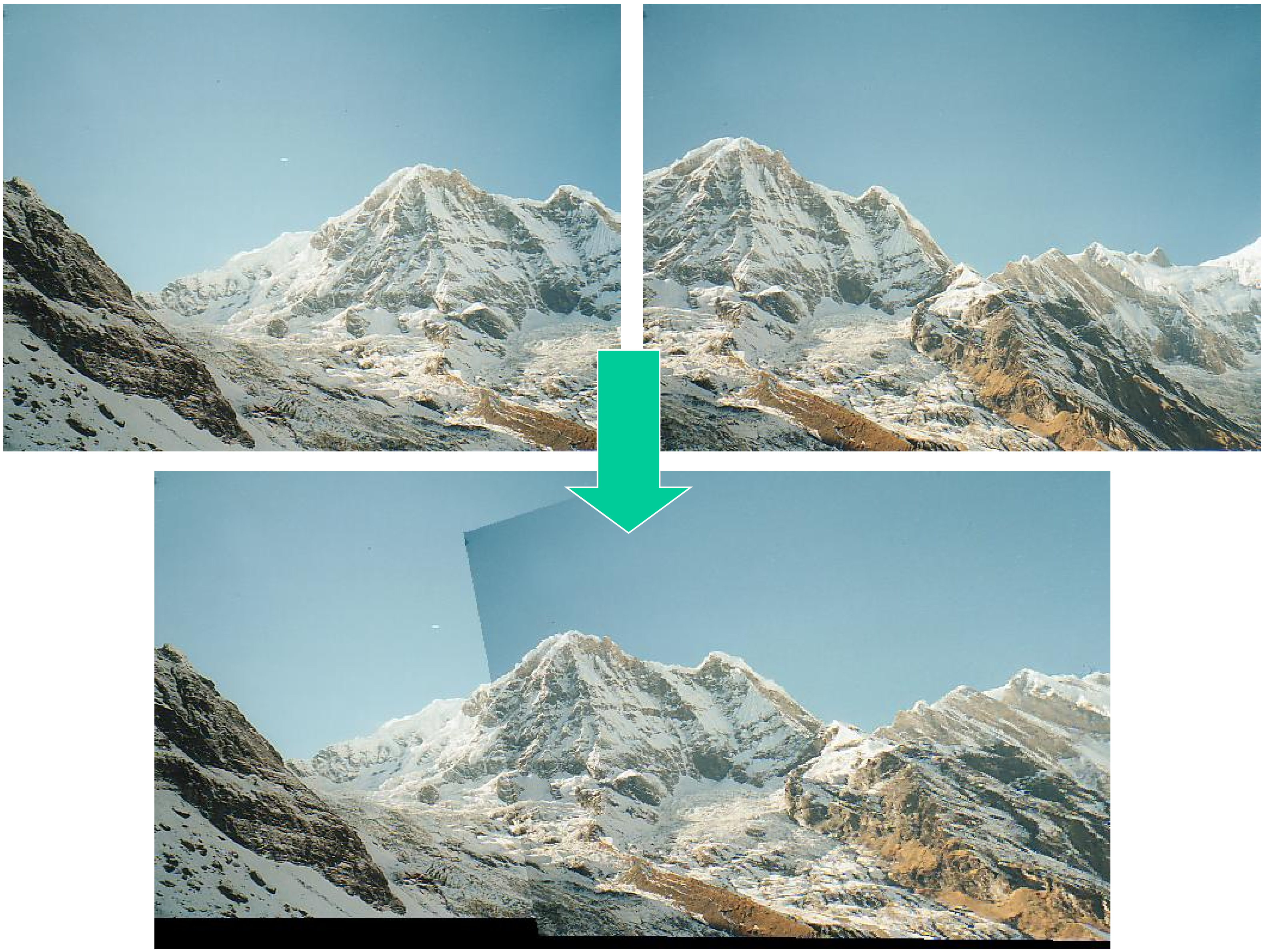
