Simulation studies for the reconstruction of a straight line in 3-D from two arbitrary perspective views using Epipolar line method

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ABSTRACT

Reconstuction of a line in 3-D space using arbitrary perspectve views involves the problem of obtaining the set of parameters representing the line. This is widely used for many applications of 3-D object recognition and machine inspection. A performance analysis of the reconstruction process in the presence of noise in the image planes is necessary in certain applications which require a large degree of accuarcy. In this paper, a methodology, which is based on the concept of epipolar line, for the reconstruction of a 3-D line, from two arbitrary perspective views is given. In this problem the points in the second image plane, which correspond to points in the first image plane are found by using epipolar line method, by considering all the points in the first image plane. Then triangulation law is used to find the points in 3-D space. Using least square regression in 3-D, the parameters of a line in 3-D space are found. This least square regression problem is solved by two different methods. Simulation study results of this epipolar line based method, in presence of noise, as well as results of error analysis are given.

Keywords: Perspective views, Least square regression, Error Analysis, Parameter, Noise

1. INTRODUCTION

Based on epipolar line method, a methodology for the reconstruction of a 3-D line from two arbitrary perspective views is given. In this problem by considering all the points of the line in the first image plane, the corresponding points in the second image plane are found by using epipolar line method. Then triangulation law is used to find the points in 3-D space. Using least square regression in 3-D, the parameters of a line in 3-D space are found. This least square regression problem is solved by two different methods. Simulation study results of this epipolar line based method in presence of noise and results of error analysis are also given. For the detailed study of reconstruction of lines, curves and surfaces one can refer the articles,[1] [3] and.[4]

Performance analysis of the method of reconstruction described in this paper is based on simulation studies. Noise with Gaussian distribution is added to the pixel coordinates of the projections of the line on the pair of image planes. This simulates the effect of noise in the sensor and signal acquisition system as well as errors in the preprocessing tools used to detect the line segments from gray level images. Error between the original and the reconstructed line is estimated and used as a criteria to analyze the performance of the system. These results provide an optimal range of values of the parameters to be used for the design of the stereo-based imaging system for best reconstruction. Certain conditions of the viewing geometry where the reconstruction process has a poor performance are also obtained from these studies. The results presented in this paper will be useful for both sensor design and error modelling of position measuring systems for computer vision and related applications.

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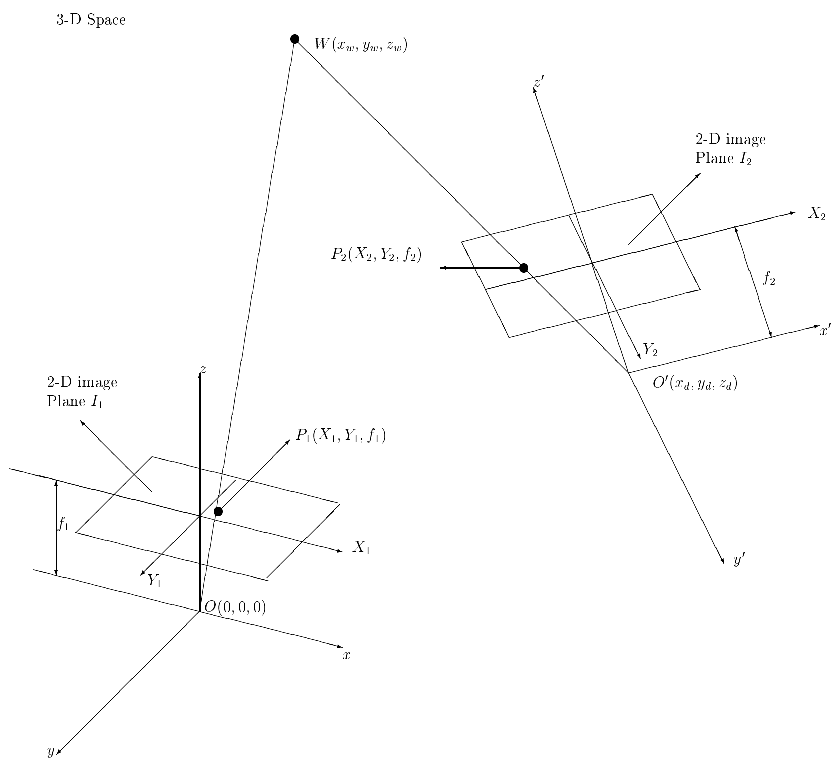

Figure 1. Imaging system with two arbitrary perspective views of a point W in 3-D space. f_1 and f_2 are the focal lengths of first and second cameras respectively.