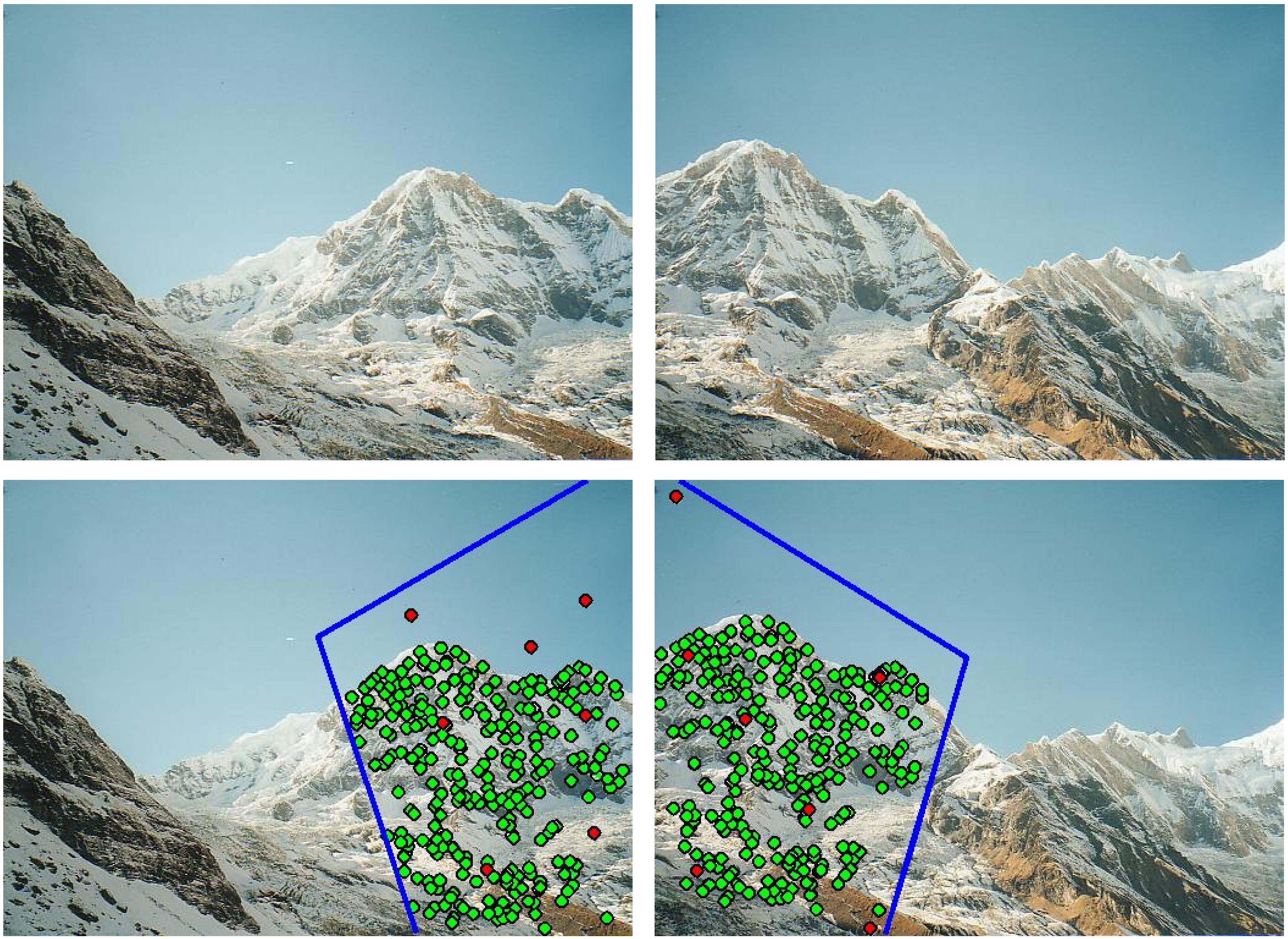


RANSAC for Homography

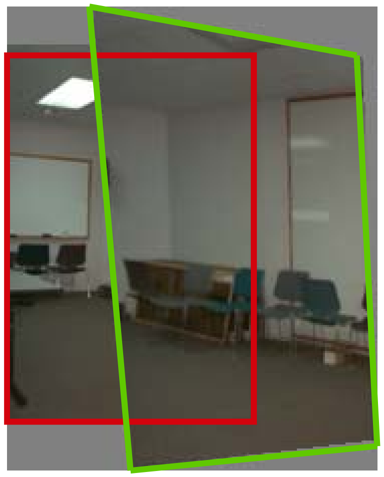


Probabilistic model for verification



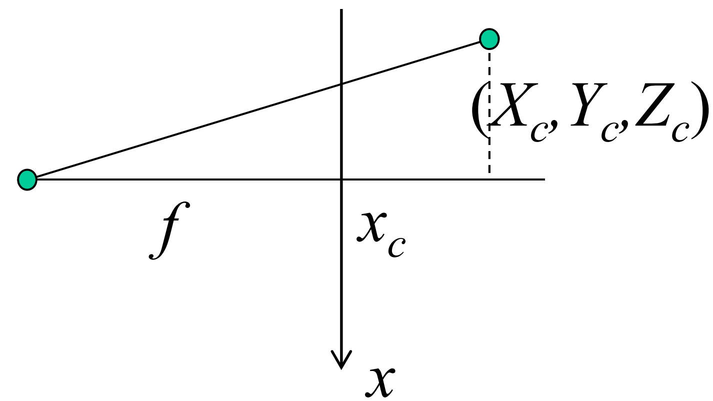
Plane perspective mosaics

- 8-parameter generalization of affine motion
  - works for pure rotation or planar surfaces
- Limitations:
  - local minima
  - slow convergence



Revisit Homography

$$\begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

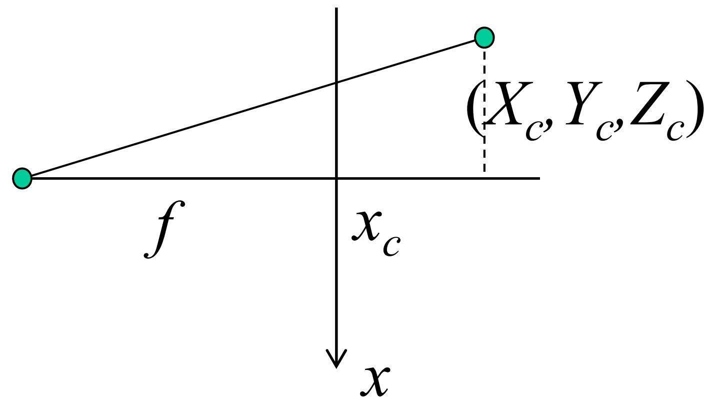


$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_c \\ 0 & f & y_c \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathbf{K}\mathbf{R}\mathbf{K}^{-1}\mathbf{x}_1 \sim \mathbf{x}_2$$

Estimate f from H?

$$\begin{pmatrix} x_1 - x_c \\ y_1 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$



$$\begin{pmatrix} x_2 - x_c \\ y_2 - y_c \\ 1 \end{pmatrix} \sim \begin{bmatrix} f_2 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

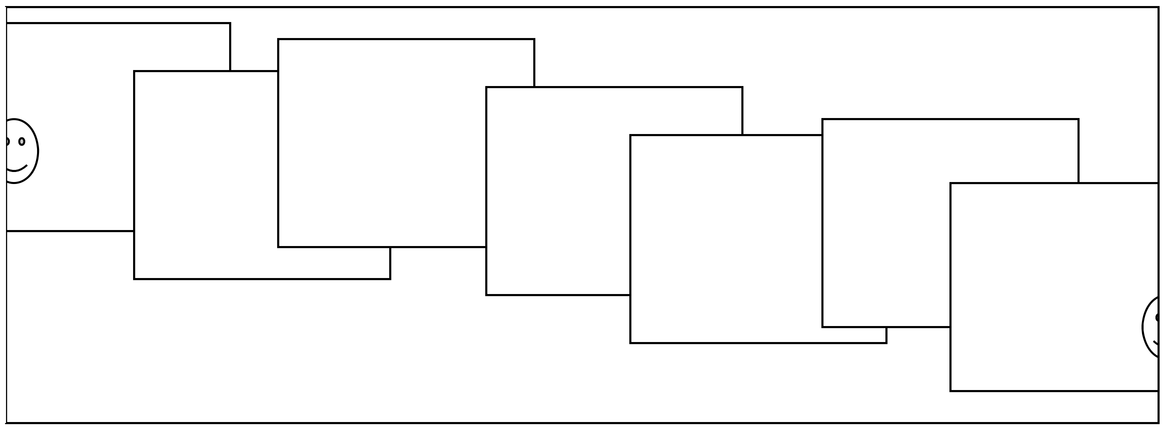
$$\mathbf{R} \sim \mathbf{K}_2^{-1} \mathbf{H} \mathbf{K}_1$$