2. BASIC MODEL OF IMAGING SETUP

Let I_1 and I_2 be the first and second image planes of the pair of cameras C_1 and C_2 respectively. Let the position and the orientation of one camera be known with respect to another and both cameras have a common field of view. Let $OXYZ$ be the rectangular cartesian frame of reference with its origin O at the center of projection of one of the cameras say C_1. A point W in 3-D space, with co-ordinates (x_w, y_w, z_w) with respect to the frame of reference at C_1, is viewed by both the cameras, C_1 and C_2. Let $O'X'Y'Z'$ be the second rectangular cartesian co-ordinate system, not necessarily parallel to $OXYZ$ system, with its origin O' at the center of projection of the second camera C_2. Let the co-ordinates of the second camera C_2 with respect to O be (x_d, y_d, z_d). Let $P_1(X_1, Y_1, f_1)$ and $P_2(X_2, Y_2, f_2)$ be the corresponding pair of projections of the point W on the pair of image planes I_1 and I_2 respectively. Let f_1 and f_2 be the respective focal lengths of the first and the second cameras. The imaging set-up using two cameras is shown in figure 1.

3. THE COLLINEARITY EQUATIONS

The collinearity equations represent the mathematical process of image formation, linking the co-ordinates of a point on an object in 3-D space to the corresponding co-ordinates of its projection in the 2-D image planes. The collinearity equations are derived using the criteria that all the three points, namely, the center of perspective projection, the image point and the object point lie on the same straight line.

The relation between the coordinates of the point $W(x_w, y_w, z_w)$ and that of the image point $P_1(X_1, Y_1, f_1)$ is given by the perspective equation[7]:

$$X_1 = f_1 \ \frac{x_w}{z_w} \quad , \qquad Y_1 = f_1 \ \frac{y_w}{z_w} \tag{1}$$

The 3-D co-ordinates $(x_w^{'}, y_w^{'}, z_w^{'})$ of the point $W(x_w, y_w, z_w)$ with respect to the second camera C_2, is given by

$$\begin{bmatrix} x_w^{'} \\ y_w^{'} \\ z_w^{'} \end{bmatrix} = \lambda \begin{bmatrix} \cos\alpha_1 & \cos\beta_1 & \cos\gamma_1 \\ \cos\alpha_2 & \cos\beta_2 & \cos\gamma_2 \\ \cos\alpha_3 & \cos\beta_3 & \cos\gamma_3 \end{bmatrix} \begin{bmatrix} (x_w - x_d) \\ (y_w - y_d) \\ (z_w - z_d) \end{bmatrix}, \tag{2}$$

where

$$\begin{aligned}
\cos\alpha_1 &= \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi, \\
\cos\alpha_2 &= \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi, \\
\cos\alpha_3 &= \sin\psi\sin\theta, \\
\cos\beta_1 &= -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi, \\
\cos\beta_2 &= -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi, \\
\cos\beta_3 &= \cos\psi\sin\theta, \\
\cos\gamma_1 &= \sin\phi\sin\theta, \\
\cos\gamma_2 &= -\sin\theta\cos\phi, \\
\cos\gamma_3 &= \cos\theta,
\end{aligned}$$

θ, ϕ and ψ being the Eulerian angles.[6]

In the above equations $\alpha_j, \beta_j, \gamma_j$, $(j = 1, 2, 3)$ are the respective direction cosines of the axes of $O'X'Y'Z'$ with respect to $OXYZ$ system. λ is a scale factor between the two reference frames and without loss of generality this is considered to be 1, in this work. Using equation (2), the relation between the object space point $W(x_w, y_w, z_w)$ and the image point $P_2(X_2, Y_2, f_2)$ is given by the perspective equations[7]:

$$X_2 = f_2 \ \frac{(x_w - x_d)\cos\alpha_1 + (y_w - y_d)\cos\beta_1 + (z_w - z_d)\cos\gamma_1}{(x_w - x_d)\cos\alpha_3 + (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3} \tag{3}$$

$$Y_2 = f_2 \ \frac{(x_w - x_d)\cos\alpha_2 + (y_w - y_d)\cos\beta_2 + (z_w - z_d)\cos\gamma_2}{(x_w - x_d)\cos\alpha_3 + (y_w - y_d)\cos\beta_3 + (z_w - z_d)\cos\gamma_3} \tag{4}$$

Equations (1), (3) and (4) are the collinearity equations for a pair of arbitrary perspective views.[7] The process of reconstruction of a line in 3-D space involves the formulation of a set of inverse perspective equations using these collinearity equations. This will give the direction cosines, namely l, m, n, of the line in 3-D space as well as the coordinates (x_w, y_w, z_w) of the point W on the line. This method is discussed in detail in the next section.