$$= \begin{bmatrix} a & b & c / f_1 \\ d & e & g / f_1 \\ h * f_2 & i * f_2 & j * \frac{f_2}{f_1} \end{bmatrix}$$

$$\underbrace{(\mathbf{K}_2 \mathbf{R} \mathbf{K}_1^{-1})}_{\mathbf{H}} \mathbf{x}_1 \sim \mathbf{x}_2$$

$$f_1 = ?, \quad f_2 = ?$$

The drifting problem



- Error accumulation
  - small errors accumulate over time

Bundle Adjustment

Associate each image i with $\mathbf{K}_i \quad \mathbf{R}_i$

Each image i has features $\mathbf{p}_{ij}$

Trying to minimize total matching residuals

$$E(\text{all } f_i \text{ and } \mathbf{R}_i) = \sum_{(i,m)} \sum_j \left\| \mathbf{p}_{ij} \sim \mathbf{K}_i \mathbf{R}_i \mathbf{R}_m^{-1} \mathbf{K}_m^{-1} \mathbf{p}_{mj} \right\|^2$$

Derive the above, from fundamentals (eqns. 2 slides back).

Rotations

- How do we represent rotation matrices?

1. Axis / angle $(\boldsymbol{n},\theta)$
   $\boldsymbol{R} = \boldsymbol{I} + \sin\theta\, [\boldsymbol{n}]_\times + (1 - \cos\theta)\, [\boldsymbol{n}]_\times^2$
   (Rodriguez Formula), with
   $[\boldsymbol{n}]_\times$ be the cross product matrix.

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Incremental rotation update

1. Small angle approximation

   $\Delta\boldsymbol{R} = \boldsymbol{I} + \sin\theta\,[\boldsymbol{n}]_\times + (1 - \cos\theta)\,[\boldsymbol{n}]_\times^2$

   $\approx \boldsymbol{I} + \theta\,[\boldsymbol{n}]_\times = \boldsymbol{I} + [\boldsymbol{\omega}]_\times$

   linear in $\boldsymbol{\omega} = \theta\boldsymbol{n}$

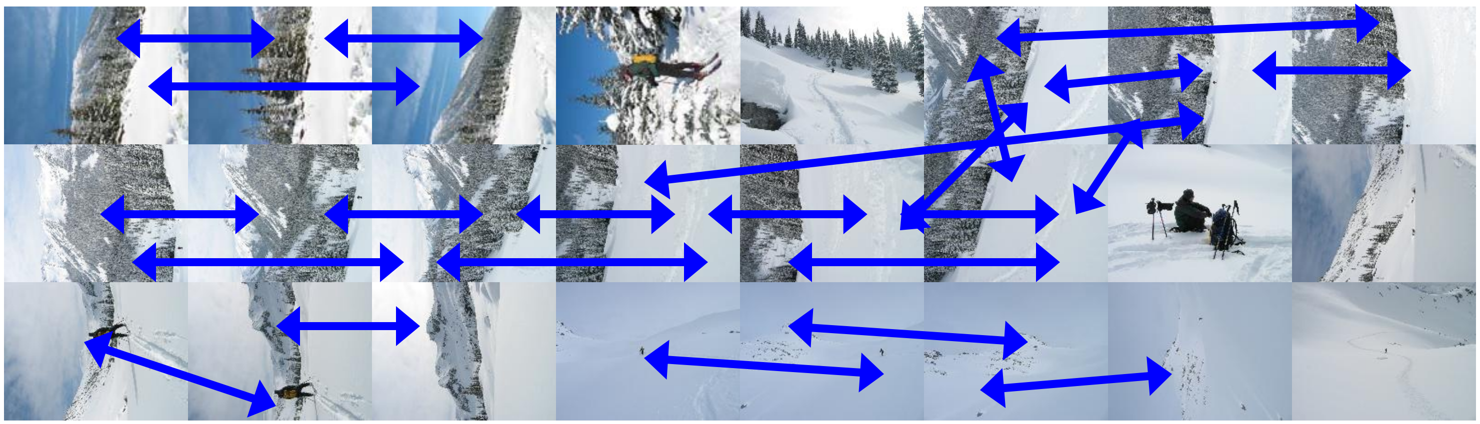
2. Update original $\boldsymbol{R}$ matrix

   $\boldsymbol{R} \leftarrow \boldsymbol{R}\,\Delta\boldsymbol{R}$

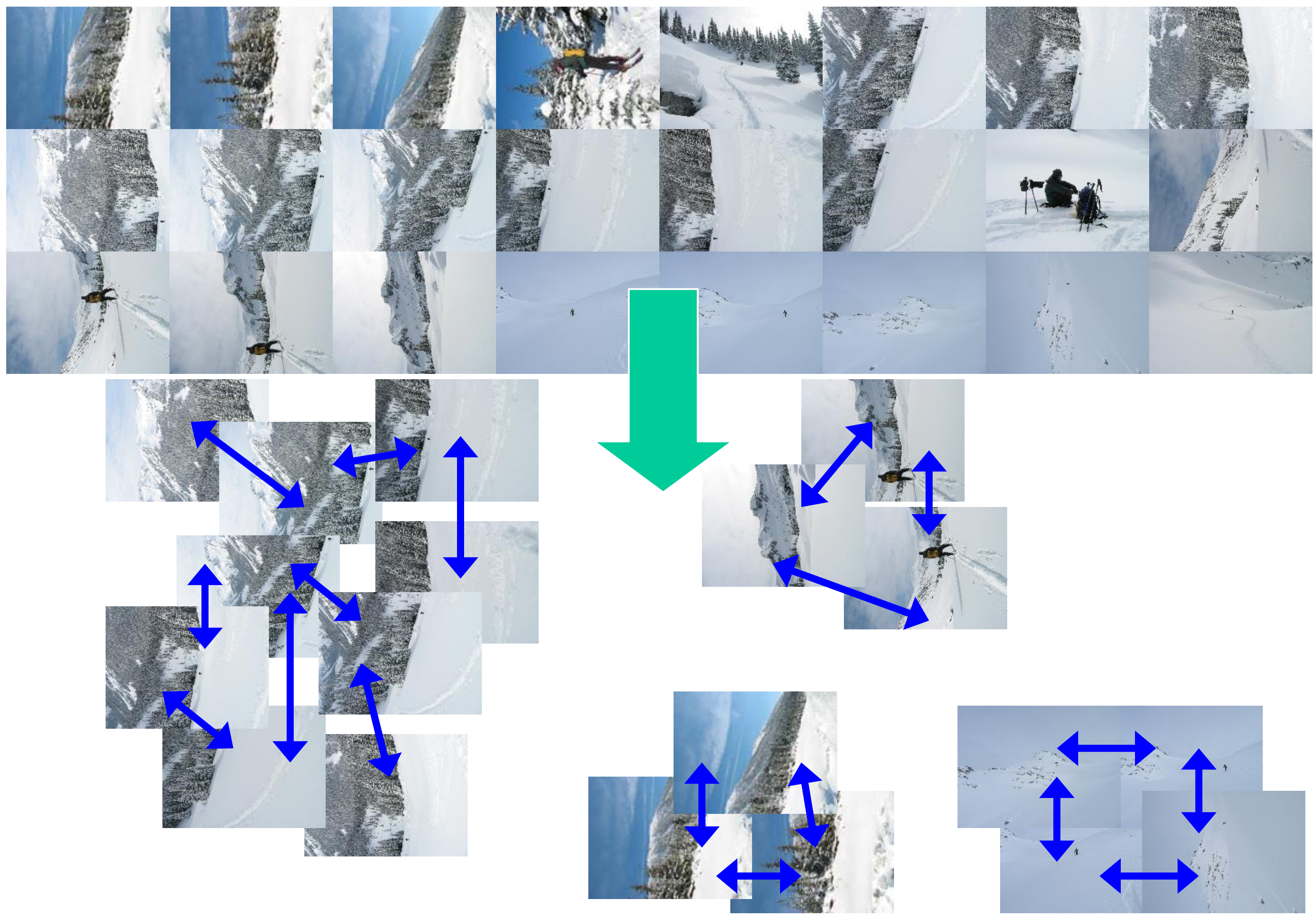
Recognizing Panoramas

[Brown & Lowe, ICCV'03]