Definition: Epipolar line
The projection of the line joining the image point $P_1(X_1, Y_1)$ and the center of projection O of the first image plane I_1 on to the second image plane is called the *epipolar line*. For a detailed study one can refer[2] and.[4]

4. EPIPOLAR LINE BASED METHODOLOGY OF RECONSTRUCTION

In order to obtain a line in 3-D space from 2-D perspective projections it is necessary to know the following input parameters.

(i) A set of pixel coordinates for the line s_1 on the first image plane I_1: $P_{1i}(X_{1i}, Y_{1i})$, $i = 1, 2, ..., N$.

(ii) A set of pixel coordinates for the corresponding line s_2 on the second image plane I_2: $P_{2i}(X_{2i}, Y_{2i})$, $i = 1, 2, ..., M$, where $N \neq M$ in general.

(iii) The parameters of the imaging setup and perspective geometry: $f_1,\ f_2,\ x_d,\ y_d,\ z_d$ and $(\alpha_i,\ \beta_i,\ \gamma_i),\ i = 1, 2, 3$.

In both the image planes use of linear least square regression is made to fit a straight line on the set of pixels which form the line. The problem is to find the parameters of a line in 3-D space from the above input parameters.

4.1. Finding correspondence for P_{1i}

Using the equation of the epipolar line as given in the articles,[2 4] with respect to O' the corresponding points $P_{2i},\ i = 1, 2, ...N$ in the second image plane I_2 can be found for each point $P_{1i},\ i = 1, 2, ...N$ in the first image plane. Once the correspondences are known, the discrete points in 3-D space are found by solving the collinearity equations. Now there are N points in the 3-D space. Let these be $w_i(x_i, y_i, z_i)$, $i = 1, 2, ...N$. Use of least square regression can be made to find the parameters of a line in 3-D space. There are two methods to solve this 3-D regression problem, which are described here.

4.2. 3-D Regression- Method I

From the set of N 3-D points $P_i(x_i, y_i, z_i),\ i = 1, 2, ..., N$ obtained in space, the centroid of these points are first computed. Let it be $G(x_p, y_p, z_p)$. Fitting of a 3D line in 3D using least square method for $(x_i, y_i, z_i),\ i = 1, ..., N$ points can be done as follows, see figure 2. The vector $\overrightarrow{P_iQ_i}$

$$P_i\vec{Q}_i = \begin{vmatrix} \hat{e_1} & \hat{e_2} & \hat{e_3} \\ l & m & n \\ x_i - x_p & y_i - y_p & z_i - z_p \end{vmatrix}$$

Any line in 3D space through the point $G(x_p, y_p, z_p)$(centroid) having their direction cosines as l, m, n is given by

$$\frac{x - x_p}{l} = \frac{y - y_p}{m} = \frac{z - z_p}{n}, \quad l^2 + m^2 + n^2 = 1$$

The perpendicular distance d_i from the point $P_i(x_i, y_i, z_i)$ to a point Q_i on the above line is given by $d_i^2 = P_iQ_i{}^2 = |P_iQ_i|^2, i = 1, 2, ..., N$. Let

$$D = \sum_{i=1}^{N} (d_i)^2 \tag{5}$$

Using the vector form of $P_i\vec{Q}_i$,

$$\begin{aligned}(P_iQ_i)^2 &= [m(z_i - z_p) - n(y_i - y_p)]^2 + [n(x_i - x_p) - l(z_i - z_p)]^2 \\ &\quad + [l(y_i - y_p) - m(x_i - x_p)]^2\end{aligned}$$

Expressing $n^2 = 1 - l^2 - m^2$ yields

$$\frac{\partial n}{\partial l} = \frac{-l}{n}, \quad \frac{\partial n}{\partial m} = \frac{-m}{n}$$

The criteria for extremum of D is given as

$$\frac{\partial D}{\partial l} = 0 \quad \text{and} \quad \frac{\partial D}{\partial m} = 0 \tag{6}$$

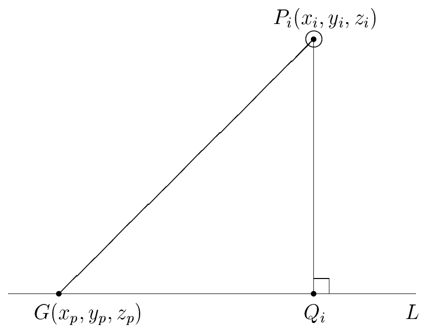

Figure 2: Regression of a 3-D line L.