Finding the panoramas



Finding the panoramas



Algorithm overview

Algorithm: Panoramic Recognition

Input: n unordered images

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I. Extract SIFT features from all n images

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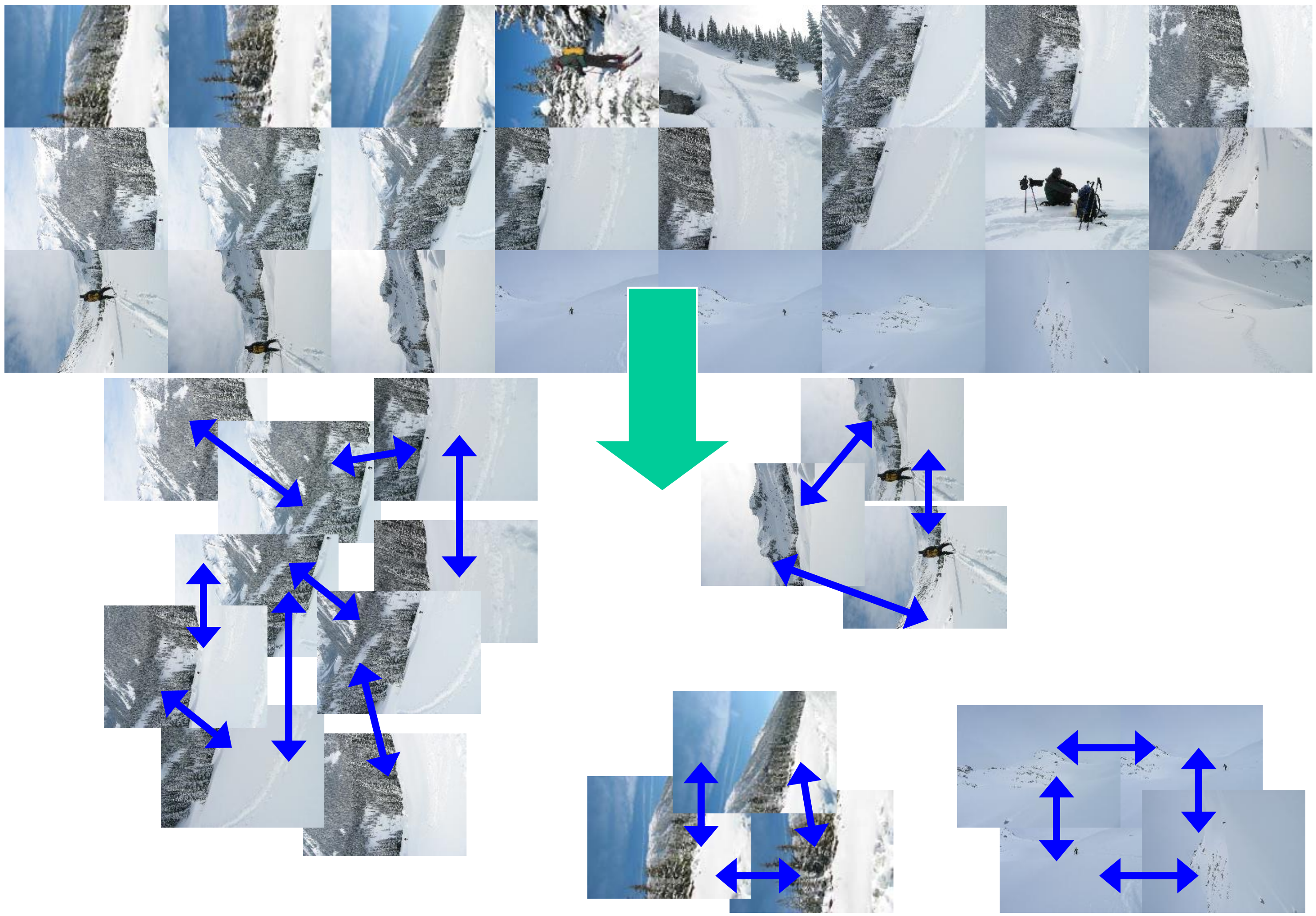
IV. Find connected components of image matches