Using (6) in (5) results in the following two non-linear equations

$$a_{11}l^2 - a_{11}n^2 + a_{12}lm + a_{13}mn + a_{14}nl = 0 \tag{7}$$

$$a_{12}m^2 - a_{12}n^2 + a_{11}lm + a_{15}mn + a_{13}nl = 0 \tag{8}$$

where

$$a_{11} = \sum_{i=1}^{N} x_i z_i - z_p \sum_{i=1}^{N} x_i - x_p \sum_{i=1}^{N} z_i + N x_p z_p$$
$$a_{12} = \sum_{i=1}^{N} y_i z_i - z_p \sum_{i=1}^{N} y_i - y_p \sum_{i=1}^{N} z_i + N y_p z_p$$
$$a_{13} = x_p \sum_{i=1}^{N} y_i + y_p \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i - N x_p y_p$$
$$a_{14} = 2x_p \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} {x_i}^2 - N x_p^2 + \sum_{i=1}^{N} {z_i}^2 - 2z_p \sum_{i=1}^{N} z_i + N z_p^2$$

and

$$a_{15} = 2y_p \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} y_i^2 - N y_p^2 - 2z_p \sum_{i=1}^{N} z_i + \sum_{i=1}^{N} z_i^2 + N z_p^2 \tag{9}$$

Squaring (7) and (8) and using the relation $l^2 + m^2 + n^2 = 1$, results in the following two non-linear equations to be solved.

$$\begin{aligned}
&(4a_{11}^2 + a_{14}^2)l^4 + (a_{11}^2 + a_{13}^2)m^4 + (4a_{11}a_{12} + 2a_{13}a_{14})l^3 m + (2a_{11}a_{12} + 2a_{13}a_{14})lm^3 \\
&+(4a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{14}^2)l^2 m^2 + (-4a_{11}^2 - a_{14}^2)l^2 + (-2a_{11}^2 - a_{13}^2)m^2 \\
&+(-2a_{11}a_{12} - 2a_{13}a_{14})lm + a_{11}^2 = 0 \qquad (10) \\
&(a_{12}^2 + a_{13}^2)l^4 + (4a_{12}^2 + a_{15}^2)m^4 + (2a_{11}a_{12} + 2a_{13}a_{15})l^3 m + (4a_{11}a_{12} + 2a_{13}a_{15})lm^3 \\
&+(4a_{12}^2 + a_{11}^2 + a_{15}^2 + a_{13}^2)l^2 m^2 + (-2a_{12}^2 - a_{13}^2)l^2 + (-4a_{12}^2 - a_{15}^2)m^2 \\
&+(-2a_{11}a_{12} - 2a_{13}a_{15})lm + a_{12}^2 = 0 \qquad (11)
\end{aligned}$$

From the above two non-linear equations l and m can be solved by any standard numerical method by choosing appropriate initial values l_0 and m_0. The $(n+1)^{th}$ iteration is given as

$$\begin{bmatrix} l_{n+1} \\ m_{n+1} \end{bmatrix} = \begin{bmatrix} l_n \\ m_n \end{bmatrix} - J^{-1}(l_n, m_n) \begin{bmatrix} f(l_n, m_n) \\ g(l_n, m_n) \end{bmatrix} \tag{12}$$

where

$$J = \begin{bmatrix} \dfrac{\partial f(l_n, m_n)}{\partial l} & \dfrac{\partial f(l_n, m_n)}{\partial m} \\ \dfrac{\partial g(l_n, m_n)}{\partial l} & \dfrac{\partial g(l_n, m_n)}{\partial m} \end{bmatrix} \tag{13}$$

Using the numerical values of l and m obtained, the numerical value of n can be found using the relation $l^2 + m^2 + n^2 = 1$. Thus the line is reconstructed in 3-D space. The parameters of this reconstructed line are (x_p, y_p, z_p, l, m, n).

4.3. 3-D Regression (Method- II)

(1) From the set of N points in 3-D in space the centroid $G(x_p, y_p, z_p)$ of the points are first computed.
(2) Shift the origin to this centroid.
(3) Obtain

$$D = \sum_{i=1}^{N} d_i^2 = \sum_{i=1}^{N} (Ax_i + By_i + z_i)^2 / (A^2 + B^2 + 1), \tag{14}$$

where, the required line, which is the least square approximation to the 3-D points, is assumed to be normal to the plane $Ax + By + z = 0$ (the plane is considered to be passing through the new origin G which is the centroid). Minimizing the distance of a point from this line is equivalent to maximizing the distance of a point from the plane $Ax + By + z = 0$.
(4) Using the following two criteria for minimizing D which are $\dfrac{\partial D}{\partial A} = 0$ and $\dfrac{\partial D}{\partial B} = 0$, reduces to

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sum x_i^2 - \lambda & \sum x_i y_i \\ \sum x_i y_i & \sum x_i^2 - \lambda \end{bmatrix} \begin{bmatrix} -\sum x_i z_i \\ -\sum x_i z_i \end{bmatrix} \tag{15}$$

where summation of i is taken from 1 to N, λ being the largest Eigenvalue of the matrix:

$$\begin{bmatrix} \sum x_i^2 - \sum d^2 & \sum x_i y_i & \sum x_i z_i \\ \sum x_i y_i & \sum y_i^2 - \sum d^2 & \sum y_i z_i \\ \sum x_i z_i & \sum y_i z_i & \sum y_i^2 - \sum d^2 \end{bmatrix} \tag{16}$$

Using standard numerical methods, the largest eigen value as well as the values of A and B can be found. Thus the equation of the required plane can be found. Then the equation of the line which is normal to this plane $Ax + By + z = 0$ can be formed, which gives the reconstructed 3-D line. Simulation studies and error analysis were carried out for this methodology. The results obtained are shown in figures from 3 to 10.

5. ERRORS IN THE RECONSTRUCTION PROCESS

The effect of noise is simulated by adding noise to the pixel coordinate values of the projection of the line on the image plane. It is also assumed that this noise has a Gaussian distribution, characterised by its variance σ with its range in [0 - 10]. The discrete set of points as projections of the line on the pair of image planes are obtained using Bresenham's algorithm Foley *et al.*,.[5] The following pair of criteria for estimating the errors in reconstruction have been used:

(i) Error $\theta_\mathbf{e}$ in orientation (angle between the original and the reconstructed lines):

$$\theta_\mathbf{e} = \sin^{-1} \sqrt{(m_1 n_2 - n_1 m_2)^2 + (n_1 l_2 - l_1 n_2)^2 + (l_1 m_2 - m_1 l_2)^2},$$

where (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of the original and the reconstructed lines respectively. If the value of $\theta_{\mathbf{e}}$ is not small enough we use

$$\theta_{\mathbf{e}} = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2),$$

to minimize errors in the computation of the inverse trigonometric functions in a digital computer.

(ii) Error $\mathbf{D_e}$ in position (Shortest Distance between the two lines):

$$\mathbf{D_e} = (x_{w1} - x_{w2})\ l'' + (y_{w1} - y_{w2})\ m'' + (z_{w1} - z_{w2})\ n''$$

$$(l'', m'', n'') = \left(\frac{(m_1 n_2 - m_2 n_1)}{\mu},\quad \frac{(n_1 l_2 - n_2 l_1)}{\mu},\quad \frac{(l_1 m_2 - l_2 m_1)}{\mu} \right)$$

$$\mu = \sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - m_1 l_2)^2}$$

where (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of the original and the reconstructed lines passing through points (x_{w1}, y_{w1}, z_{w1}) and (x_{w2}, y_{w2}, z_{w2}) respectively.

These errors are estimated using simulation studies for different combinations of the geometry of the imaging setup, parameters of the line and levels of noise added to the image feature (line). Results of performance studies, shown in figures 3-10, are obtained by taking the mean of 100 different observations of simulated experiments conducted using the parameters as specified in each corresponding figure. Each of the 3-D plots in figures 3-10, illustrate that errors vary non-linearly with respect (i) to the level of noise in the image planes, (ii) parameters of the imaging setup and (iii) the parameters of the reconstructed line. Proper visualization of such non-linear multivariate error functions, are provided by varying only one of the parameters of the imaging setup or line, keeping all other parameters constant (unless some of them are correlated with the one that is being varied).

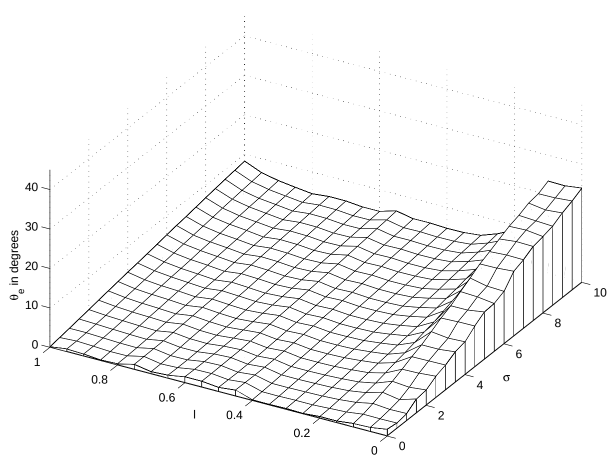

Figure 3. 3-D Plot showing the errors in reconstruction, l varies from $[0-1]$, $m = n = \sqrt{\frac{(1-l^2)}{2}}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