Finding the panoramas



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V. For each connected component:

  (i) Perform bundle adjustment to solve for the rotation $\theta_1, \theta_2, \theta_3$ and focal length f of all cameras

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III. For each image:
  (i) Select m candidate matching images (with the maximum number of feature matches to this image)
  (ii) Find geometrically conisistent feature matches using RANSAC to solve for the homography between pairs of images
  (iii) Verify image matches using probabilistic model

IV. Find connected components of image matches

V. For each connected component:
  (i) Perform bundle adjustment to solve for the rotation $\theta_1, \theta_2, \theta_3$ and focal length f of all cameras
  (ii) Render panorama using multi-band blending

Output: Panoramic image(s)

Why “Recognising Panoramas”?

1D Rotations (θ)

- Ordering $\Rightarrow$ matching images

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- 2D Rotations (θ, ϕ)
  - Ordering $\not\Rightarrow$ matching images

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- 2D Rotations (θ, ϕ)
  - Ordering $\nRightarrow$ matching images

Homography for Rotation

Parameterise each camera by rotation and focal length

$$\mathbf{R}_i = e^{[\boldsymbol{\theta}_i]_\times}, \ [\boldsymbol{\theta}_i]_\times = \begin{bmatrix} 0 & -\theta_{i3} & \theta_{i2} \\ \theta_{i3} & 0 & -\theta_{i1} \\ -\theta_{i2} & \theta_{i1} & 0 \end{bmatrix}$$

$$\mathbf{K}_i = \begin{bmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This gives pairwise

$$\tilde{\mathbf{u}}_i = \mathbf{H}_{ij}\tilde{\mathbf{u}}_j, \ \mathbf{H}_{ij} = \mathbf{K}_i\mathbf{R}_i\mathbf{R}_j^T\mathbf{K}_j^{-1}$$

Bundle Adjustment

New images initialised with rotation, focal length of best matching image

Bundle Adjustment

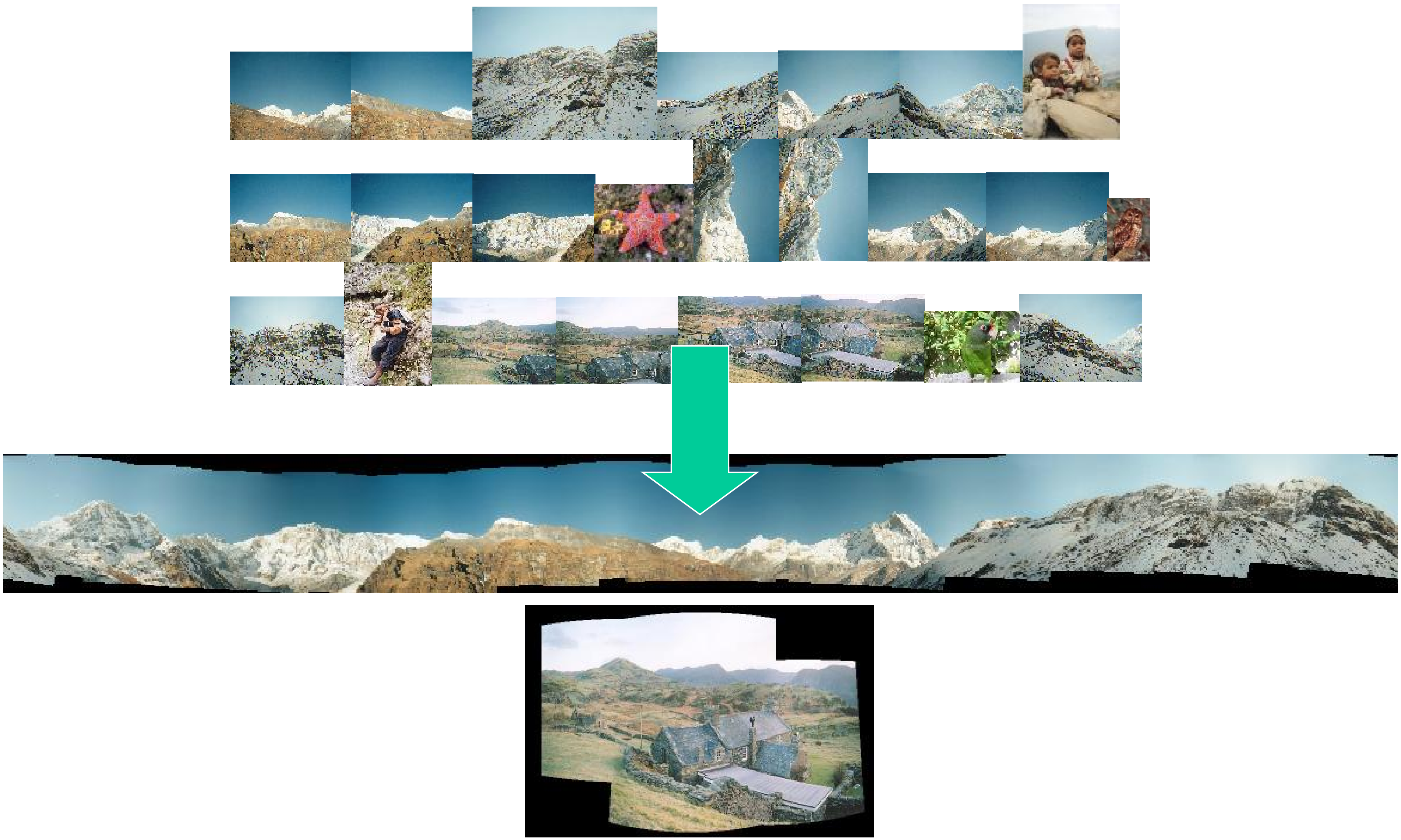
New images initialised with rotation, focal length of best matching image