Figures 3, 4, 5 and 9, illustrate the effect of the direction cosines of the line on the error in reconstruction. For the graphs in figures 3, 4, 5 and 9, the values for the various parameters of the imaging setup are chosen as : $x_w =\ y_w = 10.0,\ z_w = 200.0,\ \ x_d = 10.0,\ y_d = 20.0,\ z_d = 30.0,\ \ f_1 = f_2 = 1.0,\ N = 320$ (N is the resolution of the digital image), $\alpha_1 =\ \beta_2 = \frac{2\,\pi}{3}$ and $\gamma_3 = \frac{\pi}{6}$ (given α_1, β_2, and γ_3, the other six Eulerian angles Goldstein[6] are found using the constraint of the orthogonal matrix). Only one component of the direction cosines of the line is altered, keeping the other two identical. For example, in figure 3 as l is varied from 0 to 1 in steps of 0.05, the values of m and n are obtained as, $m = n = \sqrt{\frac{(1-l^2)}{2}}$. It is observed from figures 3, 4, 5 and 9 that the errors are generally more in reconstruction when the values of the direction cosines are near the extreme limits of the range [0 to 1]. Error in orientation of the line, $\theta_{\mathbf{e}}$, is usually more than that of the position, $\mathbf{D_e}$,

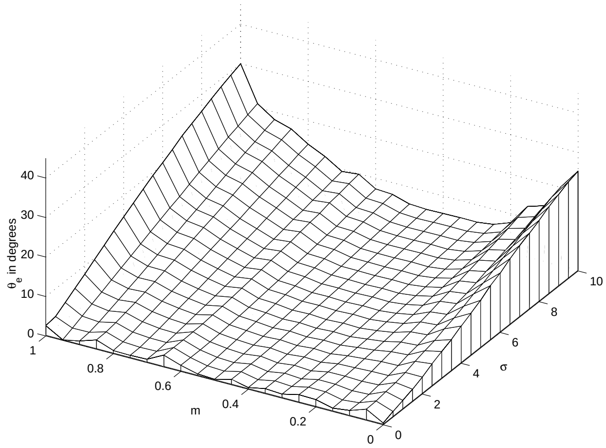

Figure 4. 3-D Plot showing the errors in reconstruction, m varies from $[0-1]$, $l = n = \sqrt{\frac{(1-m^2)}{2}}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

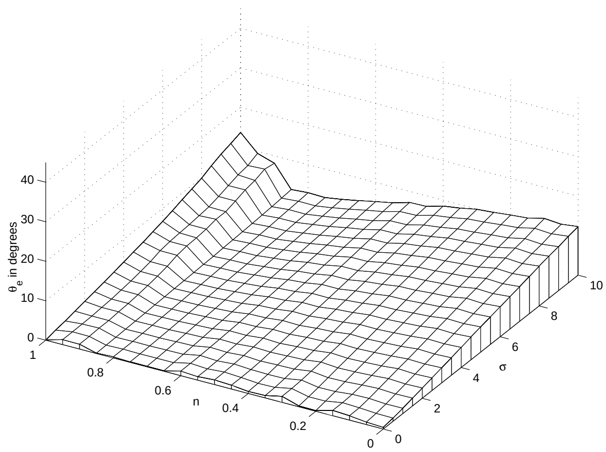

Figure 5. 3-D Plot showing the errors in reconstruction, n varies from $[0-1]$, $l = m = \sqrt{\frac{(1-n^2)}{2}}$, σ varies from $[0-10]$, $\alpha_1 = \beta_2 = \frac{2\pi}{3}$, $\gamma_3 = \frac{\pi}{6}$, $f_1 = f_2 = 1.0$, $x_d = 10.0, y_d = 20.0, z_d = 30.0$, $x_w = y_w = 10.0, z_w = 200.0$ and $N = 320$.

as shown in figures 7 and 8 respectively. For the plots in figures 7 and 8, the value of z_w is varied from 100 to 500 and $l = m = n = \frac{1}{\sqrt{3}}$. In figures 7 and 8, the parameters of the viewing geometry, α_i, β_i and γ_i, $i = 1, 2, 3$, are changed simultaneously in such a manner that the 3-D line lies within the common field of view of both the cameras.

In figure 6, the value of N (image resolution) is varied from 30 to 350 and the direction cosines are $l = m = n = \frac{1}{\sqrt{3}}$. Figure 6 shows that errors are appreciably high when the resolution of the image is low (i.e., $N \simeq 30$). Errors are negligible when the image resolution is more than 200 (typical value, as observed using our simulation studies). Figure 7 shows that error in orientation is very high when the value of depth z_w is greater than 250 and negligible when less than 200. Figure 8 shows that the error in position is high when the depth z_w, is large (350). In Figure 9, as l is varied from 0 to 1 in steps 0.05, and $m = n = \sqrt{\frac{(1-l^2)}{2}}$, it is observed that error $\mathbf{D_e}$ is very small. In Figure 10, as N is varied from 30 to 350, and $n = m = l = \frac{1}{\sqrt{3}}$, it is observed that error $\mathbf{D_e}$ is very small except for small values of $N (\leq 50)$. All 3-D graphs in figures 3-10, show that for noise levels in the range $0 \leq \sigma \leq 10$, the errors $\theta_\mathbf{e}$ and $\mathbf{D_e}$ are mostly within acceptable limits ($0^o \leq \theta_\mathbf{e} \leq 10^o$ and $0 \leq \mathbf{D_e} \leq 2$ respectively), except for certain specific conditions of the viewing geometry, orientation and position of the line. The errors are negligible for small levels of noise in the range $0 \leq \sigma \leq 2$, which is realistic. Negligible error in

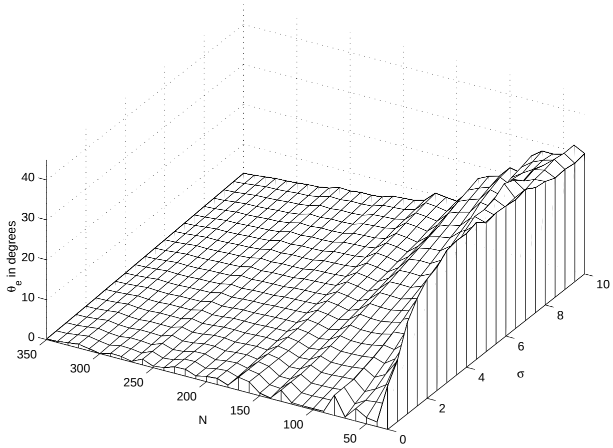

Figure 6. 3-D Plot showing the errors in reconstruction, N varies from $[30-350]$, $l=m=n=\frac{1}{\sqrt{(3)}}$, σ varies from $[0-10]$, $\alpha_1=\beta_2=\frac{2\pi}{3}$, $\gamma_3=\frac{\pi}{6}$, $f_1=f_2=1.0$, $x_d=10.0, y_d=20.0, z_d=30.0$, $x_w=y_w=10.0$ and $z_w=200.0$.

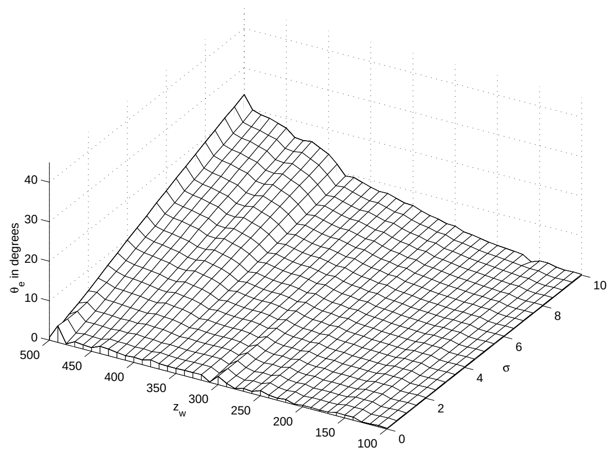

Figure 7. 3-D Plot showing the errors in reconstruction, z_w varies from $[100-500]$, $l=m=n=\frac{1}{\sqrt{3}}$, σ varies from $[0-10]$, $\alpha_1=\beta_2=\frac{2\pi}{3}$, $\gamma_3=\frac{\pi}{6}$, $f_1=f_2=1.0$, $x_d=10.0, y_d=20.0, z_d=30.0$, $x_w=y_w=10.0$ and $N=640$.

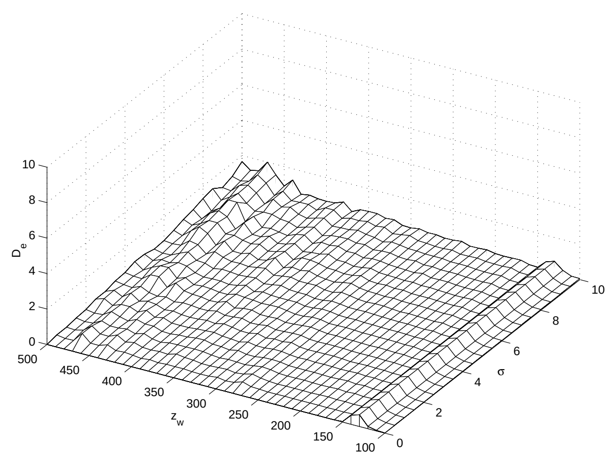

Figure 8. 3-D Plot showing the errors in reconstruction, z_w varies from $[100-500]$, $l=m=n=\frac{1}{\sqrt{3}}$, σ varies from $[0-10]$, $\alpha_1=\beta_2=\frac{2\pi}{3}$, $\gamma_3=\frac{\pi}{6}$, $f_1=f_2=1.0$, $x_d=10.0, y_d=20.0, z_d=30.0$, $x_w=y_w=10.0$ and $N=320$.