Multi-band Blending

Burt & Adelson 1983

- Blend frequency bands over range $\propto \lambda$

Results



Get you own copy!

[Brown & Lowe, ICCV 2003]
[Brown, Szeliski, Winder, CVPR'05]

How well does this work?

Test on 100s of examples…

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Test on 100s of examples…

…still too many failures (5-10%)
for consumer application

Matching Mistakes: False Positive

Matching Mistakes: False Positive

Matching Mistakes: False Negative

- Moving objects: large areas of disagreement

Matching Mistakes

- Accidental alignment
  - repeated / similar regions
- Failed alignments
  - moving objects / parallax
  - low overlap
  - "feature-less" regions

- No 100% reliable algorithm?

How can we fix these?

- Tune the feature detector
- Tune the feature matcher (cost metric)
- Tune the RANSAC stage (motion model)
- Tune the verification stage
- Use “higher-level” knowledge
  - e.g., typical camera motions

- Need a large training/test data set (panoramas)

Object Tracking Results

Gordon & Lowe 2005

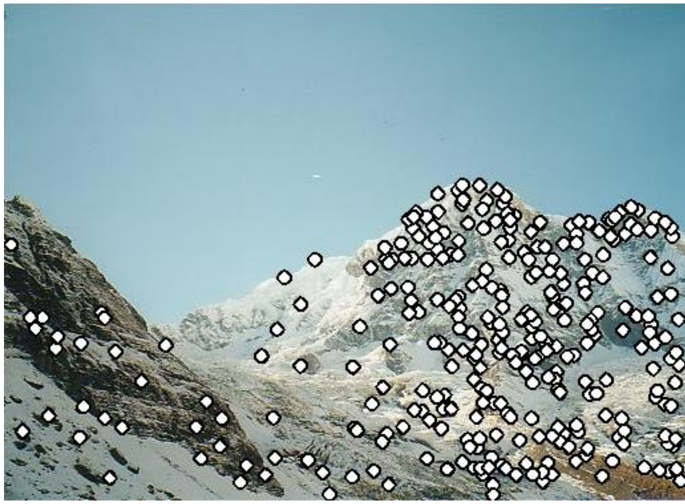
Robotics: Sony Aibo

SIFT is used for

- Recognizing charging station
- Communicating with visual cards
- Teaching object recognition

- soccer

Image Alignment



Feature Dete on and Matching

Cylinder:
Translation
2 DoF

Plane:
Homography
8 DoF