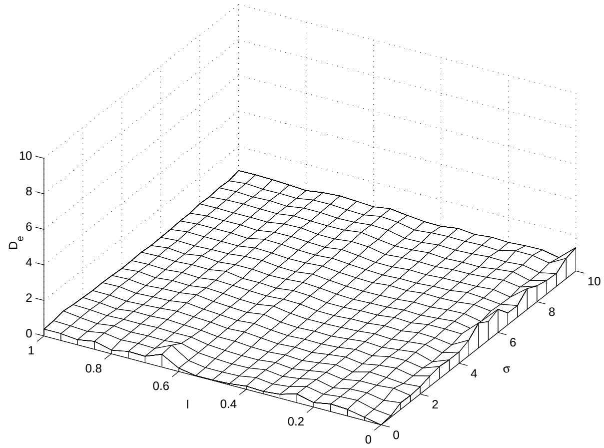

Figure 9. 3-D Plot showing the errors in reconstruction, l varies from $[0-1]$, $m=n=\sqrt{\frac{(1-l^2)}{2}}$, σ varies from $[0-10]$, $\alpha_1=\beta_2=\frac{2\pi}{3}$, $\gamma_3=\frac{\pi}{6}$, $f_1=f_2=1.0$, $x_d=10.0, y_d=20.0, z_d=30.0$, $x_w=y_w=10.0, z_w=200.0$ and $N=320$.

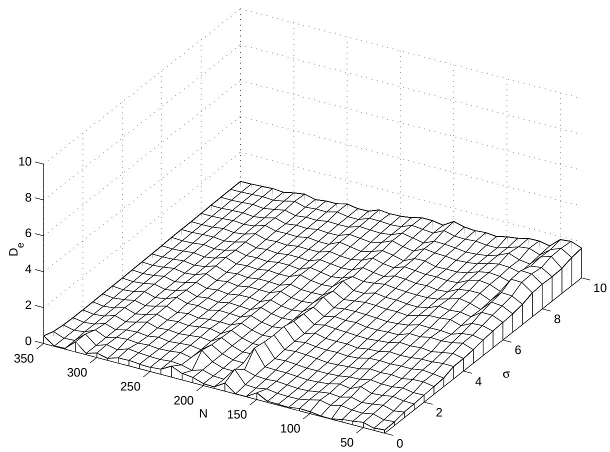

Figure 10. 3-D Plot showing the errors in reconstruction, N varies from $[30-350]$, $l=m=n=\frac{1}{\sqrt{(3)}}$, σ varies from $[0-10]$, $\alpha_1=\beta_2=\frac{2\pi}{3}$, $\gamma_3=\frac{\pi}{6}$, $f_1=f_2=1.0$, $x_d=10.0, y_d=20.0, z_d=30.0$, $x_w=y_w=10.0$ and $z_w=200.0$.

the process of reconstruction of a line in the noise free case ($\sigma=0$), is the result of digitization (sampling) of spatial coordinate values in the digital image plane. This is more if the image has low resolution as illustrated in figure 6. As the line tends to be parallel to one of the principle coordinate axis ($l, m, n \simeq 0$ or 1), the errors in reconstruction are large. With increase in the level of noise in the image planes the error in orientation, $\theta_{\mathbf{e}}$, increases rapidly than error in position, $\mathbf{D_e}$, of the line. Hence based on our studies, for best results we recommend the following range of values of the parameters of the imaging setup and line to be reconstructed, $0.2 \leq l \leq 0.9$, $0.2 \leq m \leq 0.7$, $0.2 \leq n \leq 0.8$, $z_w < 250$ and $N > 200$. Other parameters of the line and viewing geometry do not affect the accuracy of the reconstruction process to a larger extent. The dimensions of all distances used in the simulation studies are normalized with respect to the focal length of the cameras which is considered to be unity.

6. CONCLUSIONS

A software program for this method was developed in "C" language in Windows environment to verify the results obtained using the expressions derived in this paper. A rigorous performance analysis has been provided, using simulation studies, to illustrate the effect of noise, parameters of the line and the imaging setup on errors in reconstruction of a line. Results of simulation studies presented in this paper are useful for the design of an imaging system for accurate reconstruction of lines or edges of 3-D objects, as well as to evaluate the performance of such a system. Smaller resolution of the image, larger depth and certain orientations of the line in 3D have been found to produce a poor performance in the process of reconstruction. It is observed that for larger levels of noise present in the image planes, the errors in the orientation parameters of the reconstructed line are much larger than that in the position parameters.

